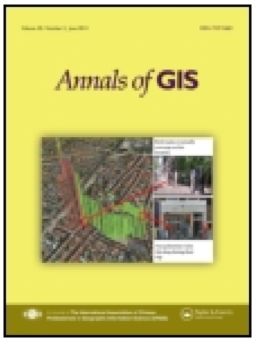

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Quadtree- and octree-based approach for point data selection in 2D or 3D

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This article describes a new automatic quadtree-/octree-based and scale-dependent generalization algorithm for point selection. The benefit toward existing point selection methods is that it preserves global as well as local characteristics of the spatial point distribution and of the spatial point density. It can be applied not only to points in 2D space but also to points in 3D space. In this article, an evaluation of the new point selection method is also provided.

Keywords: generalization; point selection; quadtree; octree

Introduction

The power of maps lies in their ability to abstract geographic space and that different levels of abstraction reveal different patterns and properties inherent among the geographic phenomena being represented. The ability to abstract data being ever more important in today's information society – in which the volume of data exceeds our insatiable appetite for more (Mackaness, Ruas, and Sarajakoski 2007). In this article a new generalization approach for point selection is introduced.

Background and objectives

Eckert (1921) already began to consider the necessity of cartographic generalization. More theoretical background of cartographic map generalization can be found in Bjørke (1996), Mustière and Moulin (2002), and Hake, Grünreich, and Meng (2002).

Research in this area draws on expertise in exploratory data analysis and visual analytics, agent-based methodologies, interface design, and cognitive ergonomics. Some of the studies within exploratory tools for analyzing point datasets can be found in Krisp and Peters (2010), Krisp et al. (2010), Krisp et al. (2009), and Peters and Krisp (2010). Investigations in this article may support data analysis and visual analytics approaches currently examined in a number of research projects, for instance, in Vismaster (2010) and in NVAC (2010).

Bertin (1983) believed that visualization is not effective enough unless it allows an immediate extraction of the essential information. Many traditional techniques in data visualization which proved to be supportive for exploratory analysis of datasets with moderate sizes fail when applied to large datasets. According to Andrienko and Andrienko (2007), two approaches can handle huge datasets. One approach is data selection which focuses on a portion of characteristic data items. The second approach is data aggregation which considers the clusters instead of the original data. But these two approaches cannot satisfy the needs of exploratory data analysis. These needs are described in Bertin (1983). Exploratory data analysis requires a consideration of the data on all levels: overall (considering a dataset as a whole), intermediate (viewing and comparing collective characteristics of arbitrary data subsets, or classes), and elementary (accessing individual data items). Therefore, a combination of data aggregation and data selection is suggested in Andrienko and Andrienko (2007), so to say to show the entire dataset and the arbitrarily defined subsets in an aggregated way. Thereby, the authors provide a solution using an adapted parallel coordinates plots which enable point data selection.

Selection is concerned with the semantics of the features rather than their locational attributes and as such represents the abstraction of the symbolic aspects of the map (Edwardes, Burghardt, and Weibel 2005). Selection involves the identification of objects to retain or remove from the database (Scott 1992). In fact, both semantic and locational attributes must be considered for selection.

The goal of point selection is also to avoid unreadability and overplotting. After point selection, the size of the displayed points can also be increased to aim an improved legibility and visibility. Past investigations in

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generalization of point datasets via point selection had focused on developing automated and assisted techniques, e.g., Weibel and Jones (1998). The algorithms developed for such applications have to be operational for use with large data volumes, but the absolute levels of performance are not critical (Burghardt, Purves, and Edwardes 2004).

Entropy in point datasets

In point selection, the main goal is to keep the information content (entropy) of the initial dataset as good as possible. What kind of information content exists in point data? In the literature, four different types of information contained in (point-) map features can be found – statistical, thematic, metric and topological information –, as described in Sukhov (1967, 1970), Neumann (1994), Bjørke (1996), Li and Huang (2002), and Yan and Weibel (2008). Statistical information of a map considers all the occurrences of the map features as unique events and all map events are equally probable. It is simply computed by counting the number of map features. But the spatial distribution is not considered. The thematic information is represented by an importance value (IV). The metric information considers the variation of the distance between map features and therefore their densities. The topological information is considered by the connectivity and adjacency between map features, e.g., neighboring points. These different types of point and point-cluster information can be quantified using different measures, shown in Ahuja (1982), Ahuja and Tuceryan (1989), Yukio (1997), Langran and Poicker (1986), Flewelling and Egenhofer (1993), and Yan and Weibel (2008).

