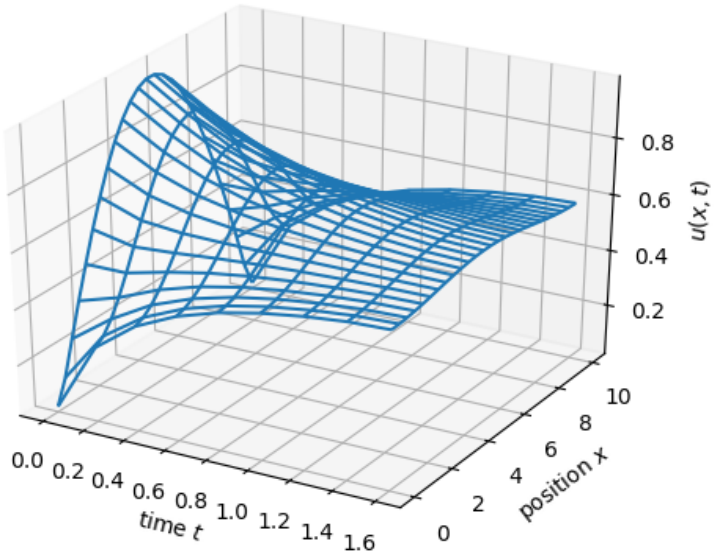
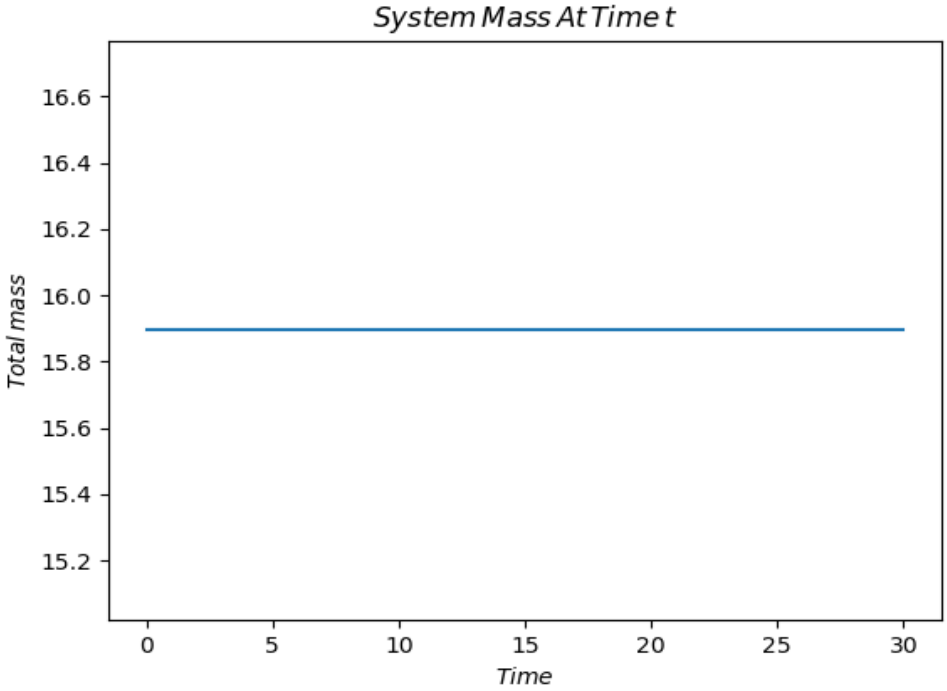
Andrew Durkiewicz Homework 11

