An Economic Analysis of the Monopoly Board Game

By Andrew Haruki Hill

Almost every American family has joined together to play the Monopoly board game. Its first version was created by Elizabeth Magie, who wanted the game to reflect her views on a prevailing tax method called "land-value tax". She was deeply invested in the current economic situation at the turn of the century and was critical of the prevailing monopolists of the time. In fact, she designed the first version of Monopoly (which she called "The Landlord's Game") "as a teaching tool to promote the economic theories of Henry George, whose progressive ideas on both taxation and women's rights remained Magie's lifelong passion."[1] A man named Charles Darrow stole the idea and sold it to the Parker Brothers, who released it as the game we know and love. A game that has such deep-rooted economic origins deserves an economic explanation.

The Game

The game lends itself to an economic study because it, along with other games, is subject to constraints and rules. First, there is the board, which has 40 spaces comprised of properties, and features. The properties are made up of three groups: Simple Properties (which are further grouped into colors), Utilities, and Railroads. The feature spaces are made up of Go, Chance/Community Chest spaces, Jail, Go To Jail, Income Tax/Luxury Tax, and Free Parking. When players pass Go, the starting space on the board, they receive $200. After landing on a Chance or Community Chest space, the player must draw a card from one of the respective decks which either monetarily penalize or help a player or send them to another space on the board. The Jail space forces the player to stay on the space for three turns and the Go To Jail space sends the player to Jail. The Tax spaces simply fine players who land on them and the Free Parking space has not effect. Players navigate through those spaces by rolling two dice and travelling the number of spaces indicated by the addition of dice. If a player rolls the same

[1] Wagner, Erica. "Do Not Pass Go: The Tangled Roots of Monopoly." *New Statesman*. 24 June 2015. Web. 24 Jan. 2016. <http://www.newstatesman.com/culture/2015/06/do-not-pass-go-tangled-roots-monopoly>.

number with each die then they get to roll again, but if they roll doubles three times in a row[2], they are sent to Jail. There are more rules, but they do not directly influence the study.

The Data

I wrote a script[3] in Python, a programming language, that simulates Monopoly. In the simulation a player rolls two dice and moves across the board while recording its location after every roll. The player is subject to all of the features that are in the game. After rolling 1,000,000,000 times, I computed the probability of landing on each space.

In the appendix, Figure 2 contains a table with the location, name, and probability of landing on each property. For easy reference, Figure 1 displays a classic Monopoly board. Below is a graphic of the probability of landing each space[4]. The graphic omits the "Go to Jail" space which has a zero percent probability of landing because it immediately sends the player to the "Jail" space.

