Under Assumption 7 and 8,

$$
\begin{aligned}
E[1\{Y_1(0,0) \leq y\}|D=1, M(1)=0] &\overset{A7,A8}{=} E[1\{Y_1(0,0) \leq y\}|\tau = n] \\
&= F_{Y_1(0,0)|\tau=n}(y),
\end{aligned} \tag{1}
$$

which proves.

0.1 Average direct effect under d = 0 on compliers

In the following, we show that

$$
\begin{aligned}
\theta_1^c(0) &= E[Y_1(1,0) - Y_1(0,0)|\tau = c], \\
&= \frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Q_{10}(Y_0) - Y_1|D=0, M=0] \\
&- \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Y_1 - Q_{00}(Y_0)|D=1, M=0].
\end{aligned}
$$

Plugging () in () under $T = 1$, we obtain

$$
\begin{aligned}
E[Y_1|D=0, M=0] = &\frac{p_n}{p_n + p_c} E[Q_{00}(Y_0)|D=1, M=0] \\
&+ \frac{p_c}{p_n + p_c} E[Y_1(0,0)|\tau = c].
\end{aligned}
$$

This allows identifying

$$
\begin{aligned}
E[Y_1(0,0)|\tau = c] = &\frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Y_1|D=0, M=0] \\
&- \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Q_{00}(Y_0)|D=1, M=0].
\end{aligned} \tag{2}
$$

Accordingly, we have to show the identification of $E[Y_1(1,0)|c]$ to finish the proof. From () we have $E[Y_1(1,0)|D=0, M=0] = E[Q_{10}(Y_0)|D=0, M=0]$. Applying