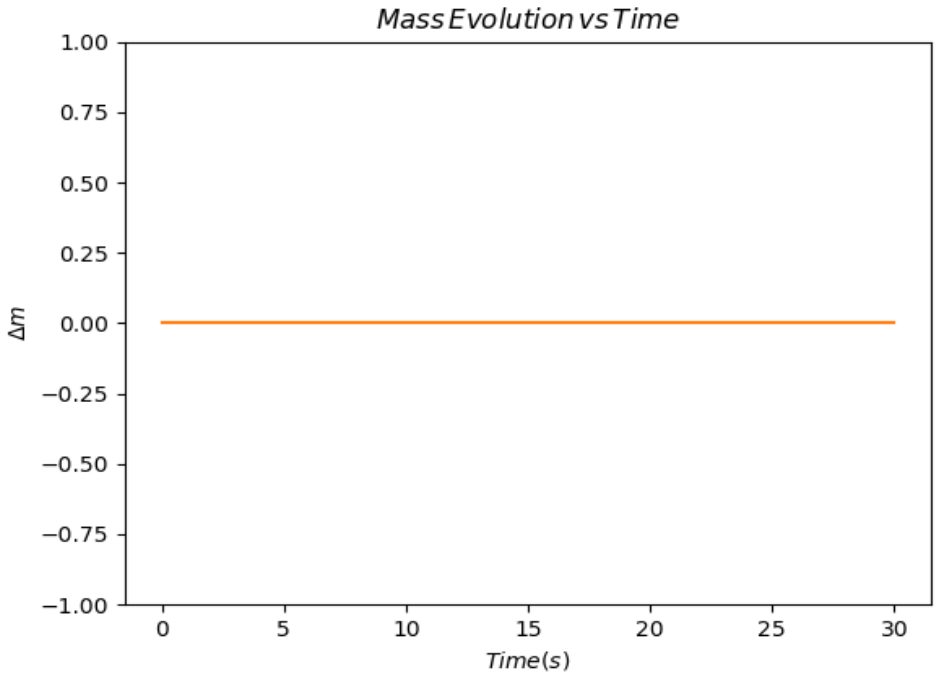
Question 1
Part 1)
Below is a 3D graph of my solution for U(x,t):



Part 2)
The mass is the total sum of all the U(x_n,t) values over the range of all n's.
Here is my plot of the total mass for each time t:



To show that the mass isn't changing, I took the difference of the i'th t value for the mass M(x,t_i) and took plotted its variance with the initial mass. So, as can be seen by the plot, there is no variance with the mass in time:

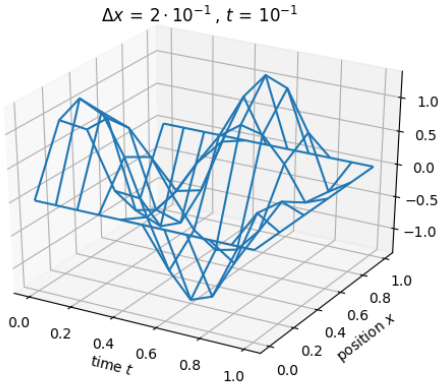
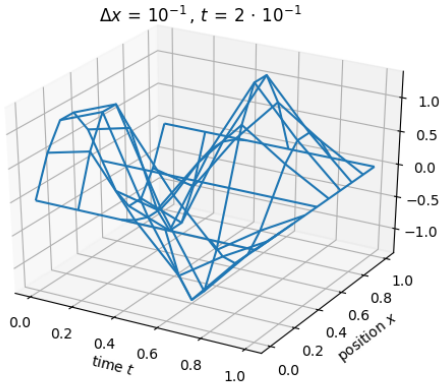
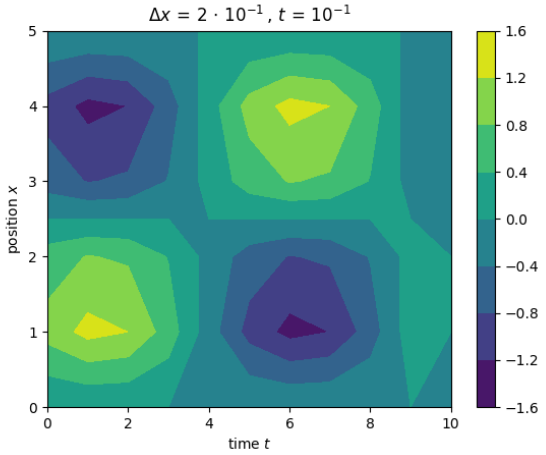
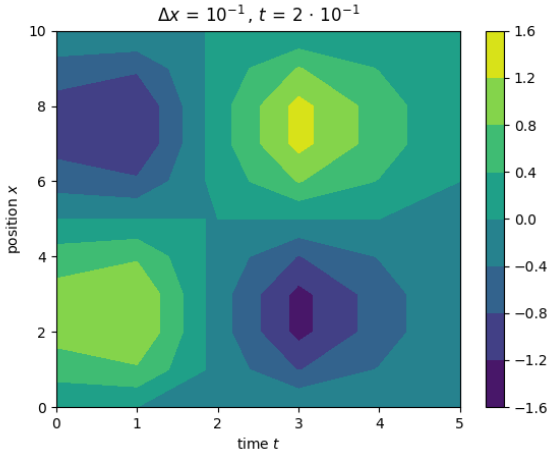


