among never-takers with $D = 1$ (as defiers do not exist). Likewise, $\theta_1^{0,1}$ corresponds to the direct effect on always-takers with $D = 0$. Indeed, the results in Table suggest that both parameters are consistently estimated with the change-in-changes model (Panel A.).

1 Proof of Theorem 1

1.1 Average direct effect under $\mathbf{d = 1}$ conditional on $\mathbf{D = 1}$ and $\mathbf{M(1) = 0}$

In the following, we prove that $\theta_1^{1,0}(1) = E[Y_1(1,0) - Y_1(0,0)|D = 1, M_i(1) = 0] = E[Y_1 - Q_{00}(Y_0)|D = 1, M = 0]$. Using the observational rule, we obtain $E[Y_1(1,0)|D = 1, M(1) = 0] = E[Y_1|D = 1, M = 0]$. Accordingly, we have to show that $E[Y_1(0,0)|D = 1, M(1) = 0] = E[Q_{00}(Y_0)|D = 1, M = 0]$ to finish the proof. Denote the inverse of $h(d, m, t, u)$ by $h^{-1}(d, m, t; y)$, which exists because of the strict monotonicity required in Assumption 1. Under Assumptions 1 and 3a, the conditional potential outcome distribution function equals

$$\begin{aligned} F_{Y_t(d,0)|D=1,M=0}(y) &\overset{A1}{=} \Pr(h(d, m, t, U) \leq y|D = 1, M = 0, T = t), \\ &= \Pr(U \leq h^{-1}(d, m, t; y)|D = 1, M = 0, T = t), \\ &\overset{A3a}{=} \Pr(U \leq h^{-1}(d, m, t; y)|D = 1, M = 0), \\ &= F_{U|10}(h^{-1}(d, m, t; y)), \end{aligned} \tag{1.1}$$

for $d, d' \in \{0, 1\}$. We use these quantities in the following. First, evaluating $F_{Y_1(0,0)|D=1,M=0}(y)$ at $h(0, 0, 1, u)$ gives

$$F_{Y_1(0,0)|D=1,M=0}(h(0, 0, 1, u)) = F_{U|10}(h^{-1}(0, 0, 1; h(0, 0, 1, u))) = F_{U|10}(u).$$

Applying $F^{-1}_{Y_1(0,0)|D=1,M=0}(q)$ to both sides, we have

$$h(0, 0, 1, u) = F^{-1}_{Y_1(0,0)|D=1,M=0}(F_{U|10}(u)). \tag{1.2}$$