1 Proof of Theorem 3

1.1 Average direct effect on the never-takers

In the following, we show that $\theta_1^n = E[Y_1(1,0)-Y_1(0,0)|\tau = n] = E[Y_1-Q_{00}(Y_0)|D = 1, M = 0]$. From (), we obtain the first ingredient $E[Y_1(1,0)|\tau = n] = E[Y_1|D = 1, M = 0]$. Furthermore, from () we have $E[Q_{00}(Y_0)|D = 1, M = 0] = E[Y_1(0,0)|D = 1, M(1) = 0]$. Under Assumption 7 and 8,

$$\begin{aligned} E[Y_1(0,0)|D = 1, M(1) = 0] &\overset{A7}{=} E[Y_1(0,0)|D = 1, \tau = n] \\ &\overset{A8}{=} E[Y_1(0,0)|\tau = n]. \end{aligned} \tag{1}$$

1.2 Quantile direct effect on the never-takers

We prove that

$$\begin{aligned} \theta_1^n(q) &= F^{-1}_{Y_1(1,0)|\tau=n}(q) - F^{-1}_{Y_1(0,0)|\tau=n}(q), \\ &= F^{-1}_{Y_1|D=1,M=0}(q) - F^{-1}_{Q_{00}(Y_0)|D=1,M=0}(q). \end{aligned}$$

This requires showing that

$$F_{Y_1(1,0)|\tau=n}(y) = F_{Y_1|D=1,M=0}(y) \text{ and} \tag{2}$$

$$F_{Y_1(0,0)|\tau=n}(y) = F_{Q_{00}(Y_0)|D=1,M=0}(y). \tag{3}$$

Under Assumptions 7 and 8,

$$\begin{aligned} F_{Y_t|D=1,M=0}(y) &= E[1\{Y_t \leq y\}|D = 1, M = 0] \\ &\overset{A7,A8}{=} E[1\{Y_t(1,0) \leq y\}|\tau = n] \\ &= F_{Y_t(1,0)|\tau=n}(y), \end{aligned} \tag{4}$$

which proves (). From (), we have

$$F_{Q_{00}(Y_0)|D=1,M=0}(y) = F_{Y_1(0,0)|D=1,M(1)=0}(y) = E[1\{Y_1(0,0) \leq y\}|D = 1, M(1) = 0].$$