CODE for question 1:

```
#!/usr/bin/env python3

import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
a = .5**2
l = 10
T = 30
dx = .4
dt = .2
ld = a/dx**2*dt #lambda
Nx = int(1+round(10/dx))
Nt = 1 + round(T/dt)
x_range = np.linspace(0,l,Nx)
t_range = np.linspace(0,T,Nt)


def u0(x):
    #initial conditions for when t = 0
    return np.sin(np.pi * x / l)

#initialize u matrix:
u = u0(x_range)

#U_{n+1} = A.u_n
```

```
A = np.zeros((Nx,Nx))
#using the matrix form of our equation:
i = np.arange(Nx)
j = np.arange(Nx-1)

A[i,i] = 1-2*ld
A[j,j+1] = ld
A[j+1,j] = ld
A[0,-1] = ld
A[-1,0] = ld

step = 20  # Plot every 10 time steps
plot_data = np.zeros((Nt//step+2, Nx))
plt_i = 0
masses = np.zeros(Nt)


for t_i in range(Nt):
    u = A.dot(u)
    masses[t_i] = np.sum(u)  # Probably off by a scalar factor
    if t_i % step == 0 or t_i == Nt-1:
        plot_data[plt_i, :] = u
        plt_i += 1
graph = 0
while graph < 3:
    if graph == 0:
        # Setup plot
        plt.plot(t_range, masses)
        plt.title(r'$System\/Mass\/At\/Time\/t$')
        plt.xlabel(r'$Time$')
        plt.ylabel(r'$Total\/mass$')
        plt.savefig('mass_at_time_t.png')
    if graph == 2:
        X, Y = np.meshgrid(range(plot_data.shape[0]), range(plot_data.shape[1]))
        Z = plot_data[X, Y]
        fig = plt.figure()
        ax = fig.add_subplot(111, projection="3d")
        ax.plot_wireframe(X*dt, Y*dx, Z)
        ax.set_xlabel(r"time $t$")
        ax.set_ylabel(r"position $x$")
        ax.set_zlabel(r'$u(x,t)$')


        plt.savefig("3d.png")
    #lets find how much the mass deviates from the initial mass:
    if graph == 1:
        def mass_change(m0,m):
            return abs(m-m0)
        change_mass = np.array([])
        for i in range(len(masses)):
            change_mass=np.append(change_mass,(mass_change(masses[0],masses[i])))
        plt.plot(t_range,change_mass)
        plt.ylim(-1,1)
        plt.xlabel(r'$Time(s)$')
        plt.ylabel(r'$\Delta m$')
        plt.title(r'$Mass\/Evolution\/vs\/Time$')
        plt.savefig('Mass_Evolution_vs_time.png')
    graph+=1
```

