Cosmological phase transitions: From perturbative particle physics to gravitational waves

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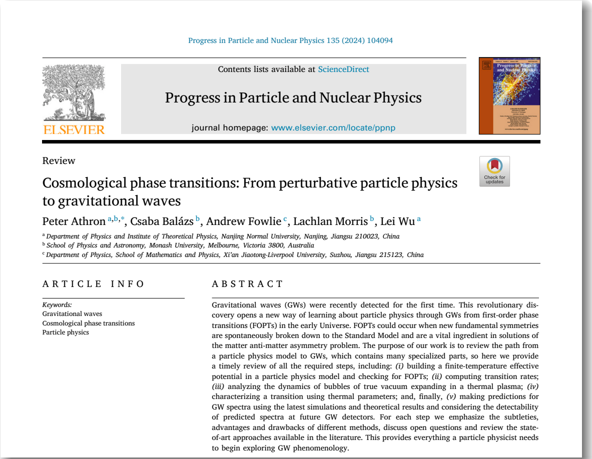

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Review

Cosmological phase transitions: From perturbative particle physics to gravitational waves

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ABSTRACT

Gravitational waves (GWs) were recently detected for the first time. This revolutionary discovery opens a new way of learning about particle physics through GWs from first-order phase transitions (FOPTs) in the early Universe. FOPTs could occur when new fundamental symmetries are spontaneously broken down to the Standard Model and are a vital ingredient in solutions of the matter anti-matter asymmetry problem. The purpose of our work is to review the path from a particle physics model to GWs, which contains many specialized parts, so here we provide a timely review of all the required steps, including: *(i)* building a finite-temperature effective potential in a particle physics model and checking for FOPTs; *(ii)* computing transition rates; *(iii)* analyzing the dynamics of bubbles of true vacuum expanding in a thermal plasma; *(iv)* characterizing a transition using thermal parameters; and, finally, *(v)* making predictions for GW spectra using the latest simulations and theoretical results and considering the detectability of predicted spectra at future GW detectors. For each step we emphasize the subtleties, advantages and drawbacks of different methods, discuss open questions and review the state-of-art approaches available in the literature. This provides everything a particle physicist needs to begin exploring GW phenomenology.

Invitation-only review article. 79 cites. ESI hot paper.

I took a speed reading course and read War and Peace in twenty minutes.

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It involves Russia.

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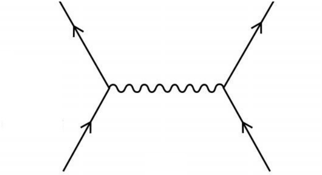
My paper involves gravitational waves.

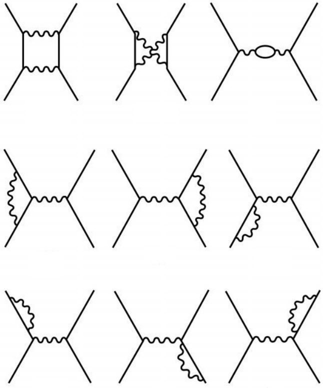
From perturbative particle physics…

- The framework is special relativity + quantum mechanics = quantum field theory description of fundamental particles
- We consider finite temperature $T \neq 0$; so we need thermodynamics as well
- We occasionally need cosmology and gravity as well; so general relativity
- We consider perturbative methods in our review (cf. lattice)

