These will turn out to be the same, so we just work with the former. Thus we have that

$$\bar{\psi}_A = \psi_A^\dagger \gamma_7 \tag{1}$$

If we choose γ_7 such that

$$(\gamma_7)^\dagger = -\gamma_7 \tag{2}$$

we can express the Hermitian conjugates of our gamma matrices as

$$\gamma_\mu^\dagger = \eta\, G^{-1} \gamma_\mu G \tag{3}$$

Importantly, with $G = G_1$ in , we have

$$\eta = -1 \tag{4}$$

This will be important in Appendix when the consistency of the symplectic Majorana condition is analyzed. For now, we just recall that the symplectic Majorana condition must take the form

$$\bar{\psi}_A = \epsilon^{AB} \psi_B^T \mathcal{C} \tag{5}$$

where

$$\mathcal{C}^2 = 1 \qquad\qquad \mathcal{C}^T = \mathcal{C} \qquad\qquad \gamma_\mu^T = -\mathcal{C}^{-1}\gamma_\mu \mathcal{C} \tag{6}$$

We now want to reduce from $d = 7$ to $d = 6$. In particular, we reduce on the time-like direction x_7. This entails finding a Euclidean induced metric on the six-dimensional surface . From the point of view of the Clifford algebra, we must remove the matrix γ_7 to get a six-dimensional Clifford algebra. However, the properties of the matrix γ^7 remain the same. In fact, we may choose

$$\gamma_7 = \gamma_0\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5 \tag{7}$$

which satisfies all of the properties ,.

1 Free differential algebra

In this Appendix, we will construct the free differential algebra (FDA) of a supergravity theory with $\mathbb{H}_6$ background in order to motivate the form of the supersymmetry variations given in . The first step of constructing the FDA is to write down the Maurer-Cartan equations (MCEs), which may be thought of as the geometrization of the (anti-)commutation relations of the superalgebra. In short, instead of defining the algebra via the (anti-)commutators of its generators, the MCEs encode the algebraic structure in integrability conditions. In