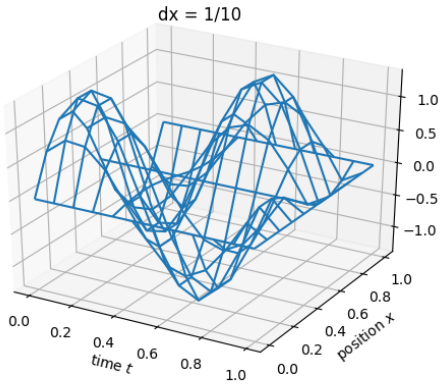
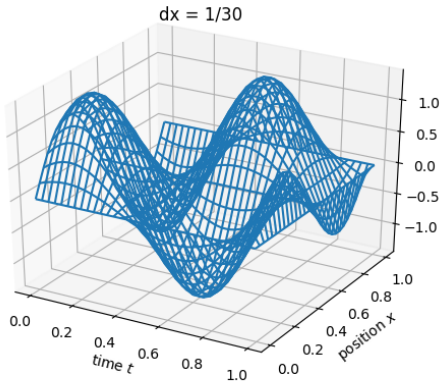
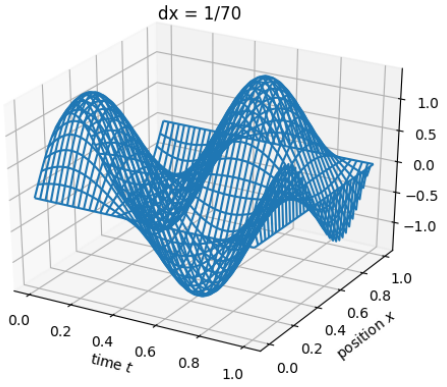
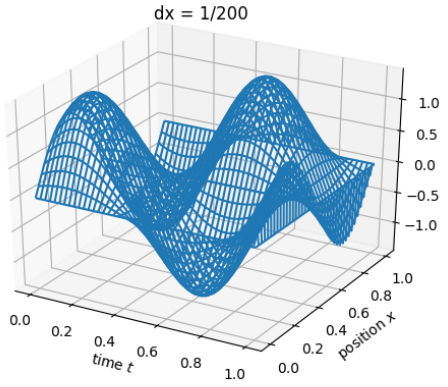
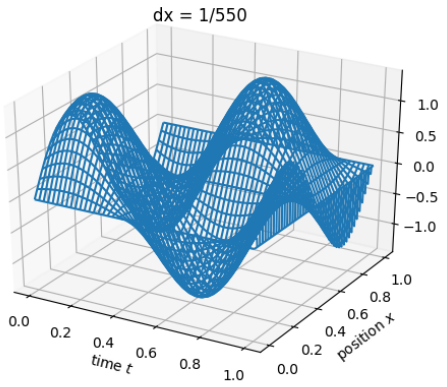
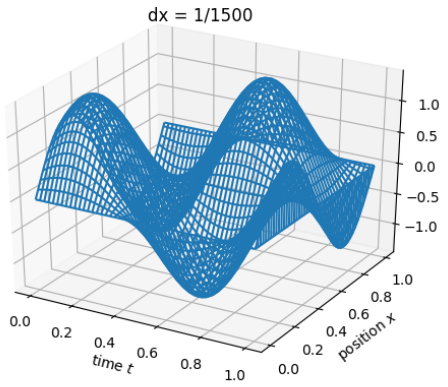
Question 2)
I solved the PDE using a 3D wireframe and used a contour:

Δx = 0.2 , Δt = 0.1	Δx = 0.1 , Δt = 0.2
	
	

The main difference that I notice is the way that peaks are occuring in U(x,t). For the δx = 0.2 and δt = 0.1, the peaks are much more smooth and less defined as the sharp peaks for δt = 0.2 and δx = 0.1.

Part C Below:

Part C)

Δx = 1/10	Δx = 1/30
	
Δx = 1/70	**Δx = 1/200**
	
Δx = 1/550	**Δx = 1/1500**
	

The accuracy order for each δx printed as such:
Order Error:
For dx = 1/10, the resulting accuracy is: 0.879133761846
For dx = 1/30, the resulting accuracy is: 1.11005002514
For dx = 1/70, the resulting accuracy is: 1.00479510211
For dx = 1/200, the resulting accuracy is: 1.00946343235
For dx = 1/550, the resulting accuracy is: 1.05517391856

