Section 1.2: Applications of Propositional Logic

Objectives: Students will be able to understand and explain:

- Translating English to Propositional Logic
- System Specifications
- Boolean Searching
- Logic Puzzles
- Logic Circuits (Introduction)

Translating English Sentences

Example: How can the following English sentences be translated into a logical expression.

a) **"You can access the Internet from campus only if you are a computer science major or you are not a freshman."**

b) **"You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."**

System Specifications

System specifications should be _____, that is, they should not contain conflicting requirements that could be used to derive a contradiction.

Example: Express the specification below using logical connectives.
"The automated reply cannot be sent when the file system is full."

Example: Determine whether these system specifications are consistent.
    a) **"The diagnostic message is stored in the buffer or it is retransmitted."**

    b) **"The diagnostic message is not stored in the buffer."**

    c) **If the diagnostic message is stored in the buffer, then it is retransmitted."**

Boolean Searches

In **Boolean searches**, the connective _____ is used to match records that contain both of two search terms, the connective _____is used to match one or both of two search terms, and the connective _____ (sometimes written as *AND NOT*) is used to exclude a particular search term.

Example of a Boolean search

Web Page Searching   Most Web search engines support Boolean searching techniques, which is useful for finding Web pages about particular subjects. For instance, using Boolean searching to find Web pages about universities in New Mexico, we can look for pages matching NEW *AND* MEXICO *AND* UNIVERSITIES. The results of this search will include those pages that contain the three words NEW, MEXICO, and UNIVERSITIES. This will include all of the pages of interest, together with others such as a page about new universities in Mexico. (Note that Google, and many other search engines, do require the use of "AND" because such search engines use all search terms by default.) Most search engines support the use of quotation marks to search for specific phrases. So, it may be more effective to search for pages matching "NEW MEXICO" *AND* UNIVERSITIES.

Logic Puzzles

Puzzles that can be solved using _____ are known as **logic puzzles**.

Example of a Logic Puzzle

A father tells his two children, a boy and a girl, to play in their backyard without getting dirty. However, while playing, both children get mud on their foreheads. When the children stop playing, the father says "At least one of you has a muddy forehead," and then asks the children to answer "Yes" or "No" to the question: "Do you know whether you have a muddy forehead?" The father asks this question twice. What will the children answer each time this question is asked, assuming that a child can see whether his or her sibling has a muddy forehead, but cannot see his or her own forehead? Assume that both children are honest and that the children answer each question simultaneously.

Logic Circuits (Introduction)

A **logic circuit** (or **digital circuit**) receives input signals ___, ___, …, ____, each a bit [either 0 (off) or 1 (on)], and produces output signals ___, ___, …, ___, each a bit. We will restrict our attention to logic circuits with a single output signal.

There are three basic circuits, called _____. The _____, or **NOT gate**, takes an input bit p, and produces as output _____. The **OR gate** takes input signals p and q, each a bit, and produces as output the signal _____. The **AND gate** takes two input signals p and q, each a bit, and produces as output the signal _____.

Inverter   OR gate   AND gate
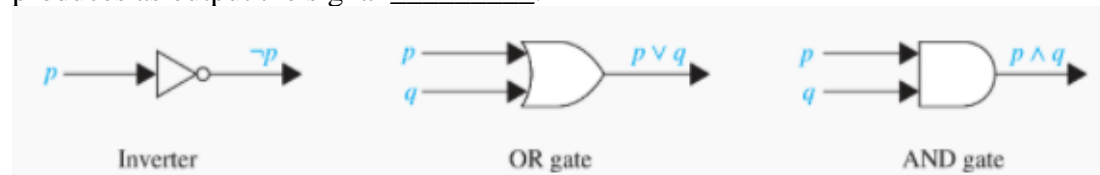
Example: Determine the output for the combinatorial circuit below.
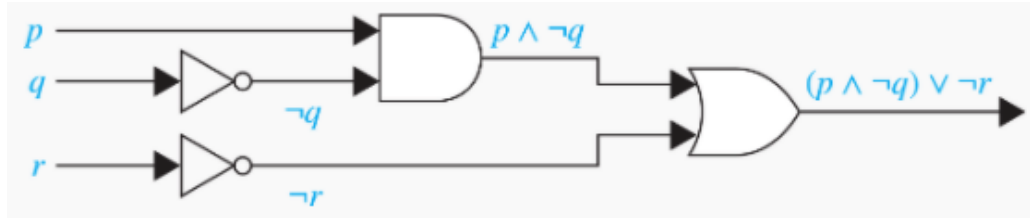
Example: Given input bits p, q, and r, build a digital circuit that produces the output
$$(p \lor \neg r) \land (\neg p \lor (q \lor \neg r))$$