Existing point selection algorithms

In the literature, a number of different cartographic generalization point selection algorithms can be found: settlement-spacing ratio algorithm, gravity-modeling algorithm, distribution-coefficient control algorithm, set-segmentation algorithm, and quadrat-reduction algorithm, among others, which all are proposed in Langran and Poicker (1986). Further algorithms are circle-growth algorithm (Yan and Weibel 2008), quadtree-based algorithm (Burghardt and Cecconi 2007), and simplification algorithm (De Berg et al. 2004). In Yan and Weibel (2008), these algorithms were summarized, compared, and evaluated. Also, a new algorithm was introduced based on the Voronoi diagram, whereby all the four types of information (statistical, metric, thematic, and topological) are transmitted in a point generalization process. The circle-growth algorithm and the settlement-spacing ratio algorithm are described and evaluated also in Li (2007). Furthermore, another algorithm for point generalization, the polarization transformation approach was introduced in Qian, Wu, and Deng (2005), Qian (2006), Qian et al. (2006), and Qian, Meng, and Zhang (2007).

In all selection, algorithms with a transmission of thematic information presuppose an IV for each point in the original dataset. What if there is no such an IV available, what if all points have the same IV or what if just many points have (almost) the same IV? Thus, the circle-growth algorithm, the Voronoi-based algorithm, the settlement-spacing ratio algorithm, the gravity-modeling algorithm, and the distribution-coefficient control algorithm do not work at all. In these cases, an IV need to exist for all points and they have to differ from each other. Point selection therefore requires variation of the IVs in these algorithms with a transmission of thematic information.

Concerning the transmission of topological information, the polarization transformation approach (Qian 2006) is the only method where the global and the local characteristics (global and local distribution; global and local density) of the generalized point dataset are retained within a point data selection in 2D.

Up to now, none of the existing point-selection algorithms dealing with points in 3D space consider both density and point distribution. Nowadays, via modern techniques in geo-data acquisition, huge amounts of static and dynamic points are captured not only in 2D space but also in 3D space. The objective of this study is to provide a solution for point data selection, in particular for 3D point datasets and global and local characteristics of the dataset.

Our approach refers to quadtrees for points in 2D space and to octrees for points in 3D space. So far, there is no appropriate method for automatic point selection in 2D or 3D based on quadtrees/octrees. In the generalization research of Burghardt, Purves, and Edwardes (2004), quadtree system is used to determinate the symbol size within geo-object exaggeration in 2D. In the set-segmentation algorithm and quadrat-reduction algorithm, Lewis (1982) made use of recursive subdivision of the plane. However, both of them require a great deal of human intervention (Yan and Weibel 2008). Furthermore, in Risi, Lehmann, and Stanley (2010) only the first subtree of a quadtree was used to get a representative selection of a 2D point dataset.

Approach and methods

Our point selection approach is based on quadtrees for points in 2D and on octrees for points in 3D. A quadtree is a tree data structure whereby each internal node has exactly four children. The region quadtree represents a partition of space in 2D by allocating the region into four equal quadrants, and so on whereas each leaf node contains data corresponding to a specific subregion. Each node in the tree can either have four children or no children (empty leaf node). If a region quadtree represent a point dataset, the regions are subdivided until each leaf contains only one single point (Finkel and Bentley 1974).

The three-dimensional analogs of quadtrees are octrees. An octree is a tree data structure whereby each internal node has eight children. Octrees are mostly used to partition a 3D space by recursively subdividing it into eight octants (Samet 2002).

In the following description, the quadrants refer to points in 2D and octants to points in 3D space.

The idea of this approach is to determine the quadtree/octree data structure of a point dataset to get information about the global and local point distributions and densities. By counting the number of points in each quadrant/octant, the global and local densities can be taken into account. By determining whether a quadrant/octant contains points or not, the global and local distribution can be considered.

Figure 1 exemplary illustrates the determination of all octants of a test dataset of 10 points in 3D. To the left, all points are displayed. Then the steps are shown, dividing the octants further into eight suboctants until each resulting octant contains only one point.

To achieve a selection that preserves global and local spatial point distributions and densities the following steps of the new quadtree-/octree-based points selection algorithm were investigated.

First of all for each step, all quadrants/octants were subdivided into 4/8 subquadrants/suboctants. Thus, in each following step, all subquadrants/suboctants were again subdivided into 4/8 subquadrants/suboctants, and so on. Step 1 contains 4 quadrants/8 octants, step 2 contains 16 quadrants/64 octants, step 3 contains 64 quadrants/512 octants, and so on.