0. Perturbative particle physics…

Tree-level + one-loop + $\cdots$



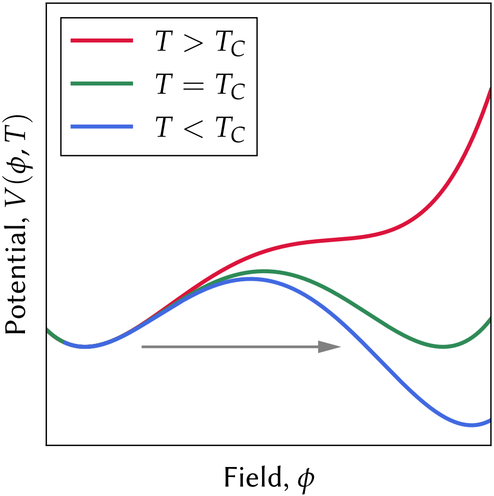


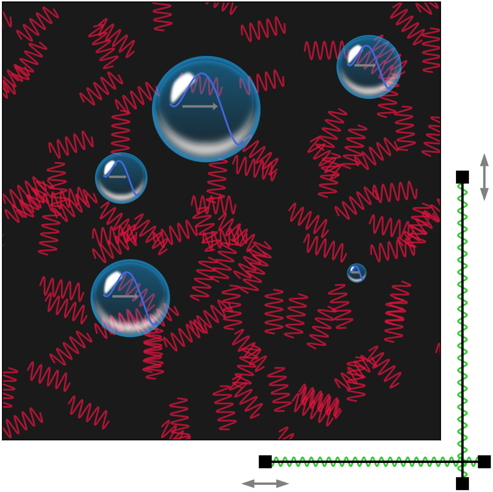
Tree-level potential

- The object we compute perturbatively is the scalar potential
- This is a function of the scalar fields — e.g., Higgs field. No derivatives
- E.g., Higgs potential at tree-level

$$V(H) = \mu^2 |H|^2 + \lambda |H|^4$$

1. First-Order Phase Transition

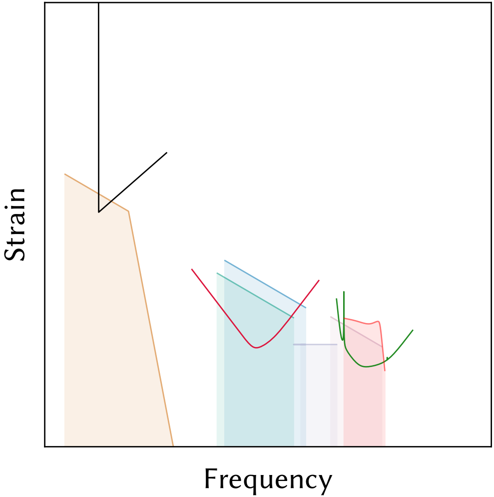




Bubbles of new phase

- In a first-order phase transition, transition occurs through bubble nucleation — think of water boiling
- Does not happen everywhere, all at once
- Stochastic process of bubbles popping into existence

3. Observable gravitational waves



Snippet 1. Shell theorem

- Do you know Gauss' law?

$$\oint \vec{E} \cdot \mathrm{d}\vec{S} = +4\pi k\, Q_{\mathrm{enc}} \qquad \text{Electromagnetism}$$

$$\oint \vec{g} \cdot \mathrm{d}\vec{S} = -4\pi G\, M_{\mathrm{enc}} \qquad \text{Gravity}$$

- Consider a spherical bubble. Choose a spherical Gaussian surface
- A growing or shrinking spherically symmetric bubble won't change the gravitational field. Outside the bubble, it looks like a point mass

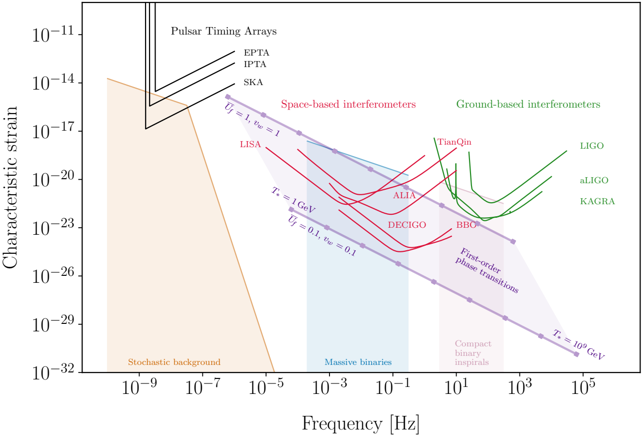
$$g\, 4\pi r^2 = -4\pi G\, M \quad \Rightarrow \quad g = -\frac{GM}{r^2}$$

