1 Gamma matrix and spinor conventions

For concreteness, we take the following basis of gamma matrices

$$\begin{aligned}
\gamma_1 &= \sigma_2 \otimes \mathbb{1}_2 \otimes \sigma_3 \\
\gamma_2 &= \sigma_2 \otimes \mathbb{1}_2 \otimes \sigma_1 \\
\gamma_3 &= \mathbb{1}_2 \otimes \sigma_1 \otimes \sigma_2 \\
\gamma_4 &= \mathbb{1}_2 \otimes \sigma_3 \otimes \sigma_2 \\
\gamma_5 &= \sigma_1 \otimes \sigma_2 \otimes \mathbb{1}_2 \\
\gamma_6 &= \sigma_3 \otimes \sigma_2 \otimes \mathbb{1}_2
\end{aligned} \tag{1}$$

These gamma matrices satisfy the Clifford algebra

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \tag{2}$$

as appropriate for a positive definite Euclidean spacetime. All matrices are purely imaginary and satisfy

$$(\gamma_\mu)^\dagger = \gamma_\mu \qquad\qquad (\gamma_\mu)^2 = \mathbb{1} \tag{3}$$

We will now be interested in a seven-dimensional Clifford algebra, which will require the introduction of a new matrix γ_7. The reason we are interested in this is that we would like to represent hyperbolic space $\mathbb{H}_6$ as a hypersurface in a seven-dimensional ambient space. This allows us to determine properties of the Dirac spinors in the Euclidean-continued $F(4)$ gauged supergravity theory with $\mathbb{H}_6$ background by first considering Dirac spinors in seven dimensions and then performing a timelike reduction. In particular, we will choose a 7D metric of signature $(+,+,+,+,+,+,-)$ for the ambient space. Then hyperbolic space $\mathbb{H}_6$ is given by the following quadratic form

$$x_1^2 + \cdots + x_6^2 - x_7^2 = -L^2 \tag{4}$$

The seven-dimensional Clifford algebra is made up of the set of matrices $\{\gamma_1, \ldots, \gamma_6, \gamma_7\}$, with γ_7 satisfying

$$(\gamma_7)^2 = -\mathbb{1} \qquad\qquad \{\gamma_\mu, \gamma_7\} = 0 \;\; \forall \mu \neq 7 \tag{5}$$

As usual, we use the notation $\gamma^7 = (\gamma_7)^{-1}$, so that by the above we have $\gamma^7 = -\gamma_7$. We now discuss Dirac spinors in $d = 7$. We define the Dirac conjugate of ψ_A to be

$$\bar{\psi}_A = \psi_A^\dagger G^{-1} \tag{6}$$

for some matrix G. There are two possible choices for G , which in the particular case of the ambient space above are

$$G_1 = \gamma^7 \qquad\qquad G_2 = \gamma^1 \ldots \gamma^6 \tag{7}$$