Then for each quadrant/octant all containing points were counted. For each step IVs are given to certain points. If a quadrant/octant contains more than one point, an IV will be given to the point most central inside the quadrant/octant. As more points were counted as smaller the IV. If a quadrant/octant contains exactly one point also, an IV will be given to this point. The IVs are sorted step after step (first all points selected in step 1, then all points chosen in step 2, and so on). For each step, the IVs are sorted in the way that first the central points, then the points laying alone in a quadrant/octant are listed.

To summarize, beginning with step 1 for each step always the most central point within each subquadrant/suboctant containing more than one point will be selected. Also, all points laying alone in a subquadrant/suboctant will be selected. In the case where the most central point was already selected in a higher step, the second (or the third, etc.) most central point within the concerning subquadrant/suboctant will be selected, presupposing no IV has not yet been given to that point.

If *k*-number of points has to be selected within a point dataset, the *k*-first points in the IV list will be chosen.

An exemplary demonstration based on a test point dataset of 12 points is shown in Figure 2 and Table 1. Thereby, the subquadrant and all therein found points for each step level are listed. Furthermore, the point closest to the subquadrant is listed under 'selected point'. The last column of Table 1 reflects the point IV. The lower the order number the earlier the point will be selected. Thus, if altogether five points have to be kept, then they will be

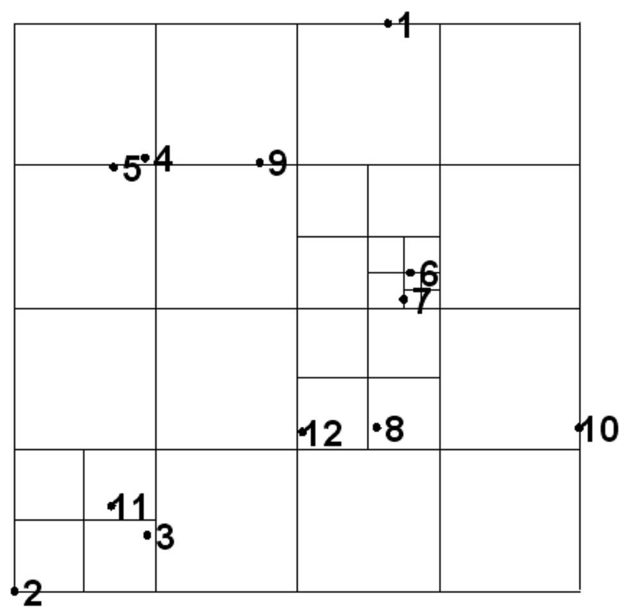

Figure 2. Test point dataset in 2D.

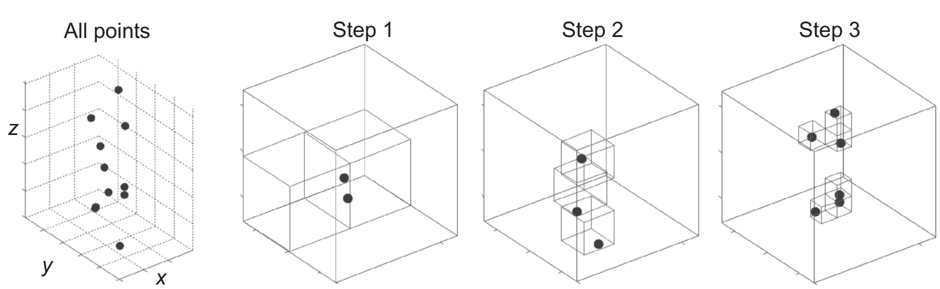

Figure 1. Exemplary stepwise determination of all octants of a test dataset of 10 points in 3D.

Table 1. Selection process of the test point dataset.

