Section 1.1: Propositional Logic

Objectives: Students will be able to understand and explain:

- The language of propositions, including connectives, truth values, and truth tables
- Applications such as translating English sentences, system specifications, logic puzzles, and logic circuits
- Logical equivalences that include important equivalences, showing equivalences, and satisfiability

Propositional Logic

A ____________ is a declarative sentence that is either true or false.

Example: Which of the following are propositions?

a) **The Moon is made of green cheese.**
b) **Sit down!**
c) **Carson City is the capital of Nevada.**
d) **What time is it?**
e) **San Francisco is the capital of California.**
f) **$1 + 1 = 2$**
g) **$x + 3 = 10$**
h) **$0 + 3 = 0$**
i) **$x + y = z$**

Constructing Propositions

We use letters (typically p, q, r, s,…) to denote _______________________ (or sentential variables).

Let p be a proposition. The *negation of p*, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

Note: Other notations for negation include:

Example: Find the negation of the proposition and express this in simple English.

a) **Max's car has four-wheel drive.**

b) **Elaina's computer is a MacBook.**

Truth Tables

The **truth value** of a proposition is true, denoted ____, if it is a true proposition, and the truth value of a proposition is false, denoted ____, if it is a false proposition.

The **Truth Table** for the Negation of a Proposition

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

The **Truth Table** for the Conjunction of Two Propositions

Note: The word "but" sometimes is used instead of "and" in a conjunction.

"The sun is shining, but it is raining." In other words…

Example: Find the conjunction of the propositions *p* and *q* where *p* is the proposition "I bought a lottery ticket this week" and *q* is the proposition "I won the million dollar jackpot."

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

The **Truth Table** for the Disjunction of Two Propositions

Note: A disjunction is true when at least one of the two propositions is true.

Example: Translate the statement "Students who have taken calculus or introductory computer science can take this class" in a statement in proposition logic.

Example: Find the disjunction of the propositions *p* and *q* where *p* is the proposition "I bought a lottery ticket this week" and *q* is the proposition "I won the million dollar jackpot."

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$ (or p XOR q), is the proposition that is true when exactly one of p and q is true and is false otherwise.

The **Truth Table** for the Exclusive Or of Two Propositions

Example: Let *p* and *q* be the propositions that state "A student can have a salad with dinner" and "A student can have soup with dinner," respectively. What is the exclusive or of *p* and *q*?

Example: Express the statement "I will use all my savings to travel to Europe or to buy an electric car" in propositional logic.

Conditional Statements

Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

Note: A conditional statement is also called a ______________.

The **Truth Table** for the Conditional Statement

Expressions for a conditional statement:

Example: Let *p* be the statement, "TY gets 100% on the final" and *q* the statement, "TY will get an A in the course". Express the statement *p*□*q* as a statement in English.

Note: In mathematical reasoning, we consider conditional statements of a more general sort than we use in English. Our definition of a conditional statement specifies its truth values; it is not based on English usage. Propositional language is an artificial language; we only parallel English usage to make it easy to use and remember.

Example: "If Ethan has a smartphone, then 2+3 = 5."

Example: "If Ethan has a smartphone, then 2+3=6."

Converse, Contrapositive, and Inverse

The proposition__________ is called the ____________ of $p \rightarrow q$.

The proposition _________ is called the _____________ of $p \rightarrow q$.

The proposition _________ is called the _____________ of $p \rightarrow q$.

- The **contrapositive**, but neither the converse or inverse, of a conditional statement is equivalent to it.

- When two compound propositions always have the same truth tables, regardless of the truth values of its propositional variables, we call them ___________.

Example: Find the contrapositive, the converse, and the inverse of the conditional statement, "The home team wins whenever it is raining."

Biconditionals

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

The **Truth Table** for the Biconditional

Expressions for a Biconditional Statement:

Example: Determine whether each of these biconditionals are true or false.

a) **2+2=4 if and only if 1+1=2.**

b) **1+1=2 if and only if 2+3=4.**

c) **1+1=3 if and only if monkeys can fly.**

d) **0>1 if and only if 2>1.**

Implicit Use of Biconditionals

We will always distinguish between the conditional statement *p*□*q* and the biconditional statement $p \leftrightarrow q$.

Why?

Truth Tables of Compound Propositions

Example: Construct the truth table of the compound proposition

$$(p \vee \neg q) \rightarrow (p \wedge q)$$

Precedence of Logical Operators

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Operator	Precedence

Example:

Logic and Bit Operations

Computers represent information using bits. A ____ is a symbol with two possible values, namely 0 (zero) and 1 (one).

Truth Value	Bit

Table for Bit Operators OR, AND, and XOR

A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

Example: 10101001 is a bit string of length ____.

Bit Strings

Example: Find the bitwise *OR,* bitwise *AND,* and bitwise *XOR* of the bit strings 01 1011 0110 and 11 0001 1101.