$$\underbrace{1000n^2 + 1000n}_{f(n)} = O(\underbrace{n^2}_{g(n)})$$

$\exists\ c, n_0$ such that:

1) $1000n^2 + 1000n \leq c \cdot n^2 \quad / \div n^2$

$1000 + \frac{1000}{n} \leq c \Rightarrow c \geq 1000 + 1000$

$\boxed{n_0 = 1}$

$c \geq 2000$

$\boxed{c = 2000}$

2) $\overbrace{1000n^2 + \underbrace{1000n}_{\leq 1000n^2\ \forall n \geq 1}}^{f(n)} \leq 1000n^2 + 1000n^2 = \underbrace{2000}_{c}n^2$

$\boxed{n_0 = 1}$ $\boxed{c = 2000}$

$100n + 5 \neq \Omega(n^2)$

Proof by contradiction:

$\exists\, c, n_0$ such that $100n + 5 \geq cn^2$

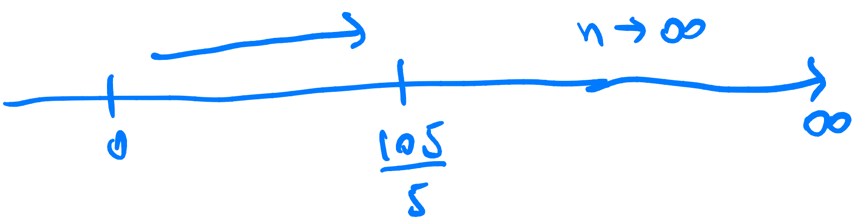
$$cn^2 \leq 100n + \underbrace{5}_{\leq 5 \cdot n \;\forall n} = 105n$$

$$cn^2 \leq 105n \Rightarrow cn^2 - 105n \leq 0$$

$$\Rightarrow \underbrace{n}_{\geq 0} \cdot \underbrace{(cn - 105)}_{\leq 0} \leq 0 \Rightarrow cn - 105 \leq 0 \Rightarrow \boxed{n \leq \frac{105}{c}}$$

contradiction

$n \to \infty$



$$\frac{n^2}{2} - \frac{n}{2} = \Theta(n^2)$$

$\exists\ c_1, c_2$ such that $c_1 n^2 \leq \frac{n^2}{2} - \frac{n}{2} \leq c_2 n^2$

1) $c_1 n^2 \leq \frac{n^2}{2} - \frac{n}{2}$ $\quad / \div n^2$

$c_1 \leq \frac{1}{2} - \frac{1}{2n} = \frac{1}{4}$ $\quad \boxed{n_0 = 2}$

$c_1 \leq \frac{1}{4} \Rightarrow \boxed{c_1 = \frac{1}{4}}$

2) $\frac{n^2}{2} - \frac{n}{2} \leq c_2 n^2$ $\quad / \div n^2$

$\frac{1}{2} - \frac{1}{2n} \leq c_2$

$$\boxed{\begin{array}{l} n_0 = 2 \\ c_2 = \frac{1}{4} \end{array}}$$