Step level	Subquadrant[a]	Found points (point nr.)	Selected point	Importance value (IV)
1	1	5,4,9	4[b]	1-997-1
	2	1,6,7	6[b]	1-997-4
	3	2,3,11	11[b]	1-997-2
	4	8,10,12	8[b]	1-997-3
	1–2	9	9	2-999-3
	1–3	5	5	2-999-2
	2–1	1	1	2-999-1
2	2–3	6,7	7[c]	2-998-2
	3–3	2,3,11	3[c]	2-997-1
	4–1	8,12	12[c]	2-998-1
	4–2	10	10	2-999-4
3	3–3–3	2	2	3-999-1

Notes: [a]Subquadrant 1 = above left; subquadrant 2 = above right; subquadrant 3 = down left; subquadrant 4 = down right.
[b]Most central point.
[c]Second central point.

points 4, 6, 11, 8, and 3 according to the smallest IV. The IV is defined as follows: the first digit of the IV represents the step level and the number between the hyphens refers to the number of all points within the respective subquadrant where the points are located. As more points are counted within the respective subquadrant as lower the number behind the hyphen (1000 minus the number of points). The number behind the second hyphen is introduced for the case that different subquadrants in the same step level contain the same number of points. Thus, the number behind the second hyphen refers to the mean distance of each selected point to the center of the respective subquadrant. The smaller the mean distance the smaller this number will be.

Töpfer's radical law (Töpfer and Pillewizer 1966) was used to determine the number of features which should retain within a selection process by defining either an achieved target scale or a wanted number of to be selected points.

Benefits of the new quadtree-/octree-based point selection approach

The benefits of this new approach are summarized in the following list:

(1) it allows an scale-dependent selection of point data;
(2) the selection is valid for point data in 2D and in 3D space;
(3) through its calculation simplicity this approach can be applicable as a Web-application;
(4) this approach can be applied to any 2D or 3D point dataset. This approach is based either on a predefined number respectively percentage of points to be selected or on a predefined output scale; and
(5) a main advantage is that this point selection approach considers both global as well as local characteristics of point data distribution and global as well as local characteristics of point data densities.

Evaluation of the quadtree-based point selection method

Two different methods can be used to evaluate a point selection, both values for 2D-point datasets. In this study, the second evaluation method was enhanced to evaluate point selections in 3D space.

Comparison of density values

In the first evaluation method, density values of the kept points are compared before and after the generalization. Also a visual comparison of the density map with all points with the density maps of the selected points can be done. For the density calculation, Scott's equation (Scott 1992) was used. Thereby, the mean distance of all original points was used for the search ratio h (kernel density bandwidth):

General kernel density function,

$$\widehat{f}_h(x) = \frac{1}{N \cdot h} \sum_{i=1}^{N} K(u) \tag{1}$$

In this work, for K, the Gaussian kernel K_G was used:

$$K_G = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}u^2\right) \tag{2}$$

where K_G is standard Gaussian function; $u = (x - x_i)/h$; $x_1, x_2, \ldots, x_N$ are points placed within the kernel radius h and h is the smoothing parameter (bandwidth).

Comparison of the relative local densities

Relative local density is used to express the density of an area over the whole study region (Yan and Weibel 2008). This area can be a Voronoi polygon (Figure 6), whereby the relative local density of each point can be determined as follows: relative local density, r_i,

$$r_i = R_i / \sum_{k=1}^{n} R_k \tag{3}$$

where R_i is absolute local density of the ith point, which is equal to $1/A_i$ for points in 2D and to $1/V_i$ for points in 3D; A_i is the area of the Voronoi polygon of the ith point; V_i is the volume of the Voronoi polyeder of the ith point and n is the number of the points.

It is assumed that R_1 is an array to capture all values of the relative density in the initial map. Thereby, r_{i1} is the

ith element of R_1. R_2 is an array for capturing all the values of the relative densities in the reduced map. Thereby r_{i2} is the ith element of R_2. The objective is to compare the relative local density for each point in the initial map with the corresponding one in the reduced map. Therefore, the following steps are necessary:

(1) Test R_1, if the ith point on the initial map had been eliminated, then eliminate r_{i1};
(2) Sort R_1 in an increasing order, then organize the elements in R_2 according to the sequences of the values of the corresponding points in R_1; and
(3) Create curves for R_1 and R_2 (see Figure 7) for a clear comparison of the change of the relative local densities.

Through a plot of the point numbers (x) and of the relative local densities (y) of the initial points as well as of the reduced map, the likeness can be visually compared. The more these both curves resemble each other, the more similar are the relative local densities of the entire and the selected point dataset.

The relative local densities are based on the Voronoi areas. For the 3D point dataset, the method was enhanced using Voronoi volumes instead of Voronoi areas in Equation (3). Thus, the relative local densities were calculated based on Voronoi volumes.