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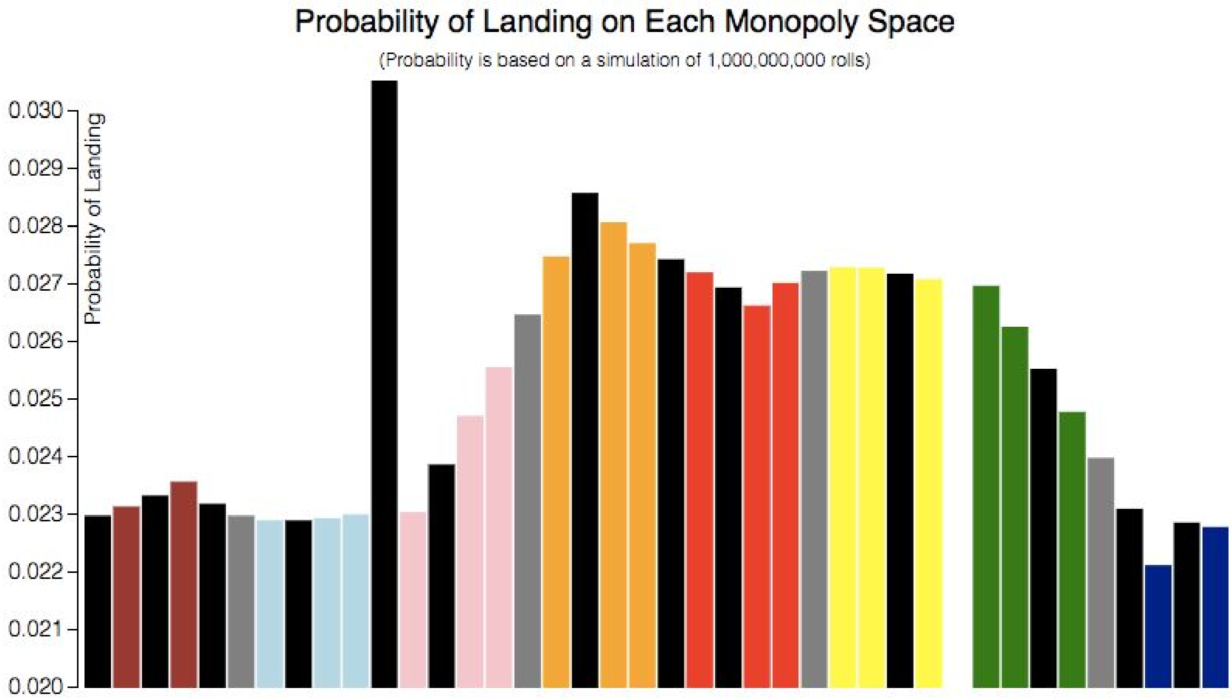
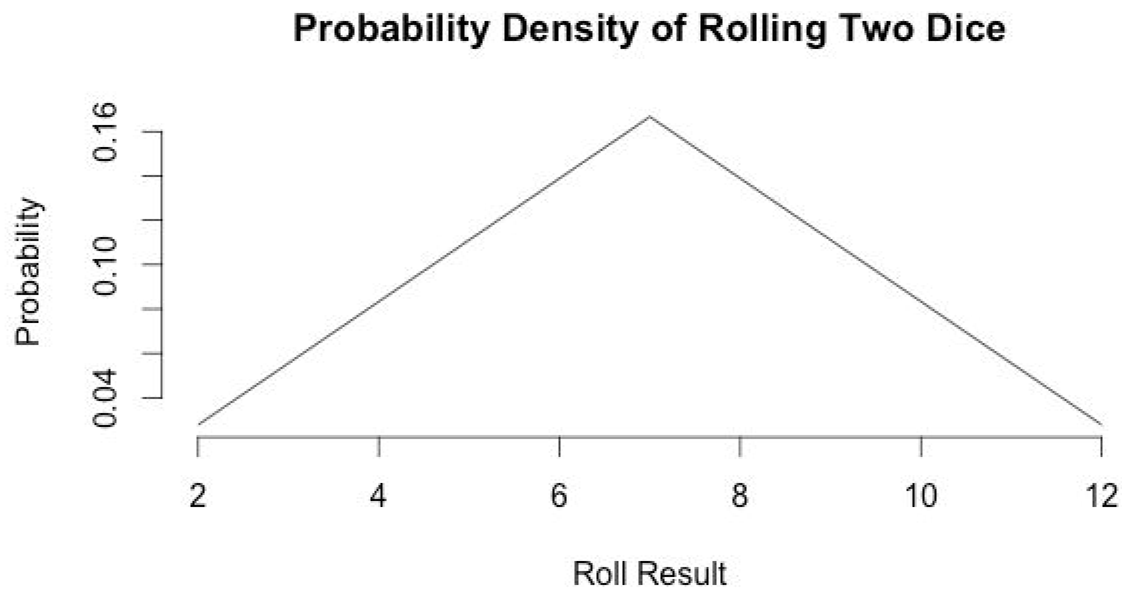
[2] A phenomenon whose theoretical probability is only 0.0046

[3] The code for the simulation can be found at: https://github.com/harukihill/monopoly-project/tree/master

[4] The interactive version of the graphic can be found at: https://bl.ocks.org/harukihill/21391f8d58be850d70a06e14b23fd556



Some interesting data points arise from the simulation. High-probability properties are mostly grouped in the northwest corner of the board. If the dice were rolled 1,000,000,000 times on a board without features, the probabilities would be equal amongst all spaces. However, the features disturb that uniform probability density. The inclusion of Jail, in which players are locked up 1 in 37.4 rolls (or 2.68% of rolls), causes more players to start their turn from the southwest corner on the tenth space. This effect is even more powerful considering there are Chance and Community Chest cards that send players to Jail. With minor variations, the probabilities of landing in the northwest corner properties closely resemble the probability density of rolling two dice, shown below, compounded with the even distribution mentioned in the board with no rules or cards.



