Second, for $F_{Y_0(0,0)|D=1,M=0}(y)$ we have

$$F^{-1}_{U|D=1,M=0}(F_{Y_0(0,0)|D=1,M=0}(y)) = h^{-1}(0,0,0;y). \tag{1}$$

Combining () and () yields,

$$h(0,0,1,h^{-1}(0,0,0;y)) = F^{-1}_{Y_1(0,0)|D=1,M=0} \circ F_{Y_0(0,0)|D=1,M=0}(y). \tag{2}$$

Note that $h(0,0,1,h^{-1}(0,0,0;y))$ maps the period 1 (potential) outcome of an individual with the outcome y in period 0 under non-treatment without the mediator. Accordingly, $E[F^{-1}_{Y_1(0,0)|D=1,M=0} \circ F_{Y_0(0,0)|D=1,M=0}(Y_0)|D = 1, M = 0] = E[Y_1(0,0)|D = 1, M = 0]$. We can identify $F_{Y_0(0,0)|D=1,M=0}(y)$ under Assumption 2, but we cannot identify $F_{Y_1(0,0)|D=1,M=0}(y)$. However, we show in the following that we can identify the overall quantile-quantile transform $F^{-1}_{Y_1(0,0)|D=1,M=0} \circ F_{Y_0(0,0)|D=1,M=0}(y)$ under the additional Assumption 3b. Under Assumptions 1 and 3b, the conditional potential outcome distribution function equals

$$\begin{aligned} F_{Y_t(d,0)|D=0,M=0}(y) &\overset{A1}{=} \Pr(h(d,m,t,U) \le y|D=0, M=0, T=t), \\ &= \Pr(U \le h^{-1}(d,m,t;y)|D=0, M=0, T=t), \\ &\overset{A3b}{=} \Pr(U \le h^{-1}(d,m,t;y)|D=0, M=0), \\ &= F_{U|00}(h^{-1}(d,m,t;y)), \end{aligned} \tag{3}$$

for $d, d' \in \{0,1\}$. We repeat similar steps as above. First, evaluating $F_{Y_1(0,0)|D=0,M=0}(y)$ at $h(0,0,1,u)$ gives

$$F_{Y_1(0,0)D=0,M=0}(h(0,0,1,u)) = F_{U|00}(h^{-1}(0,0,1;h(0,0,1,u))) = F_{U|00}(u).$$

Applying $F^{-1}_{Y_1(0,0)|D=0,M=0}(q)$ to both sides, we have

$$h(0,0,1,u) = F^{-1}_{Y_1(0,0)|D=0,M=0}(F_{U|00}(u)). \tag{4}$$