Approximate coalitional equilibria in the bipolar world

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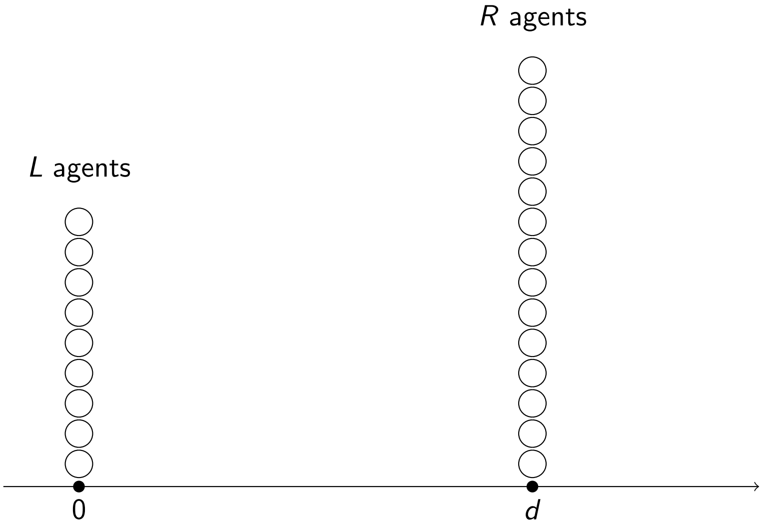
Outline

- A motivating story
- The formal model
- Analysis of stable configurations
- Analysis of approximately stable configurations

A motivating story

- A group of students wish to organize several dance parties. Some of the students prefer *L*ambada, the others prefer *R*ock'n'roll
- They could rent a dance hall, the rent price G does not depend on the number of people or music style. There is an unlimited number of dance halls to rent.
- The rent price is divided equally between the listeners.
- If a person listens a non-preferred music, then he or she gets disutility d
- If a group of students rent a hall, they decide the music style by majority voting. If there is a tie, then they may split the time between styles in any proportion
- Will they ever agree?

A bipolar world



The formal model: cost function

- There are L agents x with $p(x) = 0$ and R agents with $p(x) = d$
- A coalition partition is a representation $N = S_1 \sqcup \cdots \sqcup S_k$
- Every coalition chooses $m(S)$. If S consists of l agents with $p(x) = 0$ and r agents with $p(x) = d$, then

$$m(S) = \begin{cases} 0, & l > r; \\ d, & l < r; \\ \text{any } m \in [0, d], & l = r \end{cases}$$

- Configuration: $(S_1 \sqcup \cdots \sqcup S_k, m_1, \ldots, m_k)$, $m_i \in m(S_i)$
- The general cost function: $C(x, S, m) = \frac{G}{|S|} + t|p(x) - m|$
- The normalized cost function: $C(x, S, m) = \frac{1}{|S|} + |p(x) - m|$

The formal model: coalitional stability

- The idea: no group wants to leave their coalition in order to organize their own one
- Formally a configuration $(S_1 \sqcup \cdots \sqcup S_k, m_1, \ldots, m_k)$ is coalitionary stable if there is no pair (S, m) such that $m \in m(S)$, for all $x \in S$

$$C(x, S, m) \leq C(x, S_i, m_i),$$

and for some $x \in S$ the inequality is strict

There could be no stable configuration!

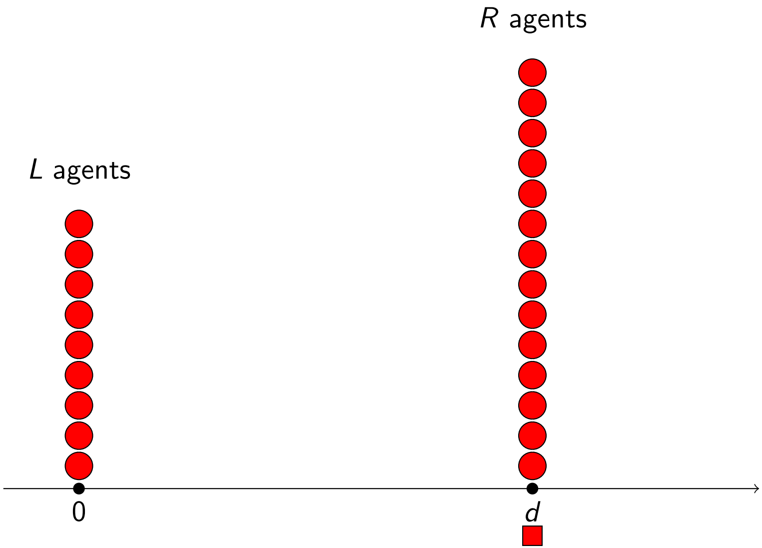
Theorem (Bogomolnaia, Le Breton, Savvateev, Weber, 2007)

