

Class Exercise 11

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Q1. The number of goals scored by a football team in a season is shown in the table below:

n = total frequency
 F = cumulative frequency before class values.
 f_m = frequency of class median
 i = class width
 l_m = lower boundary of class median.

| Number of Goals | Frequency |
|-----------------|-----------|
| 0 | 7 |
| 1 | 14 |
| 2 | 13 |
| 3 | 8 |
| 4 | 3 |
| 5 | 4 |
| 6 | 1 |

| Cumulative Frequency | Total number of goals |
|----------------------|-----------------------|
| 7 | 0 |
| 21 | 14 |
| 34 | 26 |
| 42 | 24 |
| 45 | 12 |
| 49 | 20 |
| 50 | 6 |
| $\Sigma f = 50$ | $= 102$ |
| | (class number) |

What is:

a) The mode for the number of goals scored? 1 (14 frequency) ← most frequent.b) The median number of goals scored? 2.5 $2.5 = 2$ c) The mean number of goals scored (to the nearest whole number)?

n = total frequency
 $\text{mean} = l_m + \left(\frac{\frac{n}{2} - F}{f_m} \right) i$

$$\frac{n}{2} = \frac{50}{2} = 25$$

$$F = 21$$

$$f_m = 13$$

$$l_m = 1.5$$

$$i =$$

Median: 6, 4, 5, 0, 3, 2, 1
 (7 numbers)

← order of goals by frequency.
 (7 numbers)

$\frac{50+1}{2} = 25.5$
 median

$$5 + 0 = \frac{5}{2} = 2.5$$

median
 median

Q2. Below is a frequency table of data based on survey where 89 women were asked what their shoe size was.

Calculate the mean, median, and mode of the data.

| | | | | | | | | | | | |
|-----------|---|-----|----|-----|----|-----|---|-----|---|-----|---|
| Shoe Size | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 | 8 | 8.5 | 9 |
| Frequency | 5 | 12 | 18 | 19 | 11 | 4 | 8 | 5 | 5 | 0 | 2 |

$$\text{Mean} = 4 + 4.5 + 5 + 5.5 + 6 + 6.5 + 7 + 7.5 + 8 + 8.5 + 9 = 71.5$$

$$\text{Mean} = 6.5$$

$$\text{Median} = 8.5, 9, 6.5, 4, 7.5, \underline{8}, 7, 6, 4.5, 5, 5.5 \quad (\text{11 numbers})$$

Median is The 6th number

$$\text{Median} = 8$$

$$\text{Mode} = 5.5 \quad (\text{most frequent size})$$

Q3. Consider the set of data below.

7 babies are weighed and weigh the following amounts:

2.5 kg, 3.1 kg, 3.4 kg, 3.5 kg, 3.5 kg, 4 kg, 4.1 kg

Find the interquartile range of the weights of the babies. (You need to find the lower quartile and the upper quartile.)

Babies weight

1. 2.5 kg
2. 3.1 kg
3. 3.4 kg
4. 3.5 kg
5. 3.5 kg
6. 4 kg
7. 4.1 kg

| Baby weight | f | cf |
|-------------|---|----|
| 2.5 kg | 1 | 1 |
| 3.1 kg | 1 | 2 |
| 3.4 kg | 1 | 3 |
| 3.5 kg | 2 | 5 |
| 4 kg | 1 | 6 |
| 4.1 kg | 1 | 7 |

(lower quartile)

class $Q_1 = 3$

$$3.1 + \left(\frac{1.75 - 3}{1} \right)$$

7

$\frac{7}{4} = 1.75$ define $m = 2$ therefore:

Lower quartile = 2nd from lower end = 3.1

Upper quartile = 2nd from upper end = 4

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 4 - 3.1 \\ &= 0.9 \end{aligned}$$

Q4. Consider the set of data below, shown in a grouped frequency distribution.
(Hint: work it out yourself with the formulae as discussed once you studied the slides' example)

$$\text{Median} = L_m + \left(\frac{\frac{n}{2} - F}{f_m} \right) i$$

$$L_m + \left(\frac{\frac{50}{2} - F}{f_m} \right) i$$

$i = 10$

| Time to travel to work | Frequency | Cumulative frequency |
|------------------------|-----------|----------------------|
| 1-10 | 8 | 8 |
| 11-20 | 14 | 22 |
| 21-30 | 12 | 34 |
| 31-40 | 9 | 43 |
| 41-50 | 7 | 50 |

frequency of class median.