The Community Chest space in the northwest side of the board owns the highest landing probability of all spaces, with 2.832% of all rolls. It is no mistake that it is exactly seven spaces away from Jail, matching the peak of the Probability Density graphic above.
Examination of the probability densities of two dice and how they can compound on one another help explain why players land on some spaces more than others. The highest roll probability for properties belongs to Tennessee Avenue, which is one space ahead of the northwest Community Chest. Theoretically, both the 16th and 18th space should share the same probability because the probabilities of rolling a six and an eight with two dice are equal. But, Tennessee Avenue (space 18) holds a slight edge over St. James Place (space 16) because of Chance cards that move a player to Electric Company on space twelve and Pennsylvania Railroad on space fifteen. The probability densities of those two spaces are also compounded with the two before and their peaks are much closer to Tennessee Avenue than St. James Place.

The same logic of compounded probabilities can be used to explain why there is a distinct hump on the Probability Density Graph for the third space, Baltic Avenue. The probability of landing on this space contains the compounded probabilities of Go (space 0), Boardwalk (space 39), and Short Line (space 35), all properties that Chance and Community Chest cards lead to.

The compounded probabilities can also explain why there is a noticeable dip on the 23rd space, Indiana Avenue. This property is thirteen spaces away from Jail and does not fall within the compounded probabilities because two dice can at most add up to twelve. The small bump right before the 24th space, Illinois Avenue can be explained by the Chance card that sends a player straight to the property.

The steep dropoff after the thirtieth space, Go To Jail, occurs simply because once a player lands on that space, they are immediately sent to the opposite side of the board. It is no mistake that the space with the lowest roll probability is seven spaces after Go To Jail, Park Place. The simulation suggests players should end up on Park Place a measly 2.186% of rolls. The probability density picks up right after and trends upward until the final space, Boardwalk.

One could reasonably tailor their Monopoly strategy to owning as many high-roll probability spaces as possible while avoiding low-roll probability spaces. But playing this way would ignore most of the other constraints of the game.

Four main factors directly influence a property's value. The first is a property's rent. Each property has different rents associated with it that opposing players have to pay if they land on the space. It's obvious that Boardwalk, which has a $50 rent, is more valuable, in that respect, than Mediterranean Avenue, which has a $2 rent. The second is a property's cost. Players may be hesitant to splurge on Boardwalk, which costs $400, but would have no problem acquiring Mediterranean Avenue, which costs $60. The third factor is the probability of landing on a property. The probability of landing on a space determines how many times opposing players will pay rent and give you money. Lastly, value can also be determined by the probability of monopolizing a set of properties, therefore allowing a player to build more houses and reap the benefits of astronomically high rents.

Property Value Score

In order to accurately estimate the value of properties, I created Property Value Score (PVS). PVS takes in those factors to calculate a score for each property, including Utilities and Railroads. I use a dimensionless value to compare properties because it is impossible to calculate a true dollar value for each property. Hypothetically, that true dollar value would be composed of the rent, probability of landing, and the number of rolls left in a game. However, the rules do not enforce a finite number of turns, so the game could last ad infinitum. The competitive nature and rules of Monopoly prevents the calculation and simulation of a proper estimate of game length.

The formulation of PVS uses the probability of landing, rent, cost, probability of monopolization, rent from houses and hotels, cost of houses and hotels, and number of houses or hotels as variables.

The variable R represents the rent gained from an opposing player and the same variable with a subscript, R_h , is the rent value with houses or a hotel on the property. The same variation applies to the variables C ,which represents the cost of a house, and C_h is the cost of each house. P(*land*) is the probability of landing on the space is applied to the whole equation because the main way a property can give value to its owner is if an opponent lands on it and pays rent. The probability of monopolization, P(*monopoly*), is the product of the P(*land*) of the other properties of the same color (the model assumes the probability of landing on one space is independent of the probability of landing on another). n represents the number of houses a property has in the property PVS equation and represents the number of railroads owned in the Railroad PVS equation. One peculiarity of the railroad equation is the P(*other railroads*). It represents the average probabilities of landing for three other railroads other than the railroad for whom PVS is calculated. The average is taken because it does not matter which extra railroad is owned. The average probability is raised to the power of n, the number of railroads owned, in order to appropriately weigh the rent and costs of the extra railroad. All of these equations are scaled upwards by a factor of one thousand for clarity.

Below are the mathematical formulations for the Property Value Scores of ownable spaces in Monopoly.

