The non-zero components of the Ricci tensor are

$$R_{uu} = -5\left(f'' + (f')^2\right) \qquad\qquad R_{mn} = -g_{mn}\left(f'' + 5(f')^2\right) \tag{1}$$

while the Ricci scalar is given by

$$R = -10f'' - 30(f')^2 \tag{2}$$

Furthermore, we have that $\sqrt{G} = e^{5f}\sqrt{g}$, where g is the determinant of the unit S^5 metric. Upon integration by parts, part of the Einstein-Hilbert term cancels with the Gibbons-Hawking term to give the following simple expression

$$S = \int du \int d^5x \sqrt{g}\, e^{5f}\left[-5\left((f')^2 + 4W^2\right) + 2\mathcal{L}_{\text{kin}}\right] \tag{3}$$

The restriction to the flat case was not strictly necessary so far, but it will be crucial in the next step. The gradient flow equations, together with the chain-rule, allows us to rewrite

$$\mathcal{L}_{\text{kin}} = -2\,W' \tag{4}$$

Plugging this into and using the BPS equation of the warp factor, we find

$$S = -4\int d^5x\sqrt{g}\, e^{5f}W\Big|_0^\Lambda \tag{5}$$

where Λ is the UV cutoff. Only the Λ part of the action contributes, since $e^{5f}W|_0$ vanishes due to the close-off of the geometry. Removing the UV cutoff $\Lambda \to \infty$ is equivalent to removing the cutoff ε on our asymptotic coordinate z, i.e. $\varepsilon \to 0$. From the UV asymptotics we find that in this limit the factor e^{5f} diverges like

$$e^{5f} \sim \frac{1}{\varepsilon^5} \tag{6}$$

This is the reason for the previous claims that only the terms up to $O(z^5)$ in the superpotential are relevant for obtaining counterterms. All the higher-order terms vanish as the cutoff is removed. We may thus legitimately insert the approximate superpotential into to get the counterterms,

$$S_{\text{ct}}^{(W)} = 4\int d^5x\sqrt{\gamma}\left[\frac{1}{2} + \frac{3}{4}\sigma^2 + \frac{1}{16}(\phi^0)^2 - \frac{3}{16}(\phi^3)^2 + \frac{1}{192}(\phi^0)^4 - \frac{3}{16}(\phi^0)^2\sigma\right] \tag{7}$$

where γ is the induced metric on the $z = \varepsilon$ boundary. All fields are evaluated at $z = \varepsilon$. This gives all finite and infinite counterterms for the flat domain wall solutions.