There exists a bipolar world with no stable configuration!

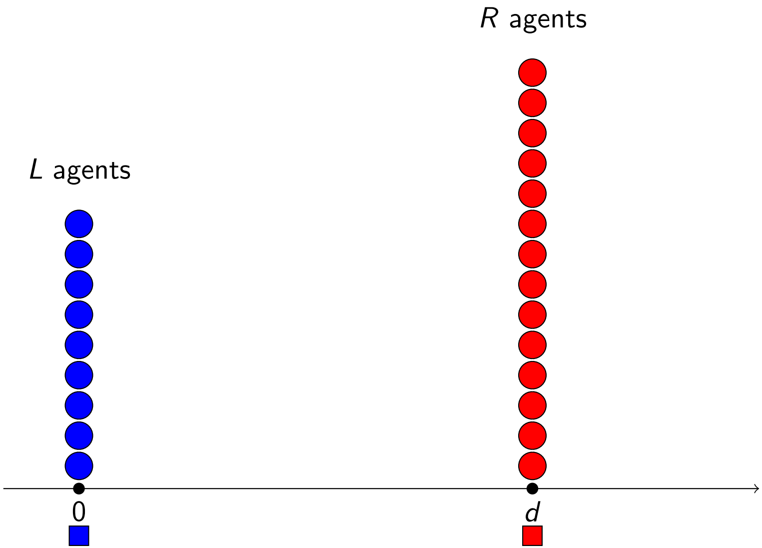
Proof.

The smallest possible example is $L = 2$, $R = 3$, $d = \frac{19}{60}$. The proof idea is shown on the next slides for $L = 9$, $R = 14$ $\square$