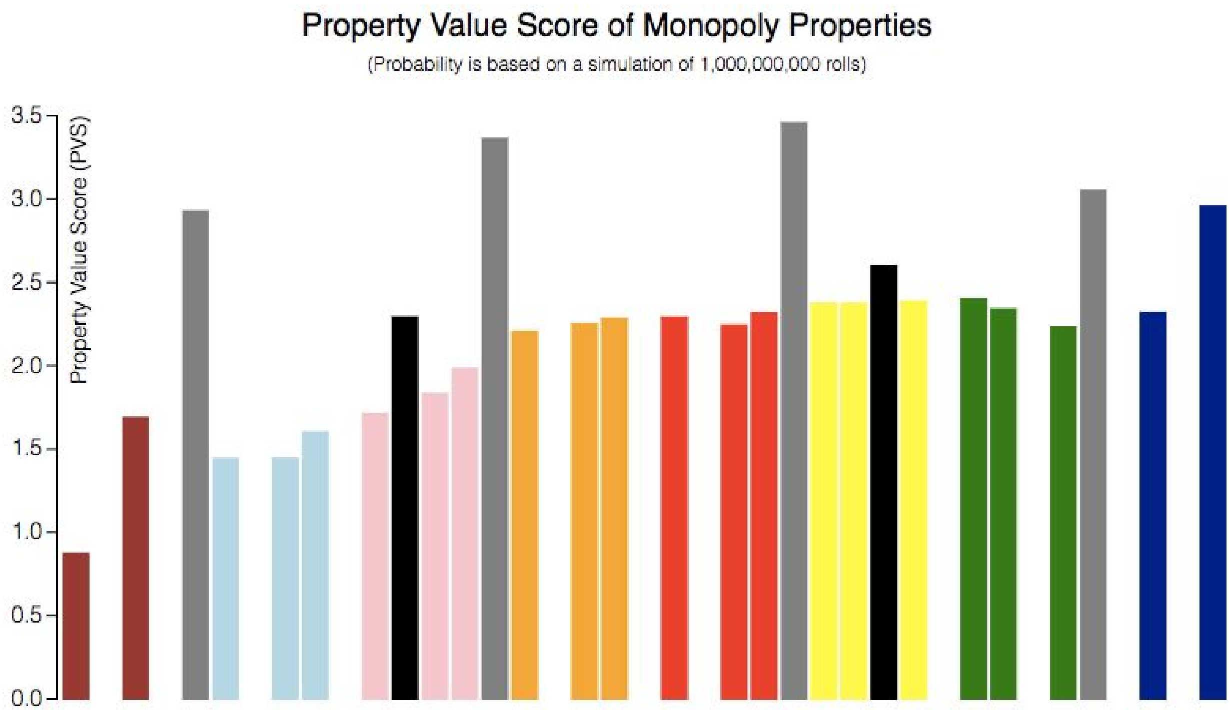
$$PVS(property) = P(land)[\frac{R}{C} + P(monopoly)\sum_{n=1}^{5}\frac{R_h}{C+nC_h}] * 1000$$

$$PVS(railroad) = P(land)\sum_{n=1}^{4} P(other\ railroads)^n \frac{25*2^{(n-1)}}{200n} * 1000$$

$$PVS(utility) = P(land)[\frac{14}{150} + P(other\ utility)\frac{70}{150}] * 1000$$

Below is a graphic[5] of the PVS for each ownable space on the board. Also, a table of the scores can be found in the appendix.

