

Solving the Granularity Problem in Portfolio Optimization

Hypothesis

Portfolios constructed with Integer Quadratic Programming (IQP) under the whole-shares constraint exhibit superior risk and return characteristics compared to those formed using Mean-Variance Efficient (MVE) portfolio optimization, which assumes fractional shares, because the rounding process in MVE to convert fractional shares to whole shares introduces unwanted exposures that may impact portfolio performance.

Experimental Design

Data Collection

- Generate a synthetic dataset of varying portfolio sizes for testing purposes
- Gather historical price and return data for stocks in the S&P 500.
- Gather historical price and return data for stocks in the Russel 3000.

MVE Implementation

- Implement a standard MVE portfolio optimization without integer constraints.

Rounding Implementations

- Create portfolios from the MVE construction but implement different rounding mechanisms for comparison.

IQP Model Implementation

- Implement the IQP model using a solver like Gurobi.
- Experiment with different objective function formulations like expected return, portfolio variance, and lambda adjusted risk and return.

Experiments

1. Scalability Analysis

- Vary the number of assets in the portfolio (e.g., 10, 50, 100, 200).
- Measure computation time and solution quality.
- Replicate research already done on the cardinality problem.
- Identify the practical limits of the IQP approach.

2. Performance Comparison

- Compare the IQP solution to the benchmark methods.
- Analyze differences in expected return and risk.

3. Budget Constraint Sensitivity

- Vary the budget constraint to simulate different investment amounts.
- Introduce additional constraints (e.g., sector exposure limits) and observe their impact.

4. Back Test Performance

- Use rolling window backtests to evaluate the performance of IQP portfolios.
- Compare Sharpe ratios, maximum drawdowns, and other performance metrics.

5. Rebalancing Strategies Analysis

- Implement periodic rebalancing using the IQP model.
- Compare different rebalancing frequencies (e.g., monthly, quarterly, annually).