Energy Evolution of Delta Function for Jet Physics

Andrew Gordeev,[1,*] Zhong-Bo Kang,[1,†] and John D. Terry[1,‡]

[1]*Department of Physics and Astronomy,*
University of California, Los Angeles, CA 90095, USA

Abstract

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*Electronic address: agordeev@ucla.edu
†Electronic address: zkang@physics.ucla.edu
‡Electronic address: johndterry@physics.ucla.edu

I. DERIVATIONS

Professor Kang gives us that the energy evolution of J is given by

$$\frac{\partial}{\partial ln\mu^2}J(z,\mu)=\frac{\alpha_s(\mu)}{2\pi}\int_z^1\frac{dx}{x}P(x)J(\frac{z}{x},\mu). \tag{1}$$

Where

$$P(x)=C_F[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)] \qquad \int_0^1 dx\frac{f(x)}{(1-x)_+}=\int_0^1 dx\frac{f(x)-f(1)}{(1-x)} \tag{2}$$

Plugging $Eq.$ (2) in $Eq.$ (1) we obtain

$$\frac{\partial}{\partial ln\mu^2}J(z,\mu)=\frac{\alpha_s(\mu)}{2\pi}\int_z^1\frac{dx}{x}C_F[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)]J(\frac{z}{x},\mu).$$

Following the chain rule,

$$\begin{aligned}\frac{\partial}{\partial\mu}J(z,\mu)&=\frac{\partial}{\partial ln\mu^2}J(z,\mu)\cdot\frac{\partial ln\mu^2}{\partial\mu}\\&=\frac{\alpha_s(\mu)}{\pi\mu}\int_z^1\frac{dx}{x}C_F\Big[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\Big]J(\frac{z}{x},\mu)\\&=\frac{\alpha_s(\mu)C_F}{\pi\mu}\int_z^1\frac{dx}{x}\frac{1+x^2}{(1-x)_+}J(\frac{z}{x},\mu)+\frac{3\alpha_s(\mu)C_F}{2\pi\mu}\int_z^1\frac{dx}{x}\delta(1-x)J(\frac{z}{x},\mu)\end{aligned} \tag{3}$$

To evaluate the left integral in $Eq.$ (3), we use the fact that

$$\begin{aligned}\int_z^1 dx\frac{f(x)}{(1-x)_+}&=\int_0^1 dx\frac{f(x)}{(1-x)_+}-\int_0^z dx\frac{f(x)}{(1-x)_+}\\&=\int_0^1 dx\frac{f(x)-f(1)}{(1-x)}-\int_0^z dx\frac{f(x)}{(1-x)}\\&=\int_z^1 dx\frac{f(x)-f(1)}{(1-x)}+\int_0^z dx\frac{f(x)}{(1-x)}-\int_0^z dx\frac{f(1)}{(1-x)}-\int_0^z dx\frac{f(x)}{(1-x)}\\&=\int_z^1 dx\frac{f(x)-f(1)}{(1-x)}+f(1)\cdot ln(1-z).\end{aligned}$$

Thus,

$$\int_z^1\frac{dx}{x}\frac{1+x^2}{(1-x)_+}J(\frac{z}{x},\mu)=\int_z^1 dx\frac{\frac{1+x^2}{x}J(\frac{z}{x},\mu)-2J(z,\mu)}{1-x}+2J(z,\mu)\cdot ln(1-z). \tag{4}$$

We evaluate the right integral in $Eq.$ (3) using the delta function:

$$\int_z^1 \frac{dx}{x}\delta(1-x)J(\frac{z}{x},\mu) = \frac{1}{2}\cdot\frac{1}{x}J(\frac{z}{x},\mu)|_{x=1} = \frac{1}{2}J(z,\mu) \tag{5}$$

Using $Eq.$ (4) and $Eq.$ (5), $Eq.$ (3) becomes

$$\frac{\partial}{\partial\mu}J(z,\mu) = \frac{\alpha_s(\mu)C_F}{\pi\mu}\Big(\int_z^1 dx\frac{\frac{1+x^2}{x}J(\frac{z}{x},\mu) - 2J(z,\mu)}{1-x} + 2J(z,\mu)\cdot ln(1-z) + \frac{3}{4}J(z,\mu)\Big). \tag{6}$$

This equation is solved using the RK4 method, setting the initial value of J as an approximation of the delta function:

$$J(z,\mu_0) = \delta(1-z) \approx \begin{cases} \frac{2}{a^2}(z-(1-a)) & \text{if } 1-a \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where a is some small constant. To find intermediate values of J in the Runge-Kutta method, we use the approximation

$$J(z,\mu+\delta\mu) \approx J(z,\mu) + \delta\mu\cdot\frac{\partial}{\partial\mu}J(z,\mu)$$

where $\frac{\partial}{\partial\mu}J(z,\mu)$ is given by $Eq.$ (6).

II. RESULTS

J was evolved from 10 GeV to 100 GeV. Figure 1 shows the results as functions of z for three values of μ.

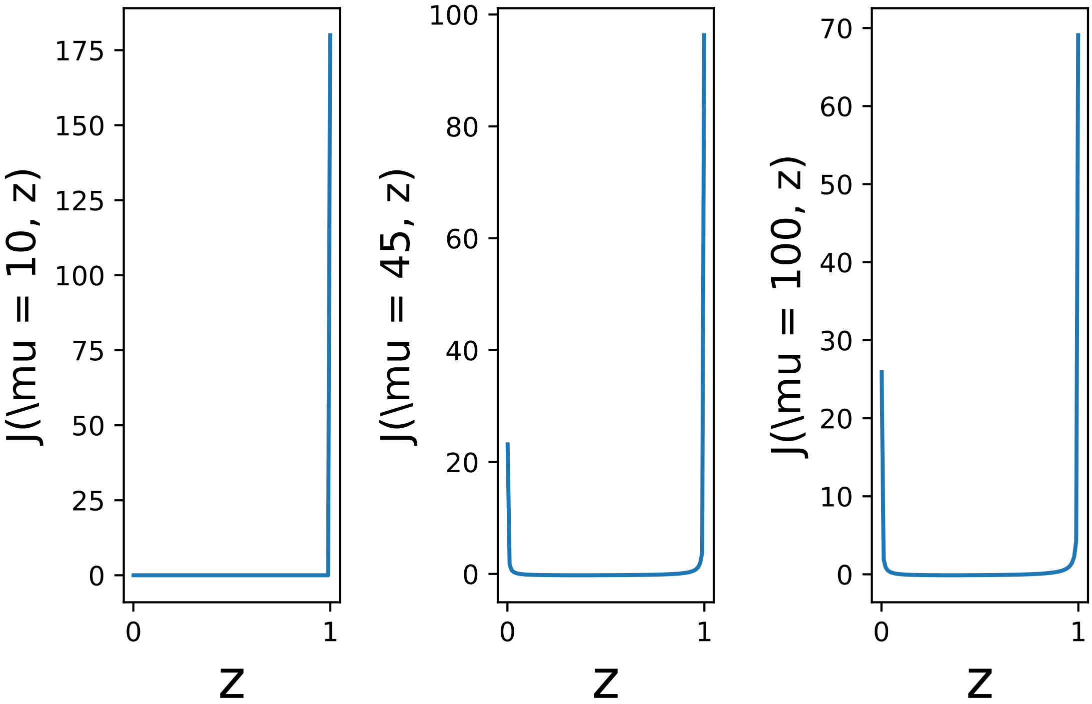

FIG. 1: Plots of J for initial, intermediate, and final values of μ.