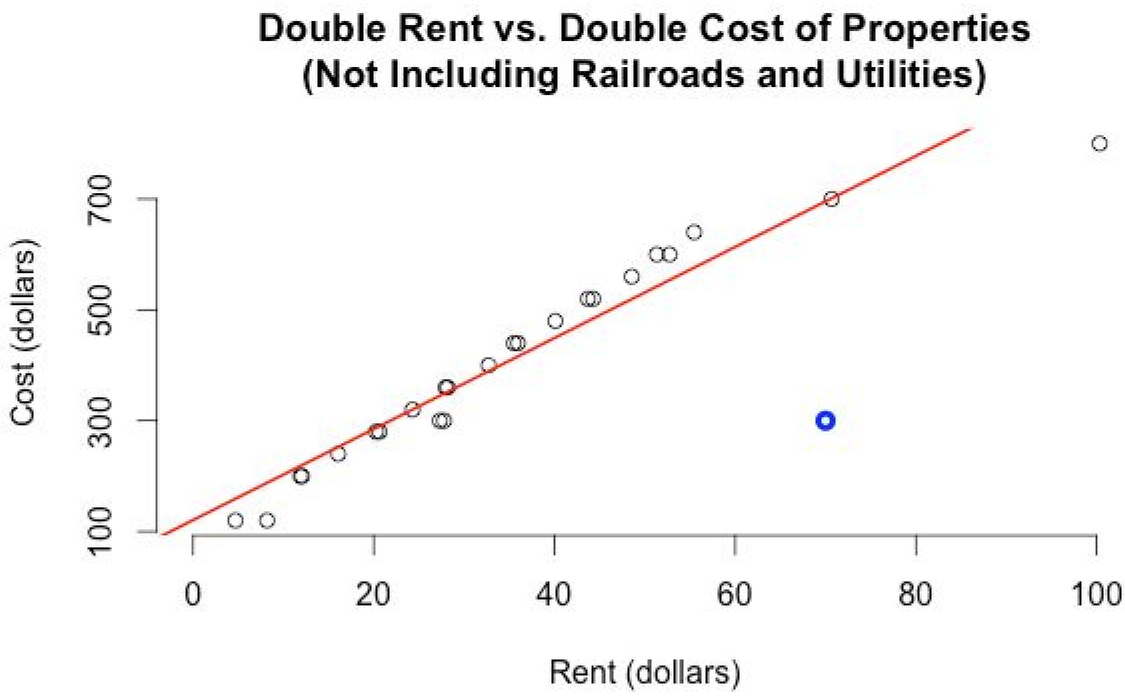
[5] The interactive version of the graphic can be found at:
https://bl.ocks.org/harukihill/a2adc68c34f86f41dc99417b7c251ec1
An additional graphic comparing Property Cost and PVS can be found at:
https://bl.ocks.org/harukihill/425fc01bb6123575767cfe319f92c251



The first and most major realization from the model is the incredible value of Railroads, colored in grey. The four Railroads are among the top five most valuable properties. A major component of their value comes from their Cost to Rent ratio. A property similar to the Railroads in cost is New York Avenue. That property's rent is $16 while the Railroads' rent are each $25. Another reason they are highly valuable is a player does not have to monopolize all the same type of property before rent is increased. With each added Railroad, the rent goes up from $25 (one Railroad) to $50 (two Railroads), $100 (three), and finally $200 (four). New York Avenue's rent jumps from $16 to $80 with one house added and then quickly eclipses the rent of four Railroads when another house is added. But a player must first funnel $560 into Orange properties before buying a house. One factor that raises the value of a Railroad that is not reflected in PVS is in a real game of Monopoly players do not go around the board as many times as the simulation. In the simulation of 10,000,000 rolls, the simulated player passes 'Go' 1,597,633 times (1 in 6.3 rolls or 15.97% of rolls). With fewer rolls, there is a smaller chance that a player will land in the same area enough to monopolize same color properties. This fact assumes only one player is playing. Railroads have the benefit of being spaced out much more evenly and a player could buy all four of them without making deals with other players.

The results also suggest one of the perennially overlooked Brown properties, Baltic Avenue (space 3), is more valuable than the the next color group of properties, which all have much larger rents. Mediterranean Avenue (space 1) and Baltic Avenue, cost $60 each and only return a rent of $2 and $4 respectively. Mediterranean Avenue backs up its low cost and rent with a measly 0.79 PVS, the lowest of all properties. But Baltic Avenue far outperforms the other Brown property with a 1.60 PVS. So what makes a property that has such a low rent and low probability of landing that valuable? One advantage Baltic Avenue is its probability of monopolization is much higher than many other properties. Probability of monopolization is a key component of PVS and is calculated by multiplying the landing probabilities of all properties of one color. There are only two Brown properties compared to three of the next color, Blue. Even though the probability of landing on a Brown property is lower than a Blue property, it is more likely that a player monopolizes the two properties in Brown than the three properties in Blue.

