

Class Exercise 4

Student's Name: Andrew CraftBoolean Algebra and Digital circuits

\vee (or) \wedge (And)
 \neg (NOT)
 \rightarrow if then (conditional)
 \leftrightarrow if and only if. (biconditional)

Question 1:

Using truth table determine whether the following two statements are logically equivalent:

$\neg(p \rightarrow q)$ and $p \wedge \neg q$
 (Not q) (Not p)

Explain how you know you are correct.

Answer: yes logically equivalent

\neg (NOT negation) $(p \rightarrow q)$ (if...then)
 Conditional

$p \wedge \neg q$
 AND Conjunction
 NOT (negation)

(Not q) \rightarrow (p \wedge \neg q)

p	q	$\neg q$	$p \wedge \neg q$
F	F	T	F
F	T	F	F
T	F	T	T
T	T	F	F

(Not p) \rightarrow (Not q)

p	q	$\neg(p \rightarrow q)$
F	F	F
F	T	F
T	F	T
T	T	F

p	q	$\neg q$	$p \wedge \neg q$
0	0	1	
0	1	0	
1	0	1	
1	1	0	

Question 2:

Make a truth table for the statement

$$\neg p \rightarrow (q \wedge r)$$

p	q	r	$(\neg p \rightarrow (q \wedge r))$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

Question 3:

Let p , q , and r be the following propositions:

p : You get an A on the final exam

q : You do every exercise in the book.

r : You get an A in this class.

Write the following formulas using p , q , and r and logical connectives.

1. You get an A in this class, but you do not do every exercise in the book.
2. To get an A in this class, it is necessary for you to get an A on the final.
3. Getting an A on the final and doing every exercise in the book is sufficient for getting an A in this class.

1. $r \wedge \neg q$

2. $r \rightarrow p$ (if)

3. $(p \wedge q) \rightarrow r$

Question 4:

State the converse and contrapositive of each of the following implications.

- a) If it snows today, I will stay home.
- b) We play the game if it is sunny.
- c) If a positive integer is a prime then it has no divisors other than 1 and itself.

a. Converse - If I stay home, it snows today
Contrapositive - If I do not stay home, it does not snow today

b. Converse - If we play the game, then it is sunny
Contrapositive - If we don't play the game, it is not sunny

c. Converse - If positive integer has no divisors other than 1 and itself
it is a prime
Contrapositive - If positive integer has divisors other than 1 and itself
then it is not a prime.

Question 5 (Next week in Week 5):

Which of the truth tables (i , ii) below represents the behaviour of NAND gate?

INPUT		OUTPUT
A	B	
0	0	1
1	0	1
0	1	1
1	1	0

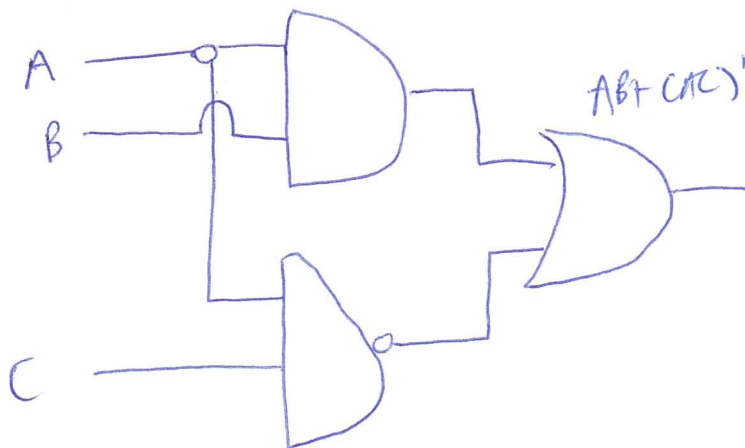
INPUT		OUTPUT
A	B	
0	0	0
1	0	0
0	1	0
1	1	1

AND Gate

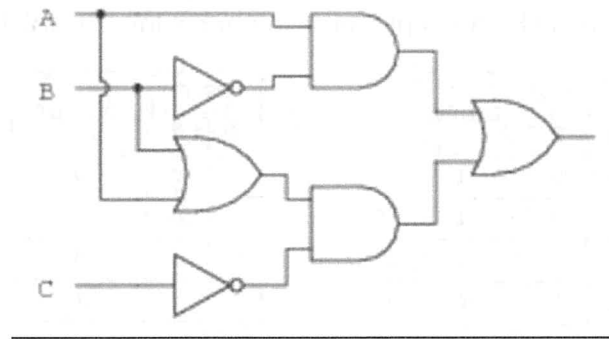
Answer: i
NAND Gate

ii

Draw a circuit diagram corresponding to the following Boolean expression
 $AB + (AC)'$



An engineer hands you a piece of paper with the following gate circuit and asks you what could be the equivalent Boolean expression on it:



$$A B' + C' (C A + B)$$

Question 6 (Next week in Week 5):

Using the laws of logic prove that the statements $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent. (Hint: first you can use conditional law and logically equivalent statement for it)

Conditional/implication law

$$\neg(p \rightarrow q)$$

implication as disjunction logically equivalent

$$\neg(\neg p \vee q)$$

de Morgan's law

$$p \wedge \neg q$$

double negation

$$\text{now: } \neg(p \rightarrow q) \equiv p \wedge \neg q$$

~~Condition law~~

Condition law $p \rightarrow q \equiv \neg p \vee q$

p	q	$\neg(p \rightarrow q)$	$p \wedge \neg q$
1	0	1	1
1	1	0	0
0	0	1	0
0	1	0	0