Determine the estimated mean, median and mode and interquartile range.

$$n = (8 + 14 + 12 + 9 + 7 = 50)$$

$$\text{Median} = 25^{\text{th}} \text{ \& } 26^{\text{th}}$$

$$F \text{ column} = 21 - 30$$

$$\text{Median} = L_m + \left(\frac{\frac{n}{2} - F}{f_m} \right) i \quad (\text{class width})$$

$$= L_m + \left(\frac{25 - 22}{12} \right) 10$$

$$= L_m (0.25) 10$$

$$\text{Median} = L_m + 2.5$$

$$= 20.5 + 2.5$$

$$\text{Median} = 23$$

$$\text{Mode} = 11-20$$

$$L_m = \text{lower boundary}$$

$$10 + 11/2 = 10.5$$

$$L_m = 10.5$$

$$\Delta_1 = (14 - 8) = 6 \quad \Delta_2 = (14 - 12) = 2$$

$$i (\text{width}) = 10$$

$$\text{mode} = 10.5 + \left(\frac{6}{6+2} \right) 10$$

$$\text{MODE} = 18$$

F = Cumulative frequency before class median

f_m = frequency of class median

$$\text{Mode} = L_m + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2} \right) i$$

lower boundary

different frequency class and frequency class before class median

Interquartile Range.

$$\text{Class } Q_1 \quad \frac{50}{4} = 12.5$$

Q1 is 2nd class 12.5 fits in (11-20)

$$Q_1 = 10.5 + \left(\frac{12.5 - 8}{14} \right) 10$$

lower boundary

$$10.5 + \left(\frac{4.5}{14} \right) 10$$

$$10.5 + 3.21428$$

$$Q_1 = 13.7142857$$

Q3 is 2nd class from bottom (4th) (31-40)

$$Q_3 = 30.5 + \left(\frac{3(50)}{4} - 34 \right) 10$$

(frequency 40)

$$IQR = Q_3 - Q_1$$

$$IQR = 34.3889 - 13.7143$$

$$3(50)$$

$$30.5 + \left(\frac{150}{4} \right) = 37.5$$

$$30.5 + \left(\frac{37.5 - 34}{9} \right) 10$$

$$0.3889$$

$$= 34.3889$$

← frequency mid point class

| Time to travel | f | x | fx |
|----------------|----|------|-------|
| 1-10 | 8 | 5.5 | 44 |
| 11-20 | 14 | 15.5 | 217 |
| 21-30 | 12 | 25.5 | 306 |
| 31-40 | 9 | 35.5 | 319.5 |
| 41-50 | 7 | 45.5 | 318.5 |

$$f \times 50 \text{ (add 110) } = 1205$$

MEAN

$$\bar{x} = \frac{\sum fx}{n} = \frac{1205}{50} = 24.1$$

$$x \leq h < 150$$

less than or equal to \underline{h}

Q5. The table below gives data on the heights, in cm, of 51 children.

| | | | | |
|----------------|--------------------|--------------------|--------------------|--------------------|
| Mid point | 145 | 155 | 165 | 175 |
| Class Interval | $140 \leq h < 150$ | $150 \leq h < 160$ | $160 \leq h < 170$ | $170 \leq h < 180$ |
| Frequency | 6 | 16 | 21 | 8 |

(a) Estimate the mean height.

(b) Find the median class.

(c) Find the modal class.

Mean height

$$\frac{8215}{51} \quad \text{(Total (Midpoint} \times \text{Frequency))}$$

| | | | | | |
|------------------------------|-----|------|------|------|--------|
| Class Interval | 145 | 155 | 165 | 175 | Total |
| Mid points | 145 | 155 | 165 | 175 | 51 |
| Frequency | 6 | 16 | 21 | 8 | |
| Mid point \times Frequency | 870 | 2480 | 3465 | 1400 | = 8215 |

$$\text{Mean} = 161 \text{ cm}$$

(b) Median class = 26th ($160 \leq h < 170$) class

(c) Modal class = 21 frequency from $160 \leq h < 170$ class.

1. The first part of the paper is devoted to a general discussion of the problem of the existence of a solution of the system of equations (1) for arbitrary values of the parameters α and β . It is shown that the system has a solution if and only if the condition $\alpha + \beta = 1$ is satisfied. In this case the solution is unique and is given by the formula

$$x = \frac{1}{\alpha + \beta} \left(\alpha x_1 + \beta x_2 \right)$$
where x_1 and x_2 are the solutions of the system of equations (1) for $\alpha = 1$ and $\beta = 0$ and $\alpha = 0$ and $\beta = 1$ respectively. The second part of the paper is devoted to a study of the properties of the solution (2) for arbitrary values of the parameters α and β . It is shown that the solution (2) is a linear function of the parameters α and β and that it is a harmonic function of the parameters α and β .