The same property of monopolization raises the value of Utilities. There are only two Utilities, Electric Company (space 12) and Water Works (space 28) and their rent increases when a player owns both. The rent of both utilities are determined by a dice roll of the landing player multiplied by a factor of four and ten depending on how many Utilities are owned. The expected value of rent when a player owns one Utility is $14, which is on par with an Orange property such as St. James Place, but it costs $30 less than that property. The expected value of rent when a player owns two Utilities is $70 and the cost of two Utilities is $300. There are no properties that have a base rent of $70 (the highest is Boardwalk which owns a $50 rent) so the costs of both cannot be compared. But, using a linear regression model with only properties, a reasonable estimate can be inferred. The rents and costs of the other properties are doubled in the graph below in order to more accurately compare them with the combined Utilities.



The blue point on the far right is the rent and cost of the combined two Utilities. The points deviate slightly from their actual values because the data is jittered in the x-direction for clarity. The red line represents the linear fit for the rent and cost of all the other properties. The line has an amazingly high correlation ($r^2 = 0.968$) and its regression model is $Cost = 120.7 + 8.2\ Rent$. The model suggests that a property with rent equalling \$70 should cost, on average, \$696, more than double the actual cost of the two combined Utilities. That predicted point can be found on the red line directly above the blue circle.

An Alternate Valuation Method

Another metric that can shed light on the true value of each property is the number of turns it takes to recoup the cost of a property. This metric uses some of the same data as before but instead of generating a dimensionless value, it returns a property's value in terms of rolls. In the appendix, Figure 4 is a table with the data.

The conventional wisdom says that as more houses are added to a property, the number of turns it takes to recoup the cost go down. But for seventeen out of twenty-two properties, the opposite is true, which suggests having four houses is sometimes more valuable than buying a hotel. All of the properties after and including St. Charles Place (space 11) adhere to the previous point.

The scarcity of houses also suggests having four houses is more valuable than owning a hotel, despite the jump in rent. There are only thirty-two houses in a standard Monopoly set and when the supply is exhausted, players can no longer buy them. This strategy also blocks opposing players from improving their properties because houses must be bought evenly amongst a color, meaning a player cannot buy two or more houses on one property while leaving the rest empty. This rule applies to buying a Hotel as well. Examples before have clearly demonstrated the power of buying houses and increasing the rent of properties, so if a player can monopolize houses, he puts himself in prime position to win.

The valuation method further substantiates PVS' results. An example is Baltic Avenue (space 4) requiring fewer rolls to recoup its cost (643.5 rolls) than all of the Blue properties. Also, Railroads prove once again to be the dominant type of property as they require far fewer rolls to recoup their costs than every other property.

Conclusion

Optimal Monopoly strategy, as determined by this study, can be summed up by the following. Spaces with the highest roll probability are clumped around the northwest corner. A player who wants their opponents to land on his or her properties should trade for or buy properties from that that area. According to PVS, players should also try to buy Railroads and Utilities. Also according to PVS, value of a property increases as a player makes their way around the board, with the exceptions of Baltic Avenue, which is more valuable than the next color of properties, and the Green properties. Lastly, players should try to monopolize houses and never buy hotels.

Although no Monopoly game should (hopefully) ever have 1,000,000,000 rolls, the information gleaned from the simulation should prove to be useful to not just Monopoly players, but also anyone wanting to simulate other board games or real-world phenomena.

Appendix

Figure 1:

(http://ecx.images-amazon.com/images/I/81oC5pYhh2L._SL1500_.jpg)

Figure 2:

Simulation Data (based on 1,000,000,000 rolls)						
Location	**Name**	**Probability**		**Location**	**Name**	**Probability**
0	Go	0.02272		20	Free Parking	0.02717
1	Mediterranean Avenue	0.02288		21	Kentucky Avenue	0.02694
2	Community Chest	0.02307		22	Chance	0.02668
3	Baltic Avenue	0.02331		23	Indiana Avenue	0.02636
4	Income Tax	0.02293		24	Illinois Avenue	0.02676
5	Reading Railroad	0.02272		25	B. & O. Railroad	0.02697
6	Oriental Avenue	0.02264		26	Atlantic Avenue	0.02703
7	Chance	0.02264		27	Ventnor Avenue	0.02702
8	Vermont Avenue	0.02268		28	Water Works	0.02692
9	Connecticut Avenue	0.02274		29	Marvin Gardens	0.02682
10	Jail	0.05416		30	Go To Jail	0
11	St. Charles Place	0.02278		31	Pacific Avenue	0.02671
12	Electric Company	0.02361		32	North Carolina Avenue	0.02600
13	States Avenue	0.02455		33	Community Chest	0.02527
14	Virginia Avenue	0.02530		34	Pennsylvania Avenue	0.02452
15	Pennsylvania Railroad	0.02621		35	Short Line	0.02372
16	St. James Place	0.02722		36	Chance	0.02284
17	Community Chest	0.02832		37	Park Place	0.02186
18	Tennessee Avenue	0.02781		38	Luxury Tax	0.02260
19	New York Avenue	0.02745		39	Boardwalk	0.02252