Results

Applying the quadtree-based point selection algorithm to a test dataset

As an original, we used lightning data provided from NOWCAST Company. These altogether 314 points represent lightning data. Each point contains the 3D-location of a cloud lightning emission point by measuring the magnetic waves via a very low frequency (VLF) network (Betz et al. 1996). In this way, everyday upto several million lightnings are detected which verify the importance of generalizing these lightning points for an improved analyzes.

First just the 2D point coordinates were used. A desired percentage of 20 percent of points to be selected was defined. That refers to a map reduction of 1/25, for instance in our case the initial map scale of 1:200,000 was reduced to 1:5,000,000. The results applying the new quadtree-based point selection method are shown in Figure 3b. Altogether 63 were kept, representing the initial point dataset.

In the second part, the 3D point coordinates were used. A predefined number of points to be kept (63 out of 314) were set. The results after applying the new octree-based point selection method are shown in Figure 4. It can be clearly seen that both spatial distribution and spatial densities are preserved. For instance, the highest point in

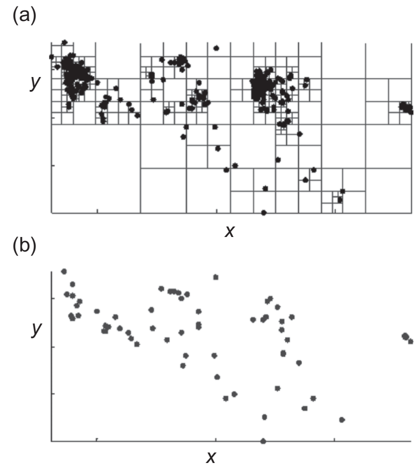

Figure 3. (a) Initial 2D points and quadrant raster; (b) selected 2D points.

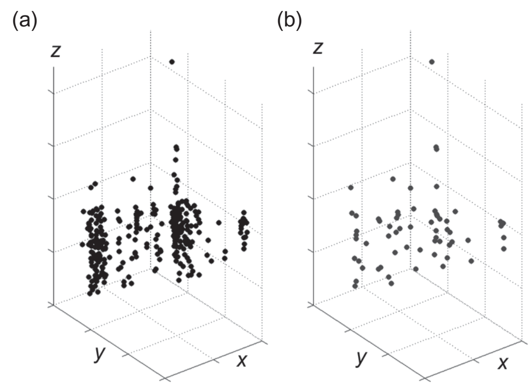

Figure 4. Small data set: (a) initial 3D points; (b) selected 3D points.

Figure 4, which is far away from all other points, is kept. In dense areas, more points are kept than in sparse areas.

Further experiments had been carried out with much larger datasets. For instance, a 3D point dataset containing 30,000 points (see Figure 5a) were used with the aim to select 20%. Thus, after applying the new algorithm, finally 6000 points (see Figure 5b) were selected.

Visual and statistical output results (consideration of density and distribution) were satisfactory in each case. By determining whether a quadrant/octant contains points or not, the global and local distribution can be considered. Even if a subquadrant/suboctant contains only 1 point, this point will be considered by giving it the highest weight within the respective level (see Figure 2, Table 1).

Evaluation by comparing the density values

Two density maps were produced of the 2D point dataset, one with all 314 points and a second one with the 63 selected points. Table 2 lists some of the point densities at positions of kept points, before and after the selection.

