

0.1 Average direct effect under $d = 1$ on compliers

In the following, we show that

$$\begin{aligned}\theta_1^c(1) &= E[Y_1(1, 1) - Y_1(0, 1)|\tau = c], \\ &= \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Y_1 - Q_{01}(Y_0)|D = 1, M = 1] \\ &\quad - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Q_{11}(Y_0) - Y_1|D = 0, M = 1].\end{aligned}$$

Plugging () in (), we obtain

$$\begin{aligned}E[Y_1|D = 1, M = 1] &= \frac{p_a}{p_a + p_c} E[Q_{11}(Y_0)|D = 0, M = 1] \\ &\quad + \frac{p_c}{p_a + p_c} E[Y_1(1, 1)|\tau = c].\end{aligned}$$

This allows identifying

$$\begin{aligned}E[Y_1(1, 1)|\tau = c] &= \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Y_1|D = 1, M = 1] \\ &\quad - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Q_{11}(Y_0)|D = 0, M = 1].\end{aligned}\tag{1}$$

From () we have $E[Y_1(0, 1)|D = 1, M = 1] = E[Q_{01}(Y_0)|D = 1, M = 1]$. Applying the law of iterative expectations, gives

$$\begin{aligned}E[Y_1(0, 1)|D = 1, M = 1] &= \frac{p_a}{p_a + p_c} E[Y_1(0, 1)|D = 1, M = 1, \tau = a] \\ &\quad + \frac{p_c}{p_a + p_c} E[Y_1(0, 1)|D = 1, M = 1, \tau = c], \\ &\stackrel{A7}{=} \frac{p_a}{p_a + p_c} E[Y_1(0, 1)|\tau = a] + \frac{p_c}{p_a + p_c} E[Y_1(0, 1)|\tau = c].\end{aligned}$$

After some rearrangements and using (), we obtain

$$E[Y_1(0, 1)|\tau = c] = \frac{p_a + p_c}{p_c} E[Q_{01}(Y_0)|D = 1, M = 1] - \frac{p_a}{p_c} E[Y_1|D = 0, M = 1].$$