Figure 3:

Property Value Scores For Ownable Spaces						
Location	**Name**	**PVS**		**Location**	**Name**	**PVS**
1	Mediterranean Avenue	0.79		21	Kentucky Avenue	2.21
3	Baltic Avenue	1.60		23	Indiana Avenue	2.16
5	Reading Railroad	2.84		24	Illinois Avenue	2.23
6	Oriental Avenue	1.36		25	B. & O. Railroad	3.37
8	Vermont Avenue	1.36		26	Atlantic Avenue	2.29
9	Connecticut Avenue	1.52		27	Ventnor Avenue	2.29
11	St. Charles Place	1.63		28	Water Works	2.52
12	Electric Company	2.21		29	Marvin Gardens	2.30
13	States Avenue	1.75		31	Pacific Avenue	2.32
14	Virginia Avenue	1.90		32	North Carolina Avenue	2.26
15	Pennsylvania Railroad	3.28		34	Pennsylvania Avenue	2.15
16	St. James Place	2.12		35	Short Line	2.97
18	Tennessee Avenue	2.17		37	Park Place	2.23
19	New York Avenue	2.21		39	Boardwalk	2.87

Figure 4:

Number of Turns to Recoup a Property's Cost							
Name	**Location**	**Base**	**1 House/ 2 Railroads/ 2 Utilities**	**2 Houses/ 3 Railroads**	**3 Houses/ 4 Railroads**	**4 Houses**	**Hotel**
Mediterranean Avenue	1	1311.2	480.8	233.1	102.0	71.0	54.2
Baltic Avenue	3	643.5	236.0	114.4	50.1	34.9	30.1
Reading Railroad	5	352.1	352.1	264.1	176.1		
Oriental Avenue	6	736.2	162.0	78.5	34.4	28.7	24.6
Vermont Avenue	8	734.9	161.7	78.4	34.3	28.7	24.6
Connecticut Avenue	9	659.6	120.9	70.4	30.8	25.4	22.6
St. Charles Place	11	614.6	140.5	76.1	35.1	32.3	32.6
Electric Company	12	453.8	90.8				
States Avenue	13	572.6	130.9	70.9	32.7	30.1	32.6
Virginia Avenue	14	527.0	105.4	57.1	28.5	26.0	27.2
Pennsylvania Railroad	15	305.2	305.2	228.9	152.6		
St. James Place	16	472.3	84.0	47.8	24.0	22.5	25.8
Tennessee Avenue	18	462.3	82.2	46.7	23.5	22.1	25.8
New York Avenue	19	455.4	72.9	43.1	21.9	20.9	24.5
Kentucky Avenue	21	453.7	86.6	53.5	27.0	28.0	33.7
Indiana Avenue	23	463.7	88.5	54.6	27.6	28.6	33.7
Illinois Avenue	24	448.4	78.5	44.8	25.4	26.7	32.2
B. & O. Railroad	25	296.6	296.6	222.5	148.3		
Atlantic Avenue	26	437.2	70.6	40.4	23.6	25.0	30.8
Ventnor Avenue	27	437.4	70.7	40.4	23.6	25.1	30.8
Water Works	28	398.0	79.6				
Marvin Gardens	29	435.0	65.2	37.3	22.4	24.0	29.5
Pacific Avenue	31	432.0	74.9	44.2	27.5	29.3	36.3
North Carolina Avenue	32	443.8	76.9	45.4	28.2	30.1	36.3
Pennsylvania Avenue	34	466.1	70.7	41.7	26.9	29.2	33.1
Short Line	35	337.3	337.3	253.0	168.6		
Park Place	37	457.5	68.0	42.1	27.4	30.3	30.9

Boardwalk	39	355.2	57.7	34.0	20.9	22.5	23.2

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