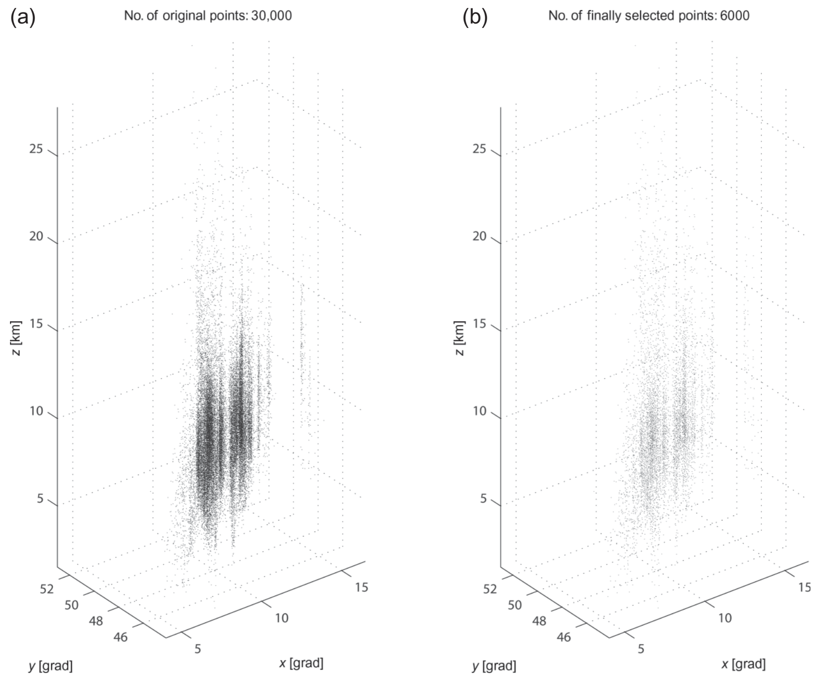

Figure 5. Large dataset: (a) initial 3D points; (b) selected 3D points.

Table 2. Density-comparison of the initial and the selected points.

Point no.-Dimension	D_all: densities using all point	D_sel: densities using only kept point	Density order D_all–D_sel
163-2D	0.2448	0.1503	4–5
178-2D	0.2115	0.1071	1–1
189-2D	0.2302	0.1090	2–2
38-2D	0.2362	0.1095	3–3
196-2D	0.2453	0.1249	5–4
159-3D	0.2048	0.0902	1–1
201-3D	0.2233	0.0985	2–3
61-3D	0.2340	0.1002	3–4
67-3D	0.2352	0.0949	4–2
187-3D	0.2440	0.1200	5–5

The order of the resulting densities remains nearly the same. For instance, points with a high density value keep the high value. Thereby, examples from both, the 2D and 3D point datasets are shown.

Figure 6 illustrates the resulting density maps, above the density map using the entire point dataset with 314 points and below the density map using only the 63 selected points in 2D. The density distribution over the study area remains.

Evaluation by comparing the relative local densities

The relative local densities of the selected points were computed and compared by using first all 314 points (R_1) and then only the 63 selected points (R_2). Figure 7, using 3D points, clearly reflects the monotonically increasing curve for R_1 (crosses) and R_2 (dots). Thus, the relative local densities of the initial and the generalized map have approximately the same values. This applied also to points in 2D space.

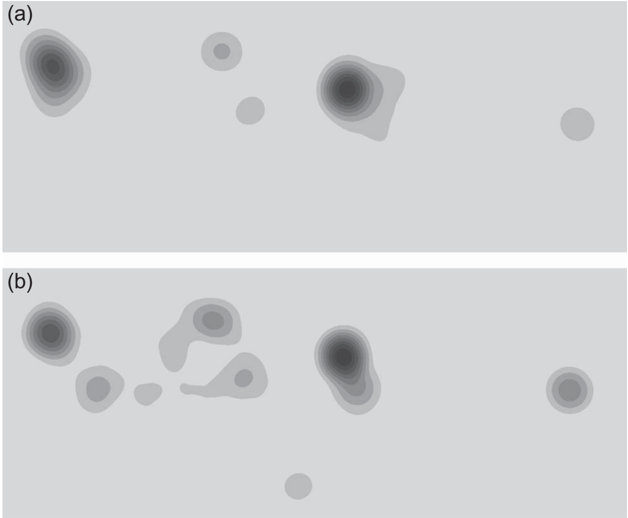

Figure 6. Density map of the original points above (a) and of the kept points below (b) in 2D.

Conclusion and future plans

Diverse fields such as Web mapping, Geo-visualization, or topographic and thematic cartography all need to consider a most faithful and recognizable representation of the real world by different map objects at different

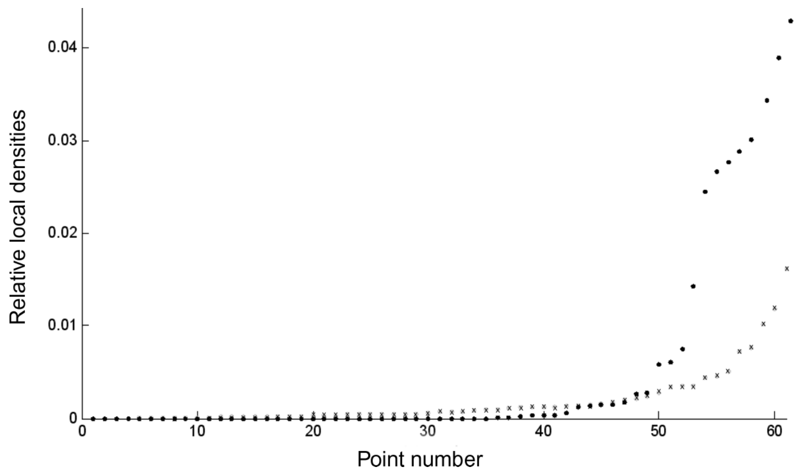

Figure 7. Curve of relative local density change of kept points (3D) before (crosses) and after (dots) generalization.

scales. In this study, a quadtree-/octree-based method for selecting/eliminating multidimensional points automatically is developed depending on the output scale or on the achieved number of points to be selected. The main advantage of this point selection approach is that global as well as local characteristics of the spatial point distributions and densities will be preserved.

