Strong first-order phase transitions in the NMSSM and methods for finding them

P. Athron, C. Balazs, A. Fowlie, G. Pozzo, G. White, and Y. Zhang, JHEP **11**, 151,
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Motivation

Why first-order phase transitions?

We know that there is an asymmetry between matter and anti-matter

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} \neq 0$$

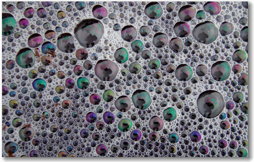
To generate this asymmetry, we must fulfil Sakharov's conditions [2] —

1. charge (C) and charge-parity (CP) violation,
2. baryon number (B) violation, and
3. departure from equilibrium

Electroweak baryogenesis with a strong EW FOPT [3–38]

Why first-order phase transitions?

Bubbles nucleate, expand and collide.



This could lead to observable gravitational wave signals.

Why first-order phase transitions?

An extended scalar sector (with a richer vacuum structure) motivated by e.g.,

- Hierarchy problem
- Dark matter
- Vacuum stability

as well as by electroweak baryogenesis.

Thus first-order phase transitions are possible, and possibly common, in many well-motivated models of new physics.

The SM is not enough

With a $125\,$GeV Higgs, the electroweak phase transition in the SM is a smooth crossover — certainly not first order.

Furthermore, the CP violating phase in the CKM matrix isn't big enough.

- ☒ charge (C) and charge-parity (CP) violation
- ☑ baryon number (B) violation
- ☒ departure from equilibrium

On the other hand, B violation provided by sphalerons.

Add more particles?

To achieve EW baryogenesis, we could add more particles

- Extra particles could alter the loop corrections to the potential
- And modify the tree-level minima
- And provide new sources of CP violation

We focus on supersymmetric models to solve the hierarchy problem at the same time.

The MSSM is not enough

- The minimal supersymmetric Standard Model (MSSM) could change the character of the EWPT by thermal corrections from stops. Light stops result in a barrier between the EW preserving and breaking phases [39, 40].
- Light stops, however, are in tension with LHC constraints.
- Thus I will present our findings from the next-to-minimal supersymmetric Standard Model (NMSSM) and our tools for finding FOPTs in arbitrary models.

Our model — NMSSM

We consider the NMSSM — the minimal supersymmetric SM extended by a complex scalar singlet. The superpotential consists of the ordinary Yukawa interactions and $\mathbb{Z}_3$ symmetric interactions involving the singlet

$$W = W_{\text{Yukawa}} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3}\kappa \hat{S}^3$$

The usual μ-term is forbidden by the $\mathbb{Z}_3$ and generated spontaneously when S acquires a VEV.

This solves the μ-problem of the MSSM by removing the scale μ.

The singlet could assist in a strong electroweak PT [10, 19, 41].

There are three contributions to the tree-level Higgs potential of the NMSSM

$$V = V_F + V_D + V_{\mathsf{soft}}$$

The F- and D-term contributions are

$$V_F = |\lambda S|^2 \left(|H_u|^2 + |H_d|^2\right) + \left|\lambda H_u \cdot H_d + \kappa S^2\right|^2$$
$$V_D = \frac{1}{8}\left(g^2 + g'^2\right)\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{1}{2} g^2 \left|H_u^\dagger H_d\right|^2$$

and the soft-breaking terms are

$$V_{\mathsf{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + \left[\lambda A_\lambda S H_u \cdot H_d + \frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}\right]$$

Matching to the THDMS

We assume that the stops are heavier than about a TeV.

The heavy stops, however, could result in large logarithms, spoiling the precision of any calculations. We resum them by matching the NMSSM to a two Higgs doublet model plus a singlet (THDMS).

1. NMSSM at $Q > M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$
2. Match to THDMS at $Q = M_{\text{SUSY}} \gtrsim 1\,\text{TeV}$
3. RG running to $Q = m_t \simeq 170\,\text{GeV}$

This is an EFT valid below M_{SUSY} with supersymmetric particles — other than the extended Higgs sector — integrated out [9, 42–44].

