Using () and rearranging the equation gives

$$F_{Y_1(1,1)|\tau=c}(y) = \frac{p_{1|1}}{p_{1|1}-p_{1|0}} F_{Y_1|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1}-p_{1|0}} F_{Q_{11}(Y_0)|D=0,M=1}(y). \tag{1}$$

From (), we have $F_{Y_1(0,1)|D=1,M(1)=1}(y) = F_{Q_{01}(Y_0)|D=1,M=1}(y)$. Applying the law of iterative expectations gives

$$\begin{aligned}
F_{Y_1(0,1)|D=1,M(1)=1}(y) &= \frac{p_a}{p_a+p_c} F_{Y_1(0,1)|D=1,M(1)=1,\tau=a}(y) \\
&\quad + \frac{p_c}{p_a+p_c} F_{Y_1(0,1)|D=1,M(1)=1,\tau=c}(y), \\
&\overset{A7}{=} \frac{p_a}{p_a+p_c} F_{Y_1(0,1)|\tau=a}(y) + \frac{p_c}{p_a+p_c} F_{Y_1(0,1)|\tau=c}(y).
\end{aligned}$$

Using () and rearranging the equation gives,

$$F_{Y_1(0,1)|\tau=c}(y) = \frac{p_{1|1}}{p_{1|1}-p_{1|0}} F_{Q_{01}(Y_0)|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1}-p_{1|0}} F_{Y_1|D=0,M=1}(y). \tag{2}$$

1 Proof of Theorem 5

1.1 Average treatment effect on the compliers

In () and (), we show that

$$\begin{aligned}
\theta_1^c &= E[Y_1(1,1) - Y_1(0,0)|\tau=c], \\
&= \frac{p_{1|1}}{p_{1|1}-p_{1|0}} E[Y_1|D=1,M=1] - \frac{p_{1|0}}{p_{1|1}-p_{1|0}} E[Q_{11}(Y_0)|D=0,M=1] \\
&\quad - \frac{p_{0|0}}{p_{0|0}-p_{0|1}} E[Y_1|D=0,M=0] + \frac{p_{0|1}}{p_{0|0}-p_{0|1}} E[Q_{00}(Y_0)|D=1,M=0].
\end{aligned}$$

1.2 Quantile treatment effect on the compliers

In () and (), we show that $F_{Y_1(1,1)|c}(y)$ and $F_{Y_1(0,0)|c}(y)$ are identified. Accordingly, $\Delta_1^c(q) = F^{-1}_{Y_1(1,1)|c}(q) - F^{-1}_{Y_1(0,0)|c}(q)$ is identified.