This approach works with large point datasets and is also suitable for real-time generalization of points in Web maps because of its computing simplicity. Via an interactive use, the user can define either the number of points to be selected or the output map scale.

The evaluation verified the similarity of point density and distribution before and after applying the selection method. In this study, an existing evaluation method for selection of point data in 2D based on Voronoi areas was extended for point data selection in 3D by using Voronoi volumes.

A next step will be to implement this new point selection method in an interactive web tool. Further research using quadtree-based point selection will have to apply the adequate point symbol size to the selected points depending on the target scale. Also, it should planned to include semantic attributes in the algorithm. Further investigation might be needed to aggregate points using quadtrees/octrees.

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References

Ahuja, N. 1982. "Dot Pattern Processing Using Voronoi Neighborhoods." *IEEE Transactions on Pattern Analysis and Machine Intelligence* 4 (3): 336–343.

Ahuja, N., and M. Tuceryan. 1989. "Extraction of Early Perceptual Structure in Dot Patterns: Integrating Region, Boundary and Component Gestalt." *Computer Vision, Graphics and Image Processing* 48 (3): 304–356.

Andrienko, G., and N. Andrienko. 2007. "Blending Aggregation and Selection: Adapting Parallel Coordinates for the Visualisation of Large Datasets." *The Cartographic Journal* 42 (1): 49–60.

Bertin, J. 1983. *Semiology of Graphics: Diagrams, Networks, Maps*. Madison, WI: University of Wisconsin Press.

Betz, H.-D., B. Eisert, A. Gerl, R. Kulzer, and W. P. Oettinger. 1996. "Measurement of True Magnetic and Electric Waveforms of VLF-Atmospherics." In *Proceedings of the 25th General Assembly URSI*, Lille, 221.

Bjørke, J. 1996. "Framework for Entroy-Based Map Evaluation." *Cartography and Geographic Information Systems* 23 (2): 78–95.

Burghardt, D., and A. Cecconi. 2007. "Mesh Simplification for Building Typification." *International Journal of Geographical Information Science* 21 (3): 283–298.

Burghardt, D., R. S. Purves, and A. J. Edwardes. 2004. "Techniques for on-the-Fly Generalisation of Thematic Point Data Using Hierarchical Data Structures." In *Proceedings of the GIS Research UK 12th Annual Conference*, Norwich, April 28–30, 2004.

De Berg, M., P. Bose, O. Cheong, and P. Morin. 2004. "On Simplifying Dot Maps." *Computational Geometry: Theory and Applications* 27 (1): 43–62.

Eckert, M. 1921. *Die Kartenwissenschaft*. Berlin: DeGruyter.

Edwardes, A., D. Burghardt, and R. Weibel. 2005. "Portrayal and Generalisation of Point Maps for Mobile Information Services." In *Map-based Mobile Services–Theories, Methods and Implementations*, edited by L. Meng, A. Zipf, and T. Reichenbacher, 11–30. Berlin: Springer-Verlag.

Finkel, R., and J. L. Bentley. 1974. "Quad Trees: A Data Structure for Retrieval on Composite Keys." *Acta Informatica* 4 (1): 1–9.

Flewelling, D. M., and M. J. Egenhofer. 1993. "Formalizing Importance: Parameters for Settlement Selection from a Geographic Database." In *Proceedings of Auto-Carto 11 (11th International Conference on Automated Cartography)*, Minneapolis, MN, 167–175.

Hake, G., D. Grünreich, and L. Meng, eds. 2002. *Kartographie. Visualisierung raum-zeitlicher Informationen*. (8. Ausgabe), Berlin: DeGruyter.

Krisp, J. M., and S. Peters. 2010. "Visualizing Dynamic 3D Densities: A Lava-lamp approach." In *Workshop on Movement Research, 13th AGILE International Conference on Geographic Information Science*, Guimarães, May 10–14, 2010.

