

We can perform the integral over w using the inversion formula which sets $y = \frac{AB^{-1}}{(pq)^{\frac{1}{2}}t^2}$, and we get

$$\begin{aligned} & (q; q)^2(p; p)^2 \Gamma_e(pqt^2 B^2) \Gamma_e(pqt^2 A^{-2}) \Gamma_e(pqt^2 A^{-1} BC^{\pm 1} D^{\pm 1}) \Gamma_e(pq) \Gamma_e(t^{-4} A^2 B^{-2}) \\ & \times \Gamma_e(pq A^{-2} B^2) \oint \frac{dw_1}{4\pi i w_1} \oint \frac{dw_2}{4\pi i w_2} \frac{\Gamma_e\left(\frac{(pq)^{\frac{1}{2}}}{t^2} w_1^{\pm 1} w_2^{\pm 1}\right)}{\Gamma_e(w_2^{\pm 2}) \Gamma_e(w_1^{\pm 2})} \Gamma_e(AB^{-1} u^{\pm 1} w_1^{\pm 1}) \\ & \times \Gamma_e((qp)^{\frac{1}{2}} t B D^{\pm 1} w_1^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t^2 A^{-1} B u^{\pm 1} w_2^{\pm 1}) \Gamma_e(t^{-1} B^{-1} D^{\pm 1} w_2^{\pm 1}) \Gamma_e(v^{\pm 1} w_1^{\pm 1}) \\ & \times \Gamma_e((qp)^{\frac{1}{2}} t A^{-1} C^{\pm 1} w_1^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t^2 v^{\pm 1} w_2^{\pm 1}) \Gamma_e(t^{-1} A C^{\pm 1} w_2^{\pm 1}). \end{aligned}$$

$\Gamma_e(pq)$ is zero but the integral over w_1 is pinched at $w_1 = v^{\pm 1}$ due to the term $\Gamma_e(v^{\pm 1} w_1^{\pm 1})$ such that the multiplication is finite and we get

$$\begin{aligned} & (q; q)(p; p) \Gamma_e(pqt^2 B^2) \Gamma_e(pqt^2 A^{-2}) \Gamma_e(pqt^2 A^{-1} BC^{\pm 1} D^{\pm 1}) \Gamma_e(t^{-4} A^2 B^{-2}) \Gamma_e(pq A^{-2} B^2) \\ & \times \Gamma_e(AB^{-1} u^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t B D^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t A^{-1} C^{\pm 1} v^{\pm 1}) \\ & \times \oint \frac{dw_2}{4\pi i w_2} \frac{1}{\Gamma_e(w_2^{\pm 2})} \Gamma_e((qp)^{\frac{1}{2}} t^2 A^{-1} B u^{\pm 1} w_2^{\pm 1}) \Gamma_e(t^{-1} B^{-1} D^{\pm 1} w_2^{\pm 1}) \Gamma_e(t^{-1} A C^{\pm 1} w_2^{\pm 1}). \end{aligned}$$

Integral over w_2 can be evaluated using the elliptic beta integral formula, and the final result is

$$\begin{aligned} & \Gamma_e(pq A^{-2} B^2) \Gamma_e(AB^{-1} u^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t B D^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t A^{-1} C^{\pm 1} v^{\pm 1}) \\ & \times \Gamma_e((pq)^{\frac{1}{2}} t B C^{\pm 1} u^{\pm 1}) \Gamma_e((pq)^{\frac{1}{2}} t A^{-1} D^{\pm 1} u^{\pm 1}). \end{aligned}$$

We glue this to another three punctured sphere closed with $a_i = A^{-1}B$ and the claim is that gluing this to a given model we get the same model. Indeed we have

$$\begin{aligned} & T_{\mathfrak{J}_C}(u) \times_u \left((T_{\mathfrak{J}_B, \mathfrak{J}_C, \mathfrak{J}_D}(w, u, v) \times_w C_{\mathfrak{J}_B}^{(0,0;AB^{-1})}(w)) \right. \\ & \quad \left. \times_v (T_{\mathfrak{J}_B, \mathfrak{J}_C, \mathfrak{J}_D}(h, z, v) \times_h C_{\mathfrak{J}_B}^{(0,0;A^{-1}B)}(h)) \right) \\ & = (q; q)^2(p; p)^2 \Gamma_e(pq(A^{-2} B^2)^{\pm 1}) \Gamma_e((pq)^{\frac{1}{2}} t B^{-1} C^{\pm 1} z^{\pm 1}) \Gamma_e((pq)^{\frac{1}{2}} t A D^{\pm 1} z^{\pm 1}) \\ & \times \oint \frac{du}{4\pi i u} \frac{\Gamma_e((qp)^{\frac{1}{2}} t^{-1} A^{\pm 1} D^{\pm 1} u^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t^{-1} B^{\pm 1} C^{\pm 1} u^{\pm 1})}{\Gamma(u^{\pm 2})} \\ & \times \Gamma_e((pq)^{\frac{1}{2}} t B C^{\pm 1} u^{\pm 1}) \Gamma_e((pq)^{\frac{1}{2}} t A^{-1} D^{\pm 1} u^{\pm 1}) T_{\mathfrak{J}_C}(u) \\ & \times \oint \frac{dv}{4\pi i v} \frac{\Gamma_e((qp)^{\frac{1}{2}} t^{-1} A^{\pm 1} C^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t^{-1} B^{\pm 1} D^{\pm 1} v^{\pm 1})}{\Gamma(v^{\pm 2})} \\ & \times \Gamma_e(AB^{-1} u^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t B D^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t A^{-1} C^{\pm 1} v^{\pm 1}) \\ & \times \Gamma_e(A^{-1} B z^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t B^{-1} D^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{2}} t A C^{\pm 1} v^{\pm 1}). \end{aligned}$$