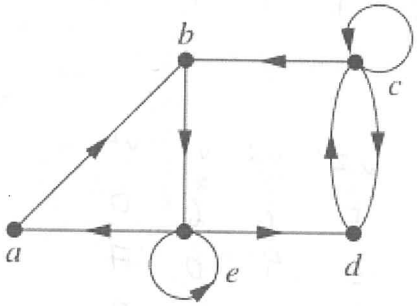
Class Exercise 6

Student's Name: Andrew Graff

Graph Theory

Q1. List the set of V for Vertices and E for Edges in following graphs. List the possible cycles in the graph (a) and (b)



(a)

$V = \begin{matrix} a=2 \\ b=3 \\ c=4 \\ d=3 \\ e=4 \end{matrix}$

$\sum_{r \in V} \deg(r) = 2|E|$

$\sum \deg(r) = \deg(a) + \deg(b) + \deg(c) + \deg(d) + \deg(e)$

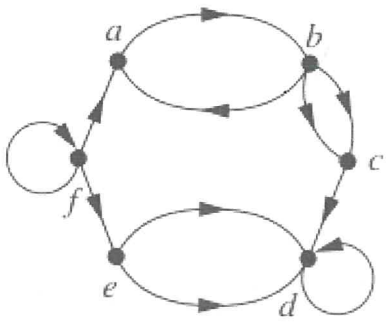
$= 2+3+4+3+4$

$= 16$

$= 2 \times 8$

$= 2|E|$

$V = 16 \quad E = 8$



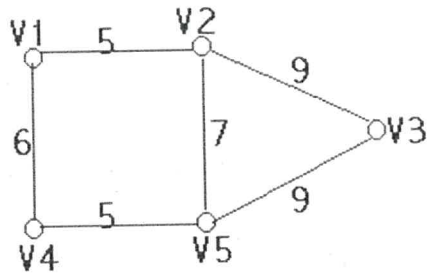
(b)

$V = \begin{matrix} a=3 \\ b=4 \\ c=3 \\ d=4 \\ e=3 \\ f=3 \end{matrix}$

$3+4+3+4+3+3$

$V = 20 \quad E = 10$

Q2. Write down the weight matrix for the weighted graph below.



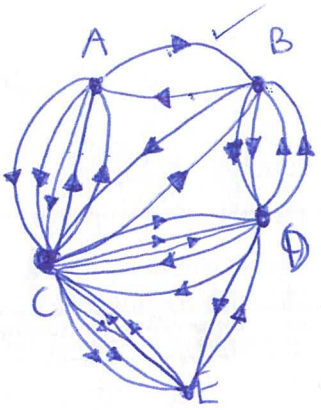
	V1	V2	V3	V4	V5
V1	0	1	0	1	0
V2	1	0	1	0	1
V3	0	1	0	0	1
V4	1	0	0	0	1
V5	0	1	1	1	0

Answer =

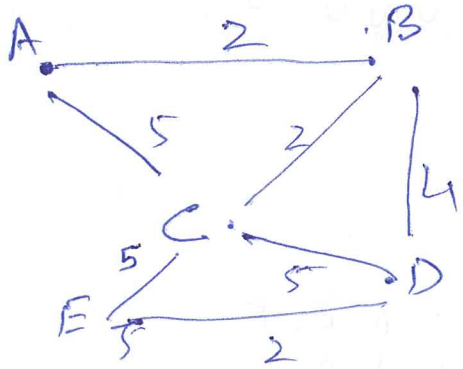
	V1	V2	V3	V4	V5
V1	0	5	0	6	0
V2	5	0	9	0	7
V3	0	9	0	0	9
V4	6	0	0	0	5
V5	0	7	9	5	0

Q3. Draw the weighted graph from the following weighted Matrix:

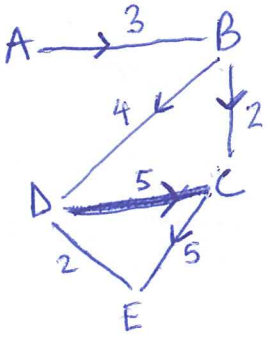
	A	B	C	D	E
A	0	2	5	0	0
B	2	0	2	4	0
C	5	2	0	5	5
D	0	4	5	0	2
E	0	0	5	2	0



answer =



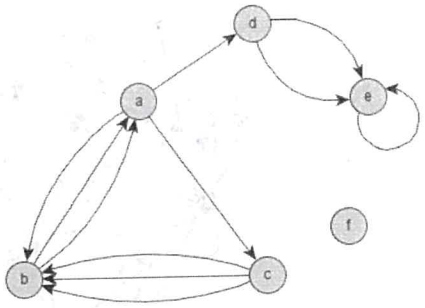
	A	B	C	D	E
A	0	3	0	0	0
B	0	0	2	4	0
C	0	0	0	0	5
D	0	0	5	0	0
E	0	0	0	2	0



Q4. Find the Adjacency Matrix:

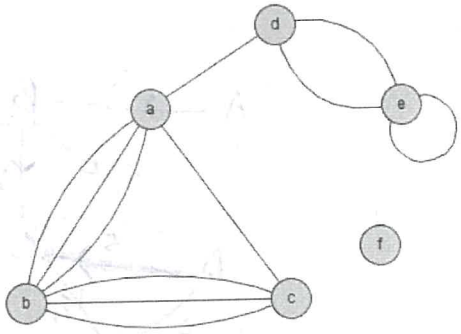
	A	B	C	D	E	F
A	0	1	1	1	0	0
B	2	0	0	0	0	0
C	0	3	0	0	0	0
D	0	0	0	0	2	0
E	0	0	0	0	1	0
F	0	0	0	0	0	0

a)

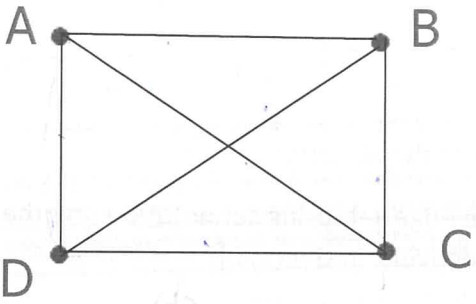


	A	B	C	D	E	F
A	0	3	1	1	0	0
B	3	0	3	0	0	0
C	1	3	0	0	0	0
D	1	0	0	0	2	0
E	0	0	0	2	1	0
F	0	0	0	0	0	0

b)



Q5. Verify for the following graph that the sum of the degrees of the vertices equals twice the number of edges.



$\sum_{v \in V} deg(v) = 2|E|$

Count vertices deg. of each

a = 3
b = 3
c = 3
d = 3

$\sum_{v \in V} deg(v) = 12 \div 2 = 6$ edges = Count edges = 6.

$= 12$

Total $\sum deg(v) = 12$

Total edges = 6

$\Rightarrow 6 \times 2 = \sum$ degrees.

This shows sum of degrees does equal twice number of edges.

Degree = 12

Edges = 6

Q6. Suppose that in a group of 5 people: A, B, C, D, and E, the following pairs of people are acquainted with each other.

A and C

A and D

Group of 5 people

A and C
A and D
B and C

C and D

C and E

a) Draw a graph G to represent this situation.

b) List the vertex set, and the edge set, using set notation. In other words, show sets V and E for the vertices and edges, respectively, in G = {V, E}.

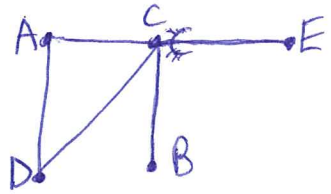
c) Draw an adjacency matrix for G.

