Minimalist Syntax and Mathematics

A practical approach to Hopf-Algebraic Minimalism in the Classroom

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Introduction and Motivations

What: Math of Linguistics at Grinnell College

- Absence of math of linguistics classes at Grinnell and comparable institutions
- Cf. integration of mathematics into Biology, Physics, Psychology.

So What: Mathematical Literacy

What ways has mathematics already found success in linguistics?

- Disciplinary level: precision of hypotheses and modeling for new hypotheses
- Subdisciplinary level:
  - Phonetics: Statistical modeling (esp. Neural Networks)
  - Typology: Clustering models and numerical analysis.
  - Syntax: Generative models, algebraic representations, etc.
  - ...and much more!

What type of literacy is still needed? What opportunities would it provide?

Now What: A Course on MCB

- A followup to a standard undergraduate syntax course, ideally after a Mathematical Linguistics course
- Critically compares two mathematical approaches to minimalist syntax: Ed Stabler's Computational Minimalism and Marcolli, Chomsky, and Berwick's Hopf-Algebraic Minimalism
- Practices new mathematical literacies concerning minimalist models; e.g. identifying/proving expected behavior from first principles
- Offers a starting point for integrating math into linguistics courses at Grinnell and elsewhere

Course Syllabus

Year Overview

Unit 1 Scientific Modeling

1. Types of Scientific Models
2. Examples of Scientific Models

Unit 2 Models of Syntax

1. Examples in Generative Linguistics
2. The Minimalist Program and its Models

Unit 3 Comparative Review

1. Chomsky's New Minimalism
2. Constituency (Merge)
3. X-Linguistic Similarities (Features)
4. Word Order (Linearization)
5. Grammaticality (Filtering)
6. Paraphrases (Possible Derivations)

Unit 1: Scientific Modeling

Takeaways:

1. A mathematical model is a type of scientific model
2. Models are dogmatic. No model is a perfect model.
3. A mathematical model can be evaluated on its expressive power, cost, and empirical soundness.
4. A mathematical model is defined by its objects and axioms.

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Unit 2: Models of Syntax

Takeaways:

1. There exist many models of syntax, particularly in generative linguistics.
2. Chomsky's Heirarchy is a method of evaluating a grammar's expressive power.
3. "Minimalism" is not one theory, but a guiding set of principles for several models of syntax.

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Unit 3: Comparative Review

Takeaways:

1. "New minimalism" is Chomsky's recent revision of the Minimalist Program.
2. Ed Stabler's "Computational Minimalism" and Marcolli, Chomsky, and Berwick's Hopf-Algebraic Minimalism are two appraoches to modelling the minimalist program.
3. Computational minimalism aligns closer to the "90s formulation" of minimalism, whereas Hopf-Algebraic minimalism aligns closer to "New minimalism."

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Optional Unit: Mathematical Prerequisites

Course equivalences:

- MAT-322 “Advanced Topics in Abstract Algebra”

Covers:

- Elementary Category Theory
- Algebras, Bialgebras, Hopf Algebras, and Universal Algebras
- Modules over Algebras

Optional Unit: Minimalist Prerequisites

This is for mathematicians who need more background on linguistics.
Course equivalences:

- LIN-216 “Syntax”
- LIN-375 “Advanced Linguistic Analysis” in Syntax
- LIN-295/LIN-395 Special Topic in Syntax

Covers:

- P&P Architecture
- Governor-Binding Theory
- 90s Minimalism

Example Unit 3

Reading Schedule for sections 3.1, 3.2

- W1 Monday: Section 7 of *Merge and the Strong Minimalist Thesis* [4, pp. 46–60]
- W1 Wednesday: "The Miracle Creed and SMT" [3]
- W1 Friday: Chapter 0 of *Mathematical Structure of Syntactic Merge* [5, pp. 1–18]
- W2 Monday: Chapter 1, Sections 1–4 of *Mathematical Structure of Syntactic Merge* [5, pp. 19–55]
- W2 Wednesday: Sections 1 and 2 of "Oxford Handbook of Linguistic Minimalism" [7, pp. 1–9]
- W2 Friday: (3.1 Essay)
- W3 Monday: Sections 1 and 3 of *Merge and the Strong Minimalist Thesis* [4, pp. 1–5, 13–27]
- W3 Wednesday: (None)
- W3 Friday: (3.2 Homework)

