$y_1 = (pq)^{-\frac{1}{2}} t^{-2} A B^{-1}$ and $y_2 = (pq)^{-\frac{1}{2}} q^{-1} t^{-2} A B^{-1}$. For y_1 the integral over w_1 is pinched at $w_1 = v^{\pm 1}$ and we proceed exactly as in appendix B to get the same result up to some overall factors

$$\frac{1}{2} \frac{\Gamma_e\big(qt^{-2}\big)\Gamma_e\big(pq^2t^2A^{-2}B^2\big)\Gamma_e\big(t^{-2}B^{-2}\big)\Gamma_e\big(t^{-2}A^2\big)\Gamma_e\big((pq)^{-1}q^{-1}t^{-4}A^2B^{-2}\big)}{\Gamma_e\big((pq)^{-1}t^{-4}A^2B^{-2}\big)}$$
$$\times \Gamma_e\big(t^{-2}AB^{-1}C^{\pm1}D^{\pm1}\big)\Gamma_e\big(AB^{-1}u^{\pm1}v^{\pm1}\big)\Gamma_e\big((qp)^{\frac{1}{2}}tBD^{\pm1}v^{\pm1}\big)\Gamma_e\big((qp)^{\frac{1}{2}}tA^{-1}C^{\pm1}v^{\pm1}\big)$$
$$\times \Gamma_e\big((pq)^{\frac{1}{2}}tBC^{\pm1}u^{\pm1}\big)\Gamma_e\big((pq)^{\frac{1}{2}}tA^{-1}D^{\pm1}u^{\pm1}\big). \tag{1}$$

Now we look at the pole y_2. the integral over w_1 is pinched at $w_1 = q^{\pm\frac{1}{2}}v^{\pm1}$. Substituting these values (**??**) now becomes

$$\frac{1}{2} \frac{\Gamma_e\big(t^{-2}\big)\Gamma_e\big(pq^3t^2A^{-2}B^2\big)}{\Gamma_e\big(pq^3t^4A^{-2}B^2\big)} \frac{\Gamma_e\big(v^2\big)}{\Gamma_e\big(qv^2\big)} \Gamma_e\Big(q^{-\frac{1}{2}}AB^{-1}u^{\pm1}\big(q^{\frac{1}{2}}v\big)^{\pm1}\Big)\Gamma_e\Big((qp)^{\frac{1}{2}}q^{\frac{1}{2}}tBD^{\pm1}\big(q^{\frac{1}{2}}v\big)^{\pm1}\Big)$$
$$\times \Gamma_e\Big((qp)^{\frac{1}{2}}q^{\frac{1}{2}}tA^{-1}C^{\pm1}\big(q^{\frac{1}{2}}v\big)^{\pm1}\Big) \oint \frac{\mathrm{d}w_2}{4\pi i w_2} \frac{1}{\Gamma_e\big(w_2^{\pm2}\big)} \Gamma_e\left(\frac{(pq)^{\frac{1}{2}}}{t^2} q^{\frac{1}{2}} v w_2^{\pm1}\right)$$
$$\times \Gamma_e\big((qp)^{\frac{1}{2}}q^{\frac{1}{2}}t^2A^{-1}Bw_2^{\pm1}u^{\pm1}\big)\Gamma_e\big(q^{-\frac{1}{2}}t^{-1}B^{-1}D^{\pm1}w_2^{\pm1}\big)$$
$$\times \Gamma_e\big((qp)^{\frac{1}{2}}q^{\frac{1}{2}}t^2w_2^{\pm1}v^{-1}\big)\Gamma_e\big(q^{-\frac{1}{2}}t^{-1}Aw_2^{\pm1}C^{\pm1}\big) + \big\{v \leftrightarrow v^{-1}\big\}.$$

The integral here can be interpreted as index of the SU(2) gauge theory with four flavors. Using the Intriligator–Pouliot duality transformation (that is $V(\underline{s}) = \prod_{1\le j<k\le 8} \Gamma_e(s_js_k) V(\sqrt{pq}/\underline{s})$ in the notations of) the expression becomes

$$\frac{1}{2} \frac{\Gamma_e\big(t^{-2}\big)\Gamma_e\big(pq^3t^2A^{-2}B^2\big)}{\Gamma_e\big(pq^3t^4A^{-2}B^2\big)} \frac{\Gamma_e\big(v^2\big)}{\Gamma_e\big(qv^2\big)} \Gamma_e\Big(q^{-\frac{1}{2}}AB^{-1}u^{\pm1}\big(q^{\frac{1}{2}}v\big)^{\pm1}\Big)\Gamma_e\Big((qp)^{\frac{1}{2}}q^{\frac{1}{2}}tBD^{\pm1}\big(q^{\frac{1}{2}}v\big)^{\pm1}\Big)$$
$$\times \Gamma_e\Big((qp)^{\frac{1}{2}}q^{\frac{1}{2}}tA^{-1}C^{\pm1}\big(q^{\frac{1}{2}}v\big)^{\pm1}\Big)\Gamma_e\big(pq^2A^{-1}Bvu^{\pm1}\big)\Gamma_e\big((pq)^{\frac{1}{2}}t^{-3}B^{-1}vD^{\pm1}\big)$$
$$\times \Gamma_e\big((pq)^{\frac{1}{2}}t^{-3}AvC^{\pm1}\big)\Gamma_e\big(pq^2t^4A^{-2}B^2\big)\Gamma_e\big((pq)^{\frac{1}{2}}tA^{-1}u^{\pm1}D^{\pm1}\big)\Gamma_e\big((pq)^{\frac{1}{2}}tBu^{\pm1}C^{\pm1}\big)$$
$$\times \Gamma_e\big(pq^2t^4A^{-1}Bv^{-1}u^{\pm1}\big)\Gamma_e\big(q^{-1}t^{-2}B^{-2}\big)\Gamma_e\big(q^{-1}t^{-2}AB^{-1}C^{\pm1}D^{\pm1}\big)$$
$$\times \Gamma_e\big((pq)^{\frac{1}{2}}tB^{-1}v^{-1}D^{\pm1}\big)\Gamma_e\big(q^{-1}t^{-2}A^2\big)\Gamma_e\big((pq)^{\frac{1}{2}}tAv^{-1}C^{\pm1}\big)\Gamma_e\big(pq^2\big)$$
$$\times \oint \frac{\mathrm{d}w_2}{4\pi i w_2} \frac{1}{\Gamma_e\big(w_2^{\pm2}\big)} \Gamma_e\big(q^{-\frac{1}{2}}t^2v^{-1}w_2^{\pm1}\big)\Gamma_e\big(q^{-\frac{1}{2}}t^{-2}AB^{-1}w_2^{\pm1}u^{\pm1}\big)$$
$$\times \Gamma_e\big((pq)^{\frac{1}{2}}q^{\frac{1}{2}}tBD^{\pm1}w_2^{\pm1}\big)\Gamma_e\big((qp)^{\frac{1}{2}}q^{\frac{1}{2}}tA^{-1}w_2^{\pm1}C^{-1}\big)\Gamma_e\big(q^{-\frac{1}{2}}t^{-2}vw_2^{\pm1}\big) + \big\{v \leftrightarrow v^{-1}\big\}.$$

We have zero multiplying the integral but the integral is pinched at $w_2 = \big(q^{\pm\frac{1}{2}}t^{-2}v\big)^{\pm1}$ due to the colliding of poles of the first and last elliptic gamma functions in the last integral. We get different contribution for each choice