The tree-level potential of a $\mathbb{Z}_3$ symmetric THDMS model is

$$V = \frac{1}{2}\lambda_1|H_d|^4 + \frac{1}{2}\lambda_2|H_u|^4 + (\lambda_3 + \lambda_4)|H_u|^2|H_d|^2 - \lambda_4|H_u^\dagger H_d|^2$$
$$+ \lambda_5|S|^2|H_d|^2 + \lambda_6|S|^2|H_u|^2 + (\lambda_7 S^{*2} H_d \cdot H_u + \text{h.c.}) + \lambda_8|S|^4$$
$$+ m_1^2|H_d|^2 + m_2^2|H_u|^2 + m_3^2|S|^2 - (m_4 S H_d \cdot H_u + \text{h.c.}) - \frac{1}{3}(m_5 S^3 + \text{h.c.}),$$

where the couplings λ_7, m_4 and m_5 may be complex. Two of the three phases, however, may be removed by re-definitions of H_u, H_d and S, leaving a single complex phase, as in the NMSSM.

Higgs sector

There are 10 real degrees of freedom in the THDMS Higgs sector. After EWSB, this results in three CP even, two CP odd, and one charged Higgs boson, and three Goldstones.

The field-dependent mass eigenvalues are found by diagonalizing the tree-level mass matrices and gauge fixing terms

$$m^2_{ij} = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} + \mathcal{O}(g\xi\phi^2)$$

The physical Higgs masses and reduced couplings to photons and gluons are found at one-loop. The observed 125 GeV Higgs could (a priori) be any of the CP even scalars.

- We assume that the vacuum never breaks charge or CP and thus may be written at all temperatures

$$\langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h_u \end{pmatrix}, \qquad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} h_d \\ 0 \end{pmatrix}, \qquad \langle S \rangle = \frac{1}{\sqrt{2}} s,$$

  where $h_{u,\,d}$ and s are real.
- As we assume that the VEVs are CP conserving, a tadpole condition forces CP violating phases in the potential to vanish. We focus on the PT and do not investigate the source of CP violation.
- Our problem contains 3 (rather than 10) dimensions. This is tractable.

We match the NMSSM to a THDMS at the scale $M_{\mathsf{SUSY}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ by the matching conditions

$$\lambda_1 = \frac{1}{4}\left(g'^2 + g^2\right), \quad \lambda_2 = \frac{1}{4}\left(g'^2 + g^2\right) + \Delta\lambda_2, \quad \lambda_3 = \frac{1}{4}\left(g^2 - g'^2\right),$$
$$\lambda_4 = \frac{1}{2}\left(2|\lambda|^2 - g^2\right), \quad \lambda_5 = \lambda_6 = |\lambda|^2, \quad \lambda_7 = -\lambda\kappa^*, \quad \lambda_8 = |\kappa|^2,$$
$$m_1^2 = m_{H_d}^2, \quad m_2^2 = m_{H_u}^2, \quad m_3^2 = m_S^2, \quad m_4 = A_\lambda \lambda, \quad m_5 = -A_\kappa \kappa$$

We furthermore included a dominant one-loop threshold correction to the matching for λ_2,

$$\Delta\lambda_2 = \frac{3 y_t^4 A_t^2}{8\pi^2 M_{\mathsf{SUSY}}^2}\left(1 - \frac{A_t^2}{12 M_{\mathsf{SUSY}}^2}\right)$$

One-loop corrections

We include the one-loop Coleman-Weinberg corrections

$$\begin{aligned}\Delta V = \frac{1}{64\pi^2}\Bigg(&\sum_h n_h m_h^4(\xi)\left[\ln\left(\frac{m_h^2(\xi)}{Q^2}\right)-3/2\right]\\&+\sum_V n_V m_V^4\left[\ln\left(\frac{m_V^2}{Q^2}\right)-5/6\right]\\&-\sum_V \tfrac{1}{3}n_V(\xi m_V^2)^2\left[\ln\left(\frac{\xi m_V^2}{Q^2}\right)-3/2\right]\\&-\sum_f n_f m_f^4\left[\ln\left(\frac{m_f^2}{Q^2}\right)-3/2\right]\Bigg).\end{aligned}$$

we work in $\overline{\text{MS}}$ in the $\xi = 1$ gauge. The masses appearing are the field-dependent tree-level mass eigenvalues.

We include the Higgs bosons (including Goldstones), the gauge bosons and the top, bottom and tau fermions.