Need more than one bubble

Interactions between bubbles, and between bubbles and the plasma avoid this

- Collisions
- Sound waves in the plasma
- Turbulence in the plasma

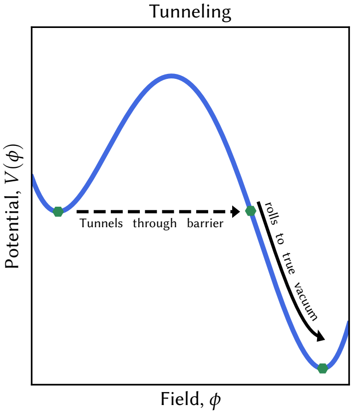
…to gravitational waves



Sensitivity of current and future GW detectors

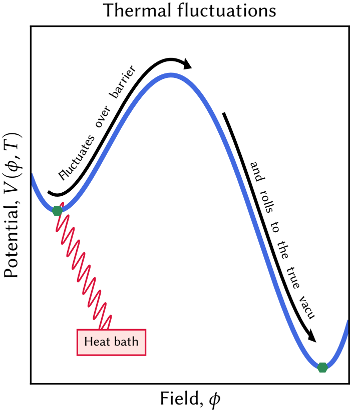
Snippet 2. — imaginary energy and instability

The field could tunnel through the barrier



Snippet 2. — imaginary energy and instability

The field could fluctuate over the barrier



How do we compute the decay rate?

Time-dependence of the wave function

$$\psi \sim e^{iEt}$$

What if energy eigenvalues are complex?

$$\psi \sim e^{iE_0 t - \frac{\Gamma}{2} t}$$

The wave function amplitude decays as

$$|\psi|^2 \propto e^{-\Gamma t}$$

Imaginary part corresponds to instability. Lifetime $1/\Gamma$

Imaginary component implies instability

- Imaginary energy in NRQM implies decay
- Imaginary mass in QFT implies particle decay, $\Gamma = 2\Im M$
- Imaginary part of vacuum energy implies vacuum decay $\Gamma = 2\Im E$
- Tachyonic $m^2 < 0$ implies symmetry breaking, e.g., Higgs mechanism

Extracting ground state energy

- We know we need the imaginary part of the ground state energy.
- Wick rotation $\tau = it$,

$$\langle q|e^{-i\hat{H}t}|q\rangle \to \langle q|e^{-\hat{H}\tau}|q\rangle = \sum e^{-E_n\tau}|\langle q|n\rangle|^2$$

- Taking $\tau \to \infty$ picks out E_0

Other consequences of Wick rotation

- No classical solution for tunnelling — trapped by conservation of energy
- Wick rotation $\tau = it$ changes equation of motion

$$\ddot{x} = -V'(x) \quad \rightarrow \quad \ddot{x} = +V'(x)$$

- Upturns the potential; no longer trapped by energy conservation
- We may now find semi-classical description of tunnelling

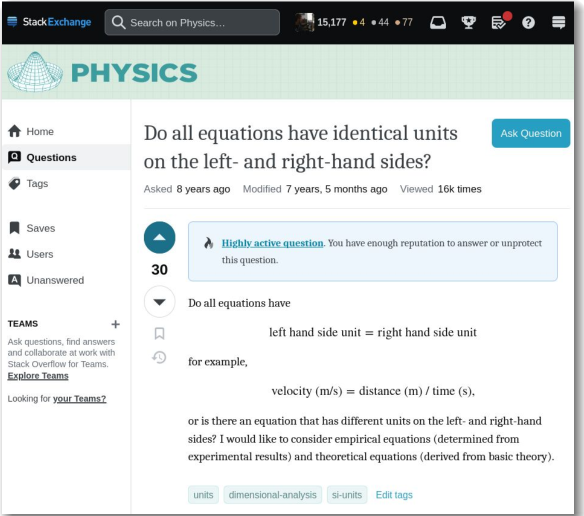
Bounce equation

- The equation of motion after Wick rotation:

$$\ddot{q}_i + \frac{3\dot{q}_i}{t} = \frac{\partial V(q)}{\partial q_i}$$

- Coupled second-order differential equations for fields q
- Unusual friction term that decays as $1/t$
- Upturned potential; field starts at rest and rolls asymptotically to top of a hill
- Fine-tuned; nasty when dimension of q more than a few

Snippet 3. — power of dimensional analysis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Do all equations have identical units on the left- and right-hand sides?

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Asked 8 years ago Modified 7 years, 5 months ago Viewed 16k times

30

Highly active question. You have enough reputation to answer or unprotect this question.

Do all equations have

$$\text{left hand side unit} = \text{right hand side unit}$$

for example,

$$\text{velocity (m/s)} = \text{distance (m) / time (s)},$$

or is there an equation that has different units on the left- and right-hand sides? I would like to consider empirical equations (determined from experimental results) and theoretical equations (derived from basic theory).

units dimensional-analysis si-units Edit tags

Snippet 3. — power of dimensional analysis

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It doesn't matter where the equation came from - a fit to experimental data or a deep string theoretic construction - or who made the equation - Albert Einstein or your next-door neighbour - if the dimensions don't agree on the left- and right-hand sides, it's nonsense.

Consider e.g. my new theory that the mass of an electron equals the speed of light. It's just meaningless nonsense from the get-go.

This isn't that restrictive - there's lots of equations with correct dimensions (though in some cases you can derive equations or estimates by so-called dimensional analysis, where you just make sure the units agree). But it is useful for checking your work. If you derive a result and the dimensions don't agree, you know you must have made a mistake.

There is a subtle distinction between unit and dimension. A dimension represents a fundamental quantity - such as mass, length or time - whereas a unit is a man-made measure of a fundamental quantity or a product of them - such as kg, meters and seconds. Arguably, one can write meaningful equations such as 60 seconds = 1 minute, with matching dimensions but mismatching units (as first noted by Mehrdad).

Energy in gravitational waves

Following arguments similar to those in Kosowsky, Turner, and Watkins 1992. Assume that energy of GWs

- must be proportional to Newton's constant, G
- depend on only the *available* vacuum energy density, $\kappa\rho$, where κ denotes the fraction of vacuum energy available for GWs
- only other relevant dimensionful scale is the characteristic timescale τ

On dimensional grounds, we must have that

$$E_{\mathrm{GW}} \sim G\kappa^2\rho^2\tau^5$$

The total liberated vacuum energy, on the other hand, is not proportional to G and on similar dimensional grounds, we must have that

$$E_V \sim \rho\tau^3$$

Thus, the fraction of vacuum energy that is in GWs must go like

$$r \equiv \frac{E_{\mathrm{GW}}}{E_V} \sim \frac{G\kappa^2\rho^2\tau^5}{\rho\tau^3} = G\kappa^2\rho\tau^2$$

Shouldn't $r < 1$? Is lifetime τ bounded? Yes!

Summary

- Long review paper that involves gravitational waves
- A few snippets of interesting physics for physicists and terrible maths for mathematicians
- Rich topics for collaboration, including percolation theory of bubbles, solving bounce equation, understanding Wick rotation tricks

References

Kosowsky, Arthur, Michael S. Turner, and Richard Watkins (1992). **"Gravitational radiation from colliding vacuum bubbles."** In: *Phys. Rev. D* 45, pp. 4514–4535. DOI: 10.1103/PhysRevD.45.4514.