Krisp, J. M., S. Peters, B. Florian, and M. Butenuth. 2010. "Visual Identification of Scattered Crowd Movement Patterns Using a Directed Kernel Density Estimation." In *SPM2010 Mobile Tartu*, Tartu, August 26–28.

Krisp, J. M., S. Peters, C. Murphy, and H. Fan. 2009. "Visual Bandwidth Selection for Kernel Density Maps." *PFG Photogrammetrie Fernerkundung Geoinformation* 5: 445–454.

Langran, G. E., and T. K. Poicker. 1986. "Integration of Name Selection and Name Placement." In *Proceedings of Second International Symposium on Spatial Data Handling*, Washington, DC, 50–64. Williamsville, NY: International Geographical Union, Commission on Geographical Data Sensing and Processing.

Lewis, G. E. 1982. "Automated Point Labeling for Geographie Data Bases." Unpublished M.S. Thesis, Department of Geography, Western Washington University, Bellingham, Washington.

Li, Z., and P. Huang. 2002. "Quantitative Measures for Spatial Information of Maps." *International Journal of Geographical Information Systems* 16 (7): 699–709.

Li, Z. L. 2007. *Algorithmic Foundation of Multi-scale Spatial Representation*, 280 p. Bacon Raton, FL: CRC Press.

Mackaness, W. A., A. Ruas, and L. T. Sarajakoski. 2007. *Generalisation of Geographic Information: Cartographic Modelling and Applications*. Boston, MA: Elsevier Science.

Mustière, S., and B. Moulin. 2002. "What Is Spatial Context in Cartographic Generalisation?" *IAPRS & SIS, Geospatial Theory, Processing and Applications* 34 (4): 274–278.

Neumann, J. 1994. "The Topological Information Content of a Map: An Attempt at a Rehabilitation of Information Theory in Cartography." *Cartographica* 31 (1): 26–34.

NVAC. 2010. National Visualization and Analytics Center. Accessed December 25, 2012. http://nvac.pnnl.gov/

Peters, S., and J. M. Krisp. 2010. "Density Calculation for Moving Points." In *13th AGILE International Conference on Geographic Information Science*, Guimarães, May 10–14.

Qian, H. 2006. "Automated Cartographic Generalization and Intelligent Generalization Process Control." PhD thesis, Information Engineering University.

Qian, H., L. Meng, F. Wu, and J. Wang. 2006. "The Generalization of Point Clusters and its Quality Assessment Based on a Polarization Approach." *Mapping and Image Science* 4: 55–63.

Qian, H., L. Meng, and M. Zhang. 2007. "Network Simplification Based on the Algorithm of Polarization Transformation." In *CD-Proceedings of the XXIII International Cartographic Conference (ICC)*, Cartographic Generalization and Multiple Representation, Moscow, August 4–10.

Qian, H., F. Wu, and H. Deng. 2005. "A Model of Point Cluster Selection with Circle Characters." *Science of Surveying and Mapping* 3: 83–86.

Risi, S., J. Lehmann, and K. Stanley. 2010. "Evolving the Placement and Density of Neurons in the HyperNEAT Substrate." In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2010)*, New York, July 7–11, 2010.

Samet, H. 2002. "Octree Approximation and Compression Methods." In *Proceedings of the 1st International Symposium on 3D Data Processing Visualization and Transmission*, Padova, June 19–21, 2002.

Scott, D. W. 1992. *Mulivariate Density Estimation*. New York, NY: Wiley.

Sukhov, V. 1967. "Information Capacity of a Map Entropy." *Geodesy and Aerophotography* 10 (4): 212–215.

Sukhov, V. 1970. "Application of Information Theory in Generalization of Map Contents." *International Yearbook of Cartography* 10 (1): 41–47.

Töpfer, F., and W. Pillewizer. 1966. "The Principles of Selection: A Means of Cartographic Generalization." *The Cartographic Journal* 3: 10–16.

Vismaster. 2010. Visual Analytics–Mastering the Information Age. Accessed September 1, 2010. http://www.vismaster.eu/

Weibel, R., and C. B. Jones. 1998. "Computational Perspective on Map Generalization." *GeoInformatica* 2: 307–314.

Yan, H., and R. Weibel. 2008. "An Algorithm for Point Cluster Generalization Based on the Voronoi Diagram." *Computers & Geosciences* 34 (8): 939–954.

Yukio, S. 1997. "Cluster Perception in the Distribution of Point Objects." *Cartographica* 34 (1): 49–61.