One-loop finite temperature corrections

We furthermore include one-loop finite-temperature corrections

$$\Delta V_T = \frac{T^4}{2\pi^2}\Bigg[\sum_h n_h J_B\left(\frac{m_h^2(\xi)}{T^2}\right) + \sum_V n_V J_B\left(\frac{m_V^2}{T^2}\right) - \sum_V \frac{1}{3} n_V J_B\left(\frac{\xi m_V^2}{T^2}\right) + \sum_f n_f J_F\left(\frac{m_f^2}{T^2}\right)\Bigg]$$

where J_B and J_F are the thermal functions.

Daisy corrections

Daisy corrections are an all-order resummation of dangerous T^2/m^2 terms that could otherwise spoil calculations at high temperature.

We include them by the Arnold-Espinosa method [45]

$$V_{\text{daisy}} = -\frac{T}{12\pi}\left(\sum_h n_h\left[\left(\bar{m}_h^2\right)^{\frac{3}{2}} - \left(m_h^2\right)^{\frac{3}{2}}\right] + \sum_V \tfrac{1}{3} n_V\left[\left(\bar{m}_V^2\right)^{\frac{3}{2}} - \left(m_V^2\right)^{\frac{3}{2}}\right]\right)$$

The barred field-dependent mass eigenvalues are the eigenvalues of the mass matrices that include additional thermal corrections

$$m_{ij}^2 \to m_{ij}^2 + c_{ij} T^2$$

where c is a model-dependent Debye coefficient.

This is our recipe!

Effective potential = match + run + one-loop + daisy corrections by Arnold-Espinosa method.

Please let's discuss refinements and missing ingredients!

Gauge invariance

The effective potential contains explicit gauge dependence (ξ). By the Nielsen identities [46]

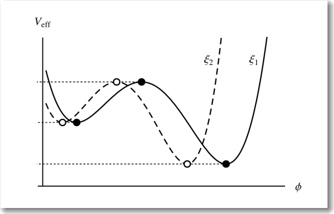
$$\frac{\partial V_{\text{eff}}(h,\xi)}{\partial \xi} \propto \frac{\partial V_{\text{eff}}(h,\xi)}{\partial h}$$

the dependence on ξ vanishes at extrema

$$\frac{dV_{\text{eff}}(h_{\text{min}},\xi)}{d\xi} = 0$$

Gauge invariance

The location of extrema, however, remains gauge dependent (figure from ref. [47]).



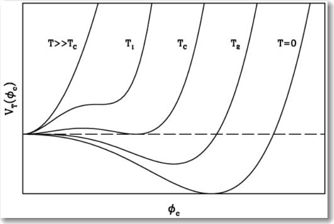
In any case, there are artificial violations of Nielsen identities due to the one-loop truncation of the effective potential (see e.g., $\hbar$ expansion [47]) and daisy terms.

Finding FOPTs

FOPT

A first-order PT requires that the vacuum contains two distinct minima separated by a barrier.

Tunnelling between the minima results in a jump discontinuity in the order parameter, e.g., $\gamma \equiv \langle \phi \rangle / T$.



The minima are degenerate at the critical temperature, T_C. The minima, x_{min}, implicitly depends on the temperature.

Tracing phases

- To find the PTs and the critical temperatures, we must track the change in the minima of the potential with temperature.
- This is an optimisation problem — we must repeatedly find the minima of a scalar function of several variables.
- We follow strategy in `CosmoTransitions` [48]. We find the expected change in a minima by solving

$$\left.\frac{\partial^2 V}{\partial x_i \partial x_j}\right|_{x_\text{min}} \frac{d {x_j}_\text{min}}{dT} = - \left.\frac{\partial^2 V}{\partial x_i \partial T}\right|_{x_\text{min}}$$

  and polish the guess

$$x_\text{min}(T + \Delta T) = x_\text{min}(T) + \frac{d x_\text{min}}{dT} \Delta T$$

Ending phases

- We reach the maximum or minimum temperature under consideration (e.g., at $T = 0$).
- The minima changes abruptly with a small change in temperature, as it indicates a jump discontinuity in the order parameter

$$\gamma = \Delta\phi / T$$

- The Hessian is singular

$$\det \left. \frac{\partial^2 V}{\partial x_i \partial x_j} \right|_{x_\text{min}} = 0$$

1. Indicates a double root, e.g., that two minima were separated by a barrier, but the barrier and a minima merged in a double root.
2. Indicates multiple solutions for $\frac{dx_\text{min}}{dT}$, which indicates a jump discontinuity in the derivative of the order parameter (i.e. a second-order phase transition in old-fashioned classification).