Example Essay Prompt from Unit 3.1

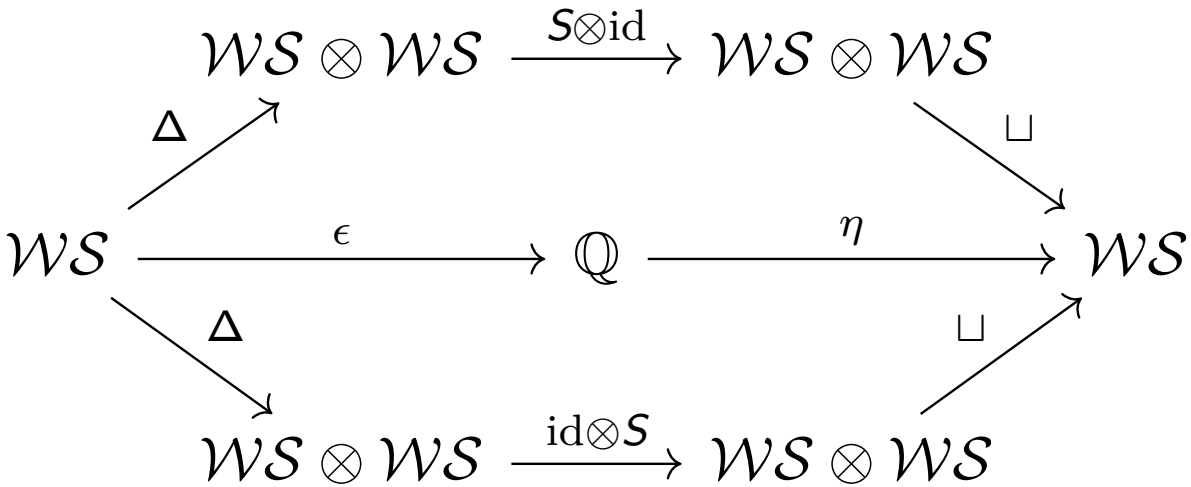
In three double-spaced pages, answer the following:

- What is the SMT? How does it relate to the miracle creed?
- Are you convinced by the concept of the miracle creed? Why or why not?
- Give one example for and one example against the existance of the miracle creed in your area of interest. E.g. in Mathematics, one may write about complex-analytic series expansions and Gödel's incompleteness theorem.

You may cite "The Miracle Creed and SMT" [3], *Merge and the Strong Minimalist Thesis* [4], or other resources should you find them useful.

Example Worksheet Questions from Unit 3.2

On the $\mathbb{Q}$-vector space of workspaces $\mathcal{WS} := \mathcal{V}(\mathcal{F}_{\mathcal{SO}_0})$, Marcolli, Chomksy, and Berwick define a product $\sqcup$, coproduct Δ, antipode S, and unit and counit ϵ, η. Prove that these operations compatibly form a Hopf Algebra. That is, show that the hexagon



commutes.

Example Lecture Notes Excerpt

Merge Constraints

- Clearly some applications of merge are ill-formed for the language.
- Two approaches:
  1) Filter now: never let an ill-formes syntactic object be formed, i.e. restrict Merge's application
  2) Filter later: let Merge apply to everything and let a future step take care of it

Note: In some senses, the first may be simpler, reducing the search tree for a valid sentence; however, the machinery required for the second is less, provided that being ill-formed in merge => being ill-formed elsewhere

- Stabler: First approach (Chomsky 1990s)
- MCB: Second approach (Externalization)

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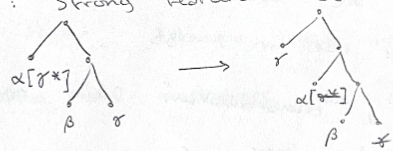
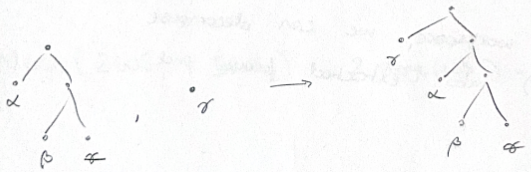
Internal Merge

- Both MCB and Stabler treat Move as a special type of Merge, called Internal Merge
  - → Stabler: Certain features trigger internal merge to be checked
  - → MCB: Merge applies to all syntactic objects "extracted" from trees and their leftovers.

Stabler: Strong feature triggers



MCB: (only combination which survives externalization and optimality)



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Future Directions

Extending this research

1. Looking at other institution's programs
2. Getting linguistics student feedback
3. Reaching out to academics in math of language community.
4. Targeting mathematicians familiar with hopf algebras who might be interested in their applications

A course for Grinnell

- Navigate the mathematical requirement
- Perhaps a more general math of language curriculum?
- Finalize syllabus into 14-week schedule

A course for the public

- GitHub repository with syllabus, this presentation, and PDF scans of lecture notes
- Please submit pull requests!!
- Everything is licensed under MIT (free to use with attribution provided)
- Let's develop mathematical literacy in linguistics as a community

It's your turn!

Figure: GitHub Repository: andrewfargo/NewMinimAlgFrm.

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