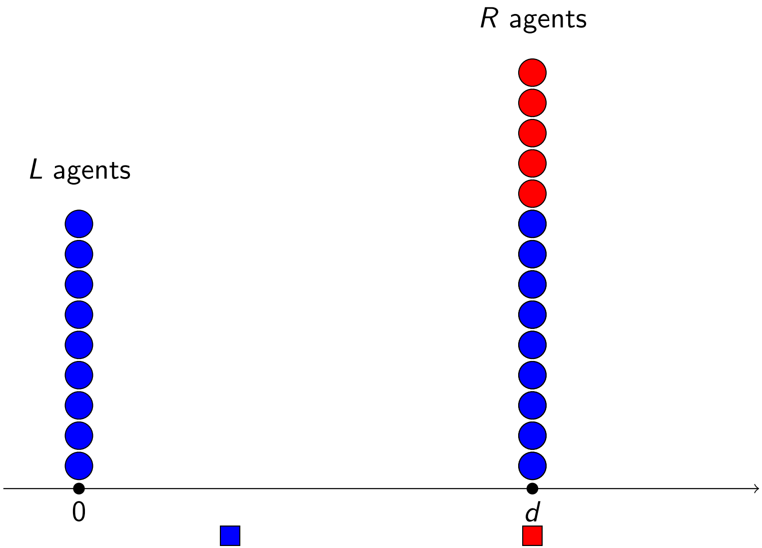
A bipolar world: union



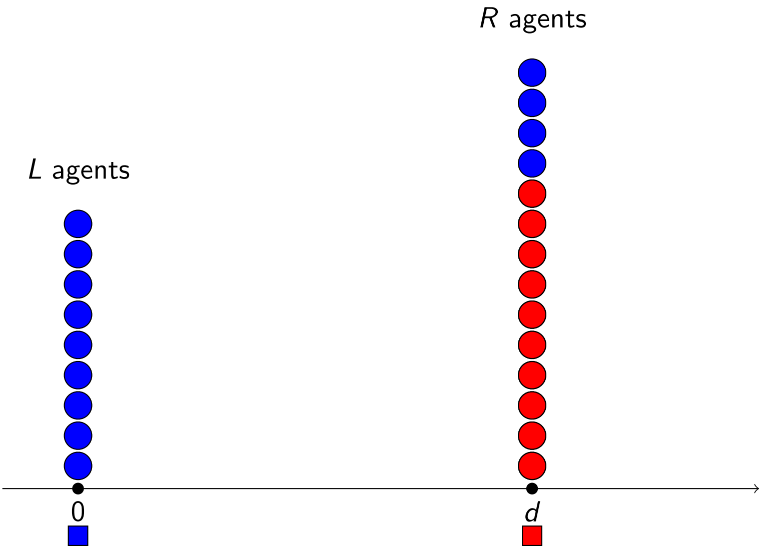
A bipolar world: federation



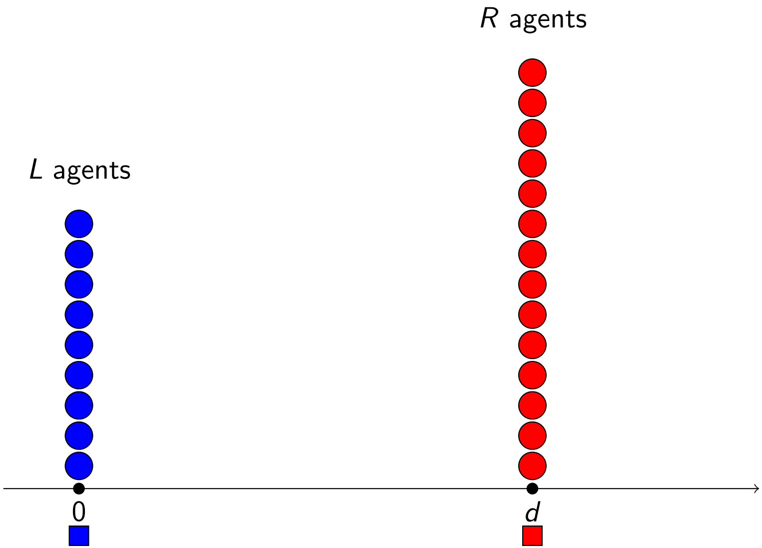
A bipolar world: mixed structure



A bipolar world: pseudofederation



A bipolar world: federation again



A bipolar world: stable configurations

Theorem (Savvateev, 2013)

In any bipolar world any stable configuration must be either a union, or a federation, or a mixed structure. (No pseudofederation may be stable!)

A bipolar world: stable configurations

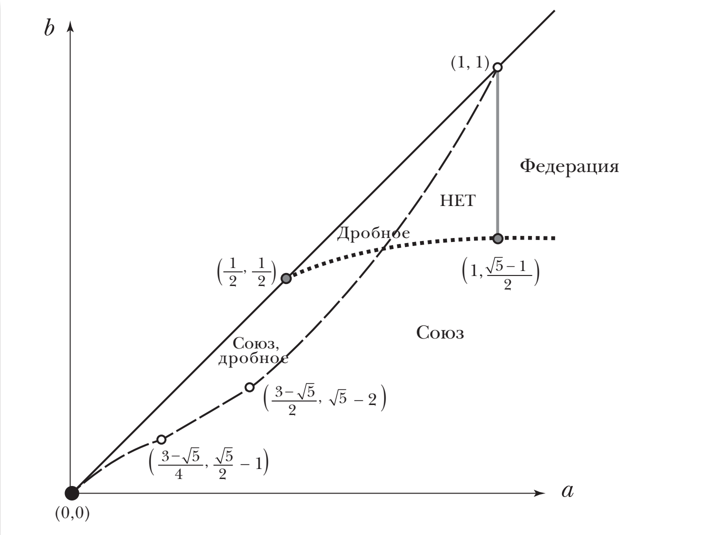


Рис. 3

Зоны устойчивости трех базовых видов разбиений

Approximate coalitional stability

Definition

An *absolute instability* of configuration $(\bigsqcup_i S_i, m_1, \ldots, m_k)$ is minimal ϵ such that no group of agents may form a new jurisdiction and thus reduce costs of all its members by at least ϵ.