Finding all phases and possible transitions

We find all phases by tracing all global minima of the potential at $T = 0$ and at $T = 1\,\text{TeV}$. Whenever we encounter the end of a phase, we trace a new phase from where the old one ended.

After finding the phases, we find possible critical temperatures by bisection, e.g., if phases x_min and y_min coexist between temperatures T_1 and T_2, and if

$$V(x_\text{min}(T_1), T_1) > V(y_\text{min}(T_1), T_1)$$

$$V(x_\text{min}(T_2), T_2) < V(y_\text{min}(T_2), T_2)$$

there must exit a critical temperature, T_C, between temperatures T_1 and T_2 at which they are degenerate,

$$V(x_\text{min}(T_C), T_C) = V(y_\text{min}(T_C), T_C).$$

which we find by bisection.

Requirements

We built our NMSSM matched to the THDMS in `FlexibleSUSY` [49, 50]. It provides precise Higgs mass calculations and field-dependent masses.

1. We checked the Higgs sector was compatible with observations with `Lilith-1.1.4_DB-17.05` [51] and `HiggsBounds—5.3.2beta` [52–56].
   This ensures that one of our Higgs is SM-like (125 GeV with sufficiently SM-like couplings) *and* that the other Higgses aren't excluded by non-SM like Higgs searches.
   Since we want to modify the EWPT, we aren't in a decoupled regime.

Requirements

We built our NMSSM matched to the THDMS in `FlexibleSUSY` [49, 50]. It provides precise Higgs mass calculations and field-dependent masses.

2. Although we don't include them in our effective theory, we don't want sparticle masses that are excluded by searches in our UV model. We avoid it by setting $M_{\text{SUSY}} \gtrsim 1\,\text{TeV}$.
   We make sure, however, that the effective Higgsino mass, $\mu_{\text{eff}} = \frac{1}{\sqrt{2}}\lambda s$, wouldn't result in $m_{\chi^\pm} \lesssim 100\,\text{GeV}$, which was excluded by LEP.

Requirements

We built our NMSSM matched to the THDMS in `FlexibleSUSY` [49, 50]. It provides precise Higgs mass calculations and field-dependent masses.

3. And we required a strong FOPT $\gamma \gtrsim 1$. We define

$$\gamma = \frac{\sqrt{(h_u - h_u')^2 + (h_d - h_d')^2}}{T_C}$$

   This isn't, in general, gauge invariant.

Scanning the parameters

Finally, we scanned the NMSSM's eight relevant parameters with `MultiNest-3.10`

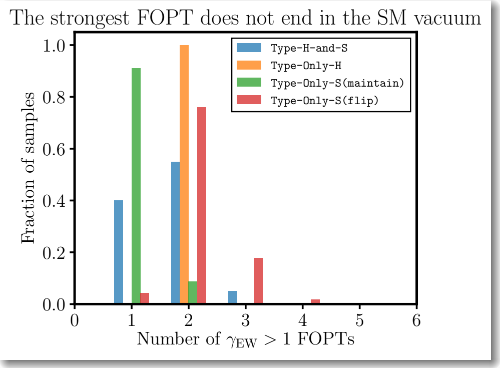
Parameter	Range	Metric
λ	$0, \pi/2$	flat
$\lvert\kappa\rvert$	$0, \pi/2$	flat
$\lvert A_\lambda\rvert$	$0, 10\,\text{TeV}$	hybrid
$\lvert A_\kappa\rvert$	$0, 10\,\text{TeV}$	hybrid
$\lvert A_t\rvert$	$0, 10\,\text{TeV}$	hybrid
M_{SUSY}	$1, 10\,\text{TeV}$	log
$\lvert v_S\rvert$	$0, 10\,\text{TeV}$	hybrid
$\tan\beta$	$1, 60$	log

We consider real parameters — we focus on the PT and don't investigate CP violation. We traded some parameter via the tadpole conditions.

Results

Millions of FOPTs!

We found more than 3,000,000 points with strong FOPTS! Many of them featured multiple strong FOPTS!



As many as four in some cases!