(b)

V = 10 E = 5 G = {10, 5}

Graph G

(a)



Vertex set

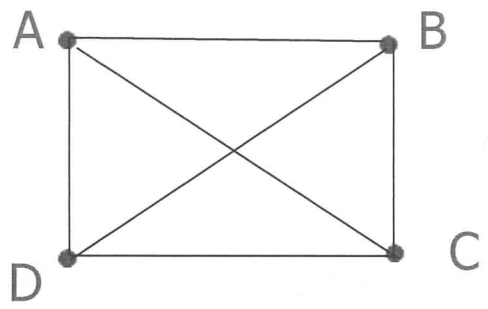
~~deg~~(a) = 2
~~deg~~(b) = 1
~~deg~~(c) = 4
~~deg~~(d) = 2
~~deg~~(e) = 1

(c)

	A	B	C	D	E
A	0	0	1	1	0
B	0	0	1	0	0
C	1	1	0	1	1
D	1	0	1	0	0
E	0	0	1	0	0

Q7. Classify each of the graphs below as Eulerian, semi-Eulerian, or neither and find an Eulerian path or Eulerian circuit if one exists.

(a)

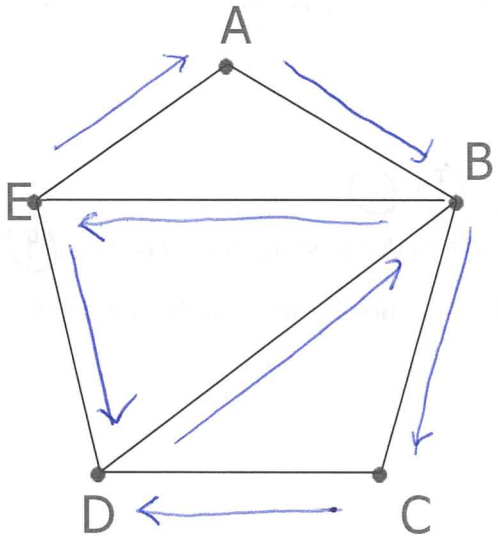


Non-Eulerian Graph

4 vertices with odd degrees.

No eulerian path.

(b)

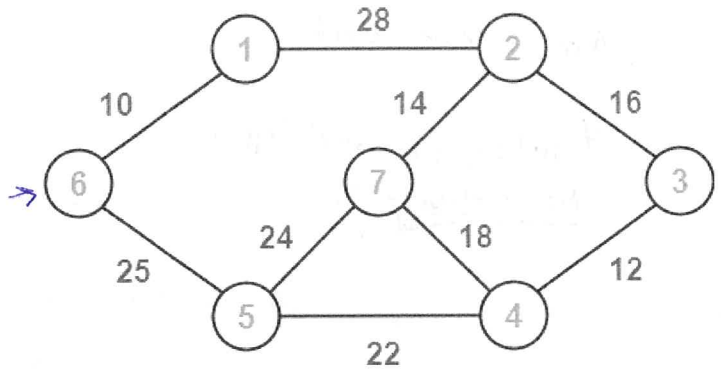


Semi-Eulerian Graph

Possible to find path from E → D.

Q8. Find a minimal spanning tree using Prims's algorithm.

Find minimum spanning tree using Prim's algorithm



Step 1:

Randomly choose any vertex. (6)

The vertex connecting to the edge having the least weight is usually selected.

Step 2:

Find all the edges that connect the tree to new vertices. (1) (5)

Find the least weight edge among those edges and include it in the existing tree. 22 → (4)

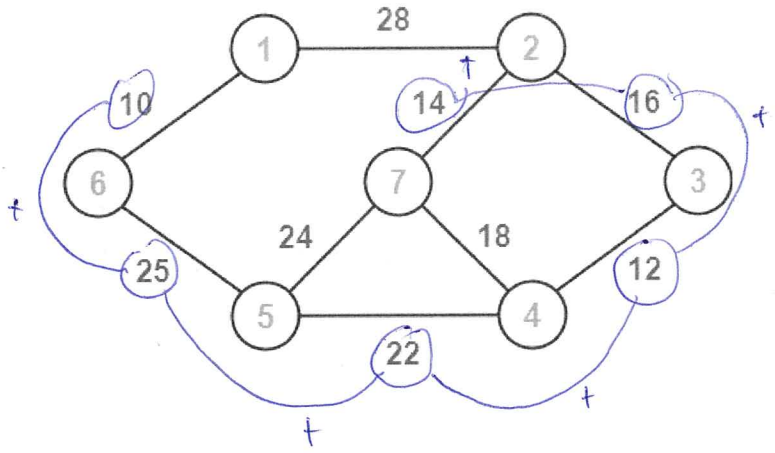
If including that edge creates a cycle, then reject that edge and look for the next least weight edge.