CODE for problem2 parts a-b:

```
#!/usr/bin/env python3

import numpy as np
from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import matplotlib.cm as cm
import matplotlib.mlab as mlab

error = []
    ## Initial conditions
T = 1
x0, x1 = 0, 1

dt = .2
dx = .1

c = 1

def u_t0(x):
    # Initial positions
    return np.sin(2 * np.pi * x)
def v_t0(x):
    # Initial velocities
    return 2 * np.pi * np.sin(2 * np.pi * x)

## Setup Parameters
intX = int(1 + round((x1-x0) / dx))
intT = 1 + round(T / dt)
x_range = np.linspace(x0, x1, intX)
t_range = np.linspace(0, T, intT)

#use our t values to make our u matrix
u_prev = u_t0(x_range)  # t_i=0
u_mat = u_prev + dt * v_t0(x_range)  # t_i=1

# u_{n+1} = A (dot) u_{n}
A = np.zeros((intX, intX))
range1 = np.arange(intX)
range2 = np.arange(intX-1)
ld = c**2 * dt**2 / dx**2
A[range1, range1] = 2 - 2*ld
A[range2, range2+1] = ld
A[range2+1, range2] = ld
A[0, -1] = ld  # Coefficients for periodic boundary condition
A[-1, 0] = ld

plot_data = np.zeros((intT, intX))
plot_data[0, :] = u_prev
```

```
def solution(x, t):
    return np.sin(2*np.pi*x) * (np.cos(2*np.pi*t) + np.sin(2*np.pi*t))
plot_our_solution = np.zeros_like(plot_data)

## Solve - iterate over time
for t_i in range(intT-1):
    u_prev[:], u_mat = u_mat, A.dot(u_mat) - u_prev

    plot_data[t_i, :] = u_mat
    plot_our_solution[t_i, :] = solution(x_range, t_range[t_i])

error.append(np.max(np.abs(plot_data - plot_our_solution)))

# Setup plot
X, Y = np.meshgrid(range(plot_our_solution.shape[0]),
range(plot_our_solution.shape[1]))
Z = plot_our_solution[X, Y]
fig = plt.figure()
ax = fig.add_subplot(111, projection="3d")
ax.plot_wireframe(X*dt, Y*dx, Z)
ax.set_xlabel(r"time $t$")
ax.set_ylabel(r"position $x$")
ax.set_title(r'$\Delta x\/=\/10^{-1}\/,\/t\/=\/2 \/ \cdot \/10^{-1}$')

plt.savefig('1b_final_true.png')
plt.show()
```

CODE for part c:

```
#!/usr/bin/env python3

import numpy as np
from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

dx_range = (1/10, 1/30, 1/70, 1/200, 1/550, 1/1500)
dx_range_string = ['1/10','1/30','1/70','1/200','1/550','1/1500']
error = []
for dx in dx_range:
    ## Initial conditions
    T = 1
    x0, x1 = 0, 1

    dt = dx * .9

    c = 1

    def u_t0(x):
        # Initial positions
        return np.sin(2 * np.pi * x)
    def v_t0(x):
        # Initial velocities
        return 2 * np.pi * np.sin(2 * np.pi * x)

    ## Setup Parameters
    intX = int(1 + round((x1-x0) / dx))
    intT = 1 + round(T / dt)
```

```
    x_range = np.linspace(x0, x1, intX)
    t_range = np.linspace(0, T, intT)

    #use our t values to make our u matrix
    u_prev = u_t0(x_range)  # t_i=0
    u_mat = u_prev + dt * v_t0(x_range)  # t_i=1

    # u_{n+1} = A (dot) u_{n}
    A = np.zeros((intX, intX))
    range1 = np.arange(intX)
    range2 = np.arange(intX-1)
    ld = c**2 * dt**2 / dx**2
    A[range1, range1] = 2 - 2*ld
    A[range2, range2+1] = ld
    A[range2+1, range2] = ld
    A[0, -1] = ld  # Coefficients for periodic boundary condition
    A[-1, 0] = ld

    plot_data = np.zeros((intT, intX))
    plot_data[0, :] = u_prev

    def solution(x, t):
        return np.sin(2*np.pi*x) * (np.cos(2*np.pi*t) + np.sin(2*np.pi*t))
    plot_our_solution = np.zeros_like(plot_data)

    ## Solve - iterate over time
    for t_i in range(intT-1):
        u_prev[:], u_mat = u_mat, A.dot(u_mat) - u_prev

        plot_data[t_i, :] = u_mat
        plot_our_solution[t_i, :] = solution(x_range, t_range[t_i])

    error.append(np.max(np.abs(plot_data - plot_our_solution)))

    # Setup plot
    if dx == 1/70:
        X, Y = np.meshgrid(range(plot_our_solution.shape[0]),
range(plot_our_solution.shape[1]))
        Z = plot_our_solution[X, Y]
        fig = plt.figure()
        ax = fig.add_subplot(111, projection="3d")
        ax.plot_wireframe(X*dt, Y*dx, Z)
        ax.set_xlabel(r"time $t$")
        ax.set_ylabel(r"position $x$")

        plt.savefig('Ex-solution.png')

print('Order Error:')
for i in range(len(error)-1):
    print("For dx = %s, the resulting accuracy is:
"%dx_range_string[i],error[i+1]/error[i] / (dx_range[i+1]/dx_range[i]))
```