Classifying phase histories

To understand them, we classified them by the nature of the first transition

1. `Type-H-and-S` — The Higgs and singlet obtain VEVs
2. `Type-Only-H` — Only (at least one) Higgs obtains a VEV
3. `Type-Only-S-maintain` — Only the singlet obtains a VEV. It maintains the same sign during the strongest PT
4. `Type-Only-S-flip` — Only the singlet obtains a VEV. It flips sign during the strongest PT

We illustrate each scenario by a benchmark. For the benchmarks, we furthermore checked the nucleation temperature

$$S(T_N) \approx 140 T_N$$

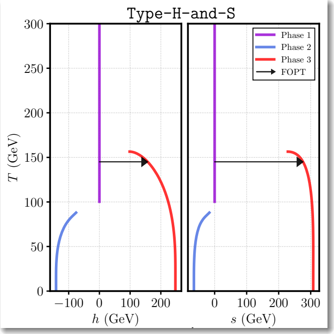
where S is the Euclidean action found from the bounce.

Benchmarks

	H-and-S	Only-S	Only-S-maintain	Only-S-flip
λ	0.618	0.607	0.601	0.935
κ	0.229	0.191	0.175	1.137
A_λ	160.1	160.5	170.0	147.4
A_κ	-93.7	-117.5	-25.2	61.4
A_t	-21.4	38.3	-24.6	-478.6
M_{SUSY}	6374.7	3463.1	5857.5	4164.3
v_S	307.9	247.5	245.7	183.1
$\tan\beta$	1.2	2.0	2.6	3.2
Strongest FOPT				
T_C	145	124	121	104
T_N	230	119	119	N/A; no nuc.
γ	1.1	1.5	1.1	2.1

Type-H-and-S

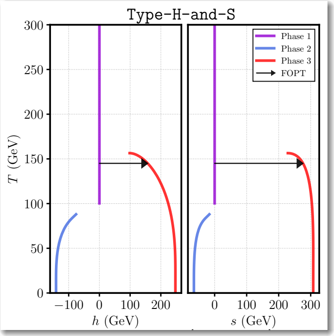
For the left-hand plot, we defined $h = \text{sign}\,(h_u h_d)\sqrt{h_u^2 + h_d^2}$



There are three phases (purple, blue and red)

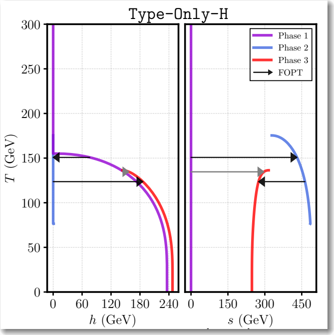
Type-H-and-S

For the left-hand plot, we defined $h = \text{sign}\,(h_u h_d)\sqrt{h_u^2 + h_d^2}$



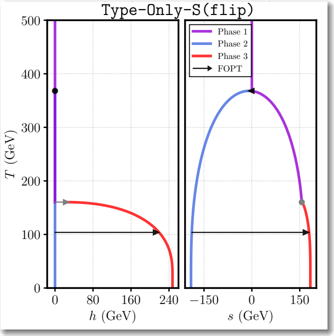
There is one FOPT from purple to red at $T_C \approx 150\,\text{GeV}$ in which h and s obtain VEVs (black arrow)

Type-Only-H



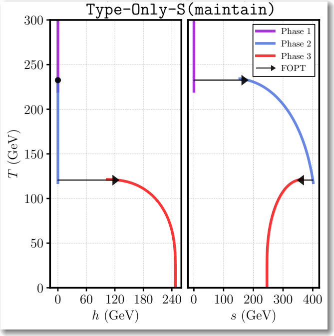
There are three FOPTs. In the first PT (which is second-order), only h obtains a VEV.

Type-Only-S-maintain



There are three possible FOPTs. In the first PT, only s obtains a VEV and it flips sign in the subsequent FOPT.

Type-Only-S-flip

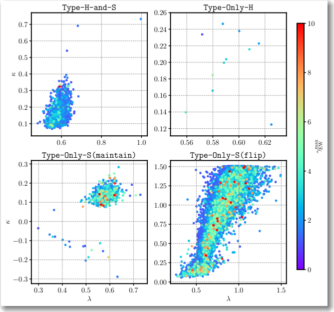


There are two FOPTs. In the first PT, only s obtains a VEV and it remains positive in the subsequent FOPT.

In this scenario, however, the strongest transition didn't completely nucleate.

Predictions for Lagrangian parameters

The quartic couplings — λ and κ – are tightly constrained in some of our scenarios.