Step 3:

Keep repeating step 2 until all the vertices are included and Minimum Spanning Tree (MST) is obtained.

Q9. Find a minimal spanning tree using Kruskal's algorithm.

Kruskal Algorithm



Step 1:

Sort all the edges from low weight to high weight.

Step 2:

Take the edge with the lowest weight and use it to connect the vertices of the graph.

If adding an edge creates a cycle, then reject that edge and go for the next least weight edge.

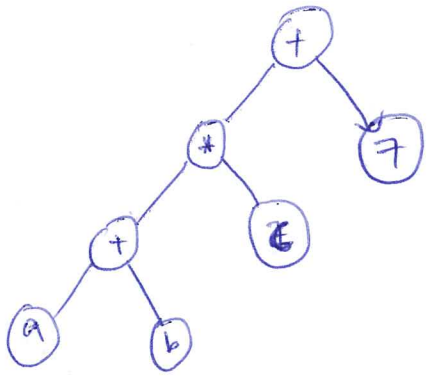
Step 3:

Keep adding edges until all the vertices are connected and a Minimum Spanning Tree (MST) is obtained.

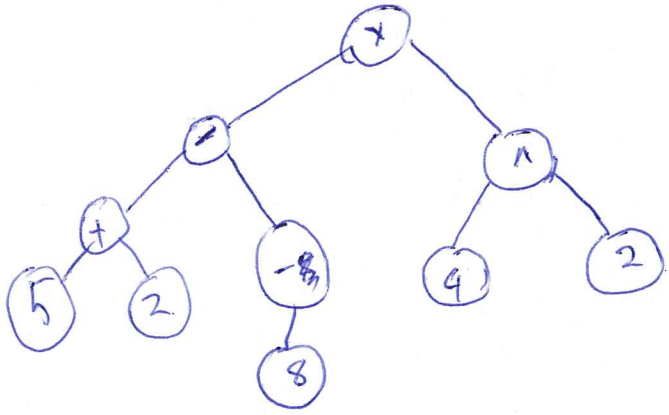
99 units

Q.10 Construct an expression tree for each of the following expressions (refer to the rooted tree slides in the Moodle)

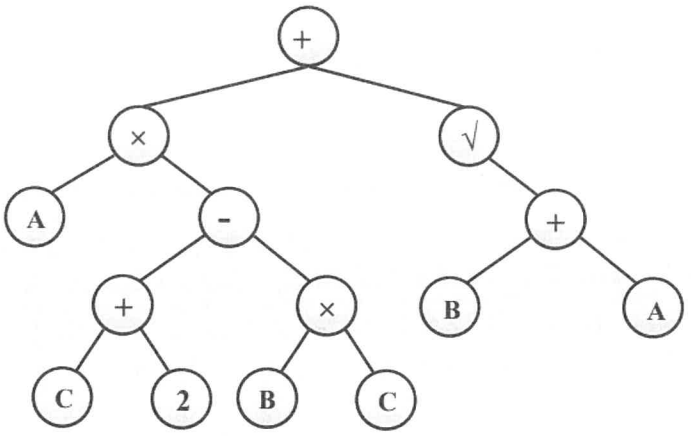
a) (a+b)*c+7



b) ((5 + z) / -8) * (4 ^ 2)



Q.11 Write the expression from the tree below and find the result.



$A((C+2)-(B*C))+\sqrt{(B+A)}$