a) and Assumption 4a, the average and quantile direct effects on never-takers are identified:

$$\theta_1^n = \theta_1^{1,0}(1) \text{ and } \theta_1^n(q) = \theta_1^{1,0}(q,1).$$

b) and Assumption 4, the average direct effect under $d = 0$ on compliers is identified:

$$\theta_1^c(0) = \frac{p_{0|0}}{p_{0|0} - p_{0|1}}\theta_1^{0,0}(0) - \frac{p_{0|1}}{p_{0|0} - p_{0|1}}\theta_1^{1,0}(1).$$

Furthermore, the potential outcome distributions under $d = 0$ on compliers are identified:

$$\begin{aligned} F_{Y_1(1,0)|\tau=c}(y) = & \frac{p_{0|0}}{p_{0|0} - p_{0|1}} F_{Q_{10}(Y_0)|D=0,M=0}(y) \\ & - \frac{p_{0|1}}{p_{0|0} - p_{0|1}c} F_{Y_1|D=1,M=0}(y), \end{aligned} \tag{1}$$

$$\begin{aligned} F_{Y_1(0,0)|\tau=c}(y) = & \frac{p_{0|0}}{p_{0|0} - p_{0|1}} F_{Y_1|D=0,M=0}(y) \\ & - \frac{p_{0|1}}{p_{0|0} - p_{0|1}} F_{Q_{00}(Y_0)|D=1,M=0}(y). \end{aligned} \tag{2}$$

Therefore, the direct quantile effect under $d = 0$ on compliers, $\theta_1^c(q,0) = F^{-1}_{Y_1(1,0)|c}(q) - F^{-1}_{Y_1(0,0)|c}(q)$, is identified.

Proof. See Appendix . **Theorem 4:** Under Assumptions 1–2, 5, 7-8,

a) and Assumption 6a, the average and quantile direct effects on always-takers are identified:

$$\theta_1^a = \theta_1^{0,1}(0) \text{ and } \theta_1^a(q) = \theta_1^{0,1}(q,0).$$

b) and Assumption 6, the average direct effect under $d = 1$ on compliers is identified:

$$\theta_1^c(1) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}}\theta_1^{1,1}(1) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}}\theta_1^{0,1}(0).$$

Furthermore, the potential outcome distributions under $d = 1$ for compliers