Strongest PT ends at SM vacuum

Sometimes close to limits on pertubativity up to the GUT scale.

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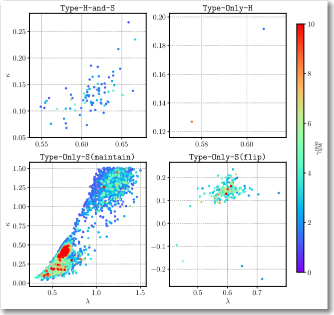
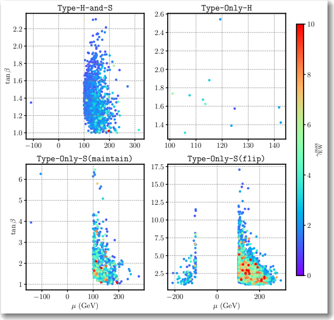


Strongest *does not* PT ends at SM vacuum

Sometimes close to limits on pertubativity up to the GUT scale.

The (μ, $\tan\beta$) plane

The requirements strongly favour small $\tan\beta$ and $\mu \lesssim 200\,\text{GeV}$



Strongest PT ends at SM vacuum

There could be light electroweakinos — visible at LHC?

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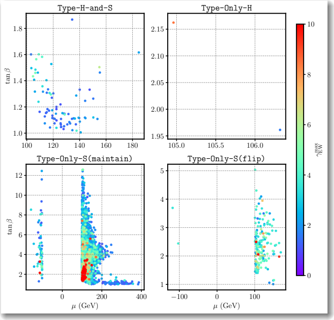
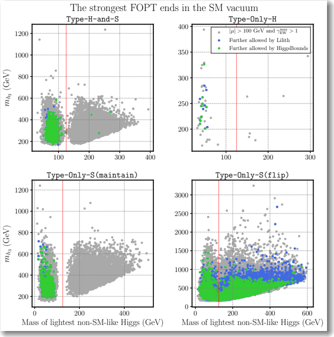


Strongest *does not* PT ends at SM vacuum

There could be light electroweakinos — visible at LHC?

We needed the singlet to assist in the EWPT. How light must it be?

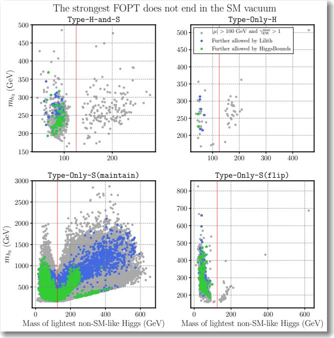


Strongest PT ends at SM vacuum

In fact, it's often lighter than the observed $125\,\mathrm{GeV}$ Higgs (i.e., to the left of the red line)!

Predictions for Higgs masses

We needed the singlet to assist in the EWPT. How light must it be?



Strongest *does not* PT ends at SM vacuum

In fact, it's often lighter than the observed 125 GeV Higgs (i.e., to the left of the red line)!

PhaseTracer

Generalising our code

Building our C++ code for the NMSSM taught us a few things. We decided to generalise it to arbitrary models in a forthcoming software package `PhaseTracer`.

The code finds and traces the minima of a potential, and calculates the critical temperatures.

And it's fast — about 100 times quicker than `CosmoTransitions`, the current leading code for this problem.

Generalising the model

- `PhaseTracer` includes a (pure virtual) class that represents an effective potential.
- The one-loop corrections to the potential in an R_ξ gauge are computed automatically once the field-dependent masses and degrees of freedom are supplied.
- It can be linked with `FlexibleSUSY`.
- Validated the results in several models with known analytic or numerical results.

Running PhaseTracer

In a few lines of code …

```
Potential::OneDimModel model;  // Construct the model

PhaseTracer::PhaseFinder pf(model);  // Construct the PhaseFinder
pf.find_phases();  // Find the phases
std::cout << pf;  // Print information about the phases

PhaseTracer::TransitionFinder tf(pf);  // Construct the TransitionFinder
tf.find_transitions();  // Find the transitions
std::cout << tf;  // Print information about the transitions
```