$$\min \epsilon : \nexists S \ \forall x \in S \ C(x, S, m) \leq C(x, S_i, m_i) - \epsilon$$

Definition

A *relative instability* of configuration $(\bigsqcup_i S_i, m_1, \ldots, m_k)$ is minimal ϵ such that no group of agents may form a new jurisdiction and thus reduce costs of all its members by at least ϵ fraction.

$$\min \epsilon : \nexists S \ \forall x \in S \ C(x, S, m) \leq C(x, S_i, m_i)(1 - \epsilon)$$

Instability of a world

Definition

An *(absolute, relative) instability* of a world $W = (L, R, d)$ is the minimal instability of its configurations.

$$min_{(\bigsqcup_i S_i, m_1, \ldots, m_k)} max_{(S', m')} min_x [C(x, S_i, m_i) - C(x, S', m')]$$

The main question: what is the maximal possible instability? Where is it achieved?

Absolute instability

Theorem (This paper)

There are only three pairs (L, R) with absolute instability greater than 0.01*. The maximal values are the following:*

L	R	d	Δ
2	3	$\frac{14}{45} \approx 0.311$	$\frac{1}{90} \approx 0.0111$
3	4	$\frac{5}{24} \approx 0.208$	$\frac{1}{48} \approx 0.0208$
4	5	$\frac{7}{40} = 0.175$	$\frac{1}{80} = 0.0125$

Relative instability

Theorem (1)

For every world there is a configuration with least instability with not more than three groups.

Theorem (2)

The maximal possible relative instability does not exceed 6.3%

Relative instability

Theorem

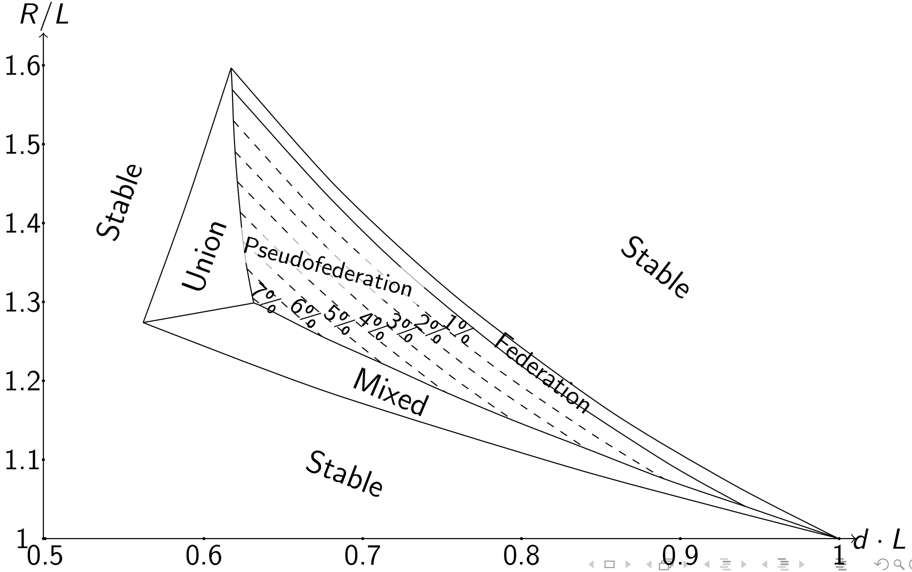
The maximal possible relative instability is between 6.15% *and* 6.3%

An example:

Configuration	Groups	Δ
Union	$(56, 73)$	0.066
Federation	$(0, 73) + (56, 0)$	0.074
Mixed	$(56, 56) + (0, 9)$	0.062
Pseudofederation	$(0, 69) + (56, 4)$	0.062

□

Which type of configuration is the most stable?



Future work

- What is the exact bound on relative instability?
- What happens if there are more than 2 points?

Thank you!

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