$H(t) = \sum_{i < j} h_{ij}(t) \|h_{ij}(t)\| \leq 1/r_{ij}^{\alpha} \|[A(t), B]\|ABXYXY|X|, |Y| = \emptyset 1 t_{si} XY t \delta = \Theta(1)ABXY\|[A(t), B]\| > \delta ?$ 

```
????non-unitarily??????
                                             a priori?dt?t = \emptyset1?t = rr?\rho d\rho/dt = (\rho)
                                             \mathbf{L}(t)\Lambda\mathcal{HB}(\mathcal{H})\mathcal{H}O \in \mathcal{B}(\mathcal{H})\langle O(t)\rangle = [O(t)\rho] = [O\rho(t)]\rho\ddot{o}\rho(t) = e^{\mathbf{L}t}\rho OO(t) = e^{\mathbf{L}^{\dagger}t}O\mathbf{L}^{\dagger}\mathbf{L}^{\dagger}O = +i[H,O] + \sum_{i}\left[L_{i}^{\dagger}OL_{i} - \frac{1}{2}\{L_{i}^{\dagger}OL_{i} - \frac{1}{2}\{
                                               HL_i
                                             LL = \sum_{Z \subset \Lambda} L_Z i, j \sum_{Z \ni i,j} \|L_Z\|_{\infty} \sup_{O \in \mathcal{B}(\mathcal{H})} \frac{\|L_{ZO}\|}{\|O\|} \le \frac{1}{dist(i,j)} \|\cdot\|_{\infty} \|\cdot\|_{\infty} \infty \infty \infty dist(i,j)ij
??> dA \in \mathcal{B}(X)XK_Y \in L_Y Y e^{L^{\dagger}t}_Y (e^{L^{\dagger}t}A) \le C \|K_Y\|_{\infty} AXY \left(\frac{e^{vt}-1}{r^{\alpha}}\right), rd(X,Y)Cv \emptyset 1K_Y K_Y(O_X) = i[O_X, O_Y] \|K_Y\|_{\infty} 2x^{\alpha} \|K_Y\|_{\infty} \|K_
                                             v \propto 2\infty?_Y(e^{\mathbf{L}^{\dagger}t}A) \le C||K_Y||_{\infty}AXY\left(\frac{e^{vt}}{[(1-\mu)r]^{\alpha}}\right)
 + e^{vt - \mu r} \Big), \mu \in (0, 1) vv?k?? \le d?N \Lambda JJ = N^{1 - /d} < dJ = \log N = d??_Y(e^{\mathbf{L}^{\dagger} t} A) \le C \|K_Y\|_{\infty} AXY\left(\frac{e^{Jt} - 1}{Jr^{\alpha}}\right).N^{-1} + e^{vt - \mu r} \Big)
                                             > 2d?_Y(e^{\mathbf{L}^{\dagger}t}A) \le C||K_Y||_{\infty}A\frac{t^{\alpha-d}}{r^{\alpha-2d}}.
                                               d = 1 > 3 > 3d = 1?1/r1/r^{\alpha - 2}?H = \sum_{ij} H_{ij}{}^{iHt}Ae^{-iHt}, B] \le CAB\frac{t}{r}, B \in \mathcal{B}(Y)YL = \sum_{ij} L_{ijY}(e^{\mathbf{L}^{\dagger}t}A) \le CK_{Y\infty}A\frac{t}{r^{\alpha - 2 - \alpha}}
                                               L^{\dagger}AX \in rX \in truncated).
    AXL^{\dagger}L^{\dagger}rXC(r,t)
                                                 AX \in LLrX\tilde{A}(t)AL^{\dagger}A(t)\tilde{A}(t)\|A(t) - \tilde{A}(t)\| \le K\|A\|C(r,t),KC(r,t)r\alpha > d
    \leq dN||r \propto N^{1/d}
    _{t}runcated._{t}runcated follows straightforwardly from the open-system Lieb-Robinson bounds detailed in sec: open-LR. In part of the contract of the contr
                                               \stackrel{L}{\lambda} > 0 \stackrel{primitive}{}{}
                                               fg
 f_t g_t f_g \mathcal{L}^{\dagger}_{s??} static
\lambda \mathbf{L} f_t, g_t \\ [\sigma f_t] = [\sigma f] f_{\sigma}(f, g) - Cov_{\sigma}(f_t, g_t) |
= \frac{1}{2} (|[(fg - f_t g_t)\sigma] + [(gf - g_t f_t)\sigma]|)
    = \frac{1}{2}(|[((fg)_t - f_tg_t)\sigma] + [((gf)_t - g_tf_t)\sigma]|)
    \leq \frac{1}{2}(\|(fg)_t - f_t g_t\| + \|(gf)_t - g_t f_t\|)
   \leq Kfg\mathcal{C}(r,t), rd(X,Y)\mathcal{C}(r,t)_t runcated. Specifically, we use the following Lemma, which is itself are statement of Corollary 7 in f, gXY d\Lambda Ls \sigma \lambda c > 0 \lambda, v
   t
   \lambda' = 2\lambda, K' = K/4
                                             \mathcal{C}(r,t)dh/dt = 0_{c}orr_{m}inimization leads to a minimum at time \bar{t} = -\left(\frac{1}{\lambda'+v}\right)\log\left(\frac{K'v}{\lambda r^{\alpha-d}}\right).(9)\bar{t}) = \left(\frac{K'v}{\lambda'r^{\alpha-d}}\right)^{\frac{\lambda'}{\lambda'+v}} + \frac{K'}{r^{\alpha-d}}\left(\frac{K'v}{\lambda'r^{\alpha-d}}\right)^{\frac{\lambda'}{\lambda'+v}} + \frac{K'}{r^{\alpha-d}}\left(\frac{K'v}{\lambda'r^{\alpha-d}}\right)^{\frac{\lambda'}{\lambda'+v}} + \frac{K'v}{r^{\alpha-d}}\left(\frac{K'v}{\lambda'r^{\alpha-d}}\right)^{\frac{\lambda'}{\lambda'+v}} + \frac{K'v}{r^{\alpha-d}}\left(\frac{K'v}{\lambda'r^{
   \leq c \left(r^{\alpha-d}\right)^{\frac{-2\lambda}{v+2\lambda}} c\lambda, v, Kf, gT_{\sigma}(X:Y)\alpha > d
```

$$\begin{split} t^* &= 1 + \log(r^{\beta})\mathcal{C}(r,t) \propto t^{\alpha-d+1}/r^{\alpha-3d}t^*) = e^{-\lambda(1+\log(r^{\beta}))} + K\frac{(1+\log(r^{\beta}))^{-d+1}}{r^{-3d}} \\ &= \frac{e^{-\lambda}}{r^{\lambda\beta}} + K\frac{(\beta\log(r))^{-d+1}}{r^{-3d}} + \mathcal{O}\frac{\log^{-d}(r)}{r^{-3d}}. \\ \beta &= (-3d)/\lambda > 3d^*) = \frac{e^{-\lambda} + K\left(\frac{\alpha-3d}{\lambda}\log r\right)^{\alpha-d+1}}{r^{\alpha-3d}} + \mathcal{O}\frac{\log^{-d}(r)}{r^{-3d}} \\ &= K\left(\frac{-3d}{\lambda_*}\right)^{-d+1}\frac{\log^{-d+1}(r)}{r^{-3d}} \end{split}$$