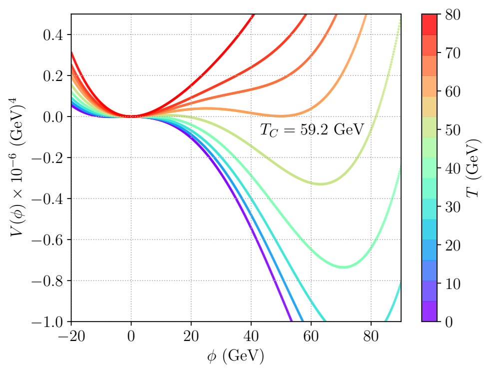
... you go from the model ...

```
class OneDimModel : public Potential {
  public:
    // Implement our scalar potential - this is compulsory
    double V(x_type x, double T) const override {
      return (0.1 * square(T) - square(100.)) * square(x[0]) - 10. * cube(x[0]) + 0.1 * pow_4(x[0]);
    }
    // Declare the number of scalars in this model - this is compulsory
    size_t get_n_scalars() const override { return 1; }
};
```

This is a trivial one-dimensional model

$$V = (0.1\,T^2 - 100^2)\,x^2 - 10x^3 + 0.1x^4$$

Running PhaseTracer



We know analytically that $T_C \simeq 59.16$ between $x = 0$ and 50.

Running PhaseTracer

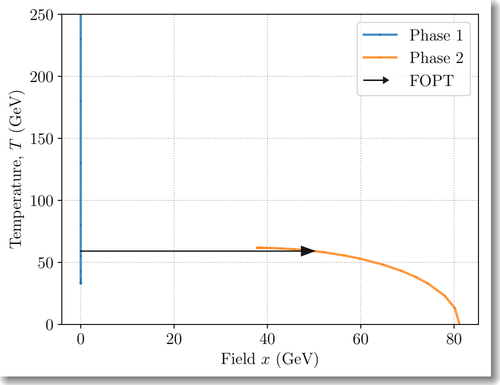
... to the results (about 0.01 seconds for 1 or 2 fields) ...

```
found 2 phases

=== phase key = 0 ===
Maximum temperature = 1000
Minimum temperature = 33.1513
Field at tmax = [1.48461e-05]
Field at tmin = [1.52403e-05]
Potential at tmax = 2.20185e-05
Potential at tmin = 2.29965e-09
Ended at tmax = Reached tstop
Ended at tmin = Jump in fields indicated end of phase

=== phase key = 1 === ...
```

```
found 1 transition

=== transition from 0 to 1 ===
true vacuum = [50.0003]
false vacuum = [1.09314e-05]
changed = [true]
TC = 59.1608
gamma = 0.845159 ...
```

Running PhaseTracer

… to a plot that shows what's happening.



This is completely automated.

Outstanding technical challenges

- We work in the R_ξ gauge — gauge dependent. We could use $\hbar$-expansion [47]. We worry it trades gauge invariance for an order in $\hbar$, as it would mean we looked only at tree-level minima.
- The Debye coefficients are model-dependent and we haven't implemented an efficient method for calculating them in an arbitrary model.
- The thermal functions oscillate when $m^2 < 0$. We truncate them. What is the correct treatment? What is the correct treatment of the imaginary part of the effective potential at $m^2 < 0$?
- Our treatment is perturbative — how can we indicate the uncertainties in our results?

Our philosophy: flexible and fast solutions for arbitrary models for the phenomenological community

FlexibleCOSMOS?!

FlexibleCOSMOS

The connection to FlexibleSUSY opens the door to making extremely flexible programs that can go automatically from the Lagrangian to

- The Feynman rules
- The mass spectrum and mixing angles
- The LHC and DM phenomenology, etc
- The presence of FOPTs by PhaseTracer
- The Euclidean action by another one of our codes BubbleProfiler [57]

Completing the chain

We want to complete this tool-chain in a flexible way by automatically computing

- Constraints on new CP violating phases (e.g., EDMs)
- The baryon asymmetry parameter
- GW phenomenology

This is ambitious but opens the door to fast, reliable calculations in any model. This makes a bridge between GWs, baryogenesis and particle phenomenology communities, and could allow rapid progress at the interface of these topics.

Conclusions

1. We carefully built the NMSSM effective potential — including EFT resummation of large logs
2. There are many possibilities for first order phase transitions and a variety of symmetry breaking patterns
3. The $125\,$GeV isn't usually the lightest one!
4. We are packaging our routines into a public software program — `PhaseTracer`
5. We have an ambition to create `FlexibleCOSMOS` — a flexible program for computing observables related to phase transitions in an arbitrary model

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