

$$h(1, 0, 1, u) = F_{Y_1(1,0)|D=1,M=0}^{-1}(F_{U|10}(u)). \quad (1)$$

Second, for  $F_{Y_0(1,0)|D=0,M=0}(y)$  we have

$$F_{U|10}^{-1}(F_{Y_0(1,0)|D=1,M=0}(y)) = h^{-1}(1, 0, 0; y), \quad (2)$$

using (). Combining () and () yields,

$$h(1, 0, 1, h^{-1}(1, 0, 0; y)) = F_{Y_1(1,0)|D=1,M=0}^{-1} \circ F_{Y_0(1,0)|D=1,M=0}(y). \quad (3)$$

The left sides of () and () are equal. In contrast to (), () contains only distributions that can be identified from observable data. In particular,  $F_{Y_t(1,0)|D=1,M=0}(y) = \Pr(Y_t(1,0) \leq y | D = 1, M = 0) = \Pr(Y_t \leq y | D = 1, M = 0)$ . Accordingly, we can identify  $F_{Y_1(1,0)|D=0,M=0}^{-1} \circ F_{Y_0(1,0)|D=0,M=0}(y)$  by  $Q_{10}(y) \equiv F_{Y_1|D=1,M=0}^{-1} \circ F_{Y_0|D=1,M=0}(y)$ . Parsing  $Y_0$  through  $Q_{10}(\cdot)$  in the non-treated group without mediator gives

$$\begin{aligned} & E[Q_{10}(Y_0) | D = 0, M = 0] \\ &= E[F_{Y_1|D=1,M=0}^{-1} \circ F_{Y_0|D=1,M=0}(Y_0) | D = 0, M = 0], \\ &= E[F_{Y_1(1,0)|D=1,M=0}^{-1} \circ F_{Y_0(1,0)|D=1,M=0}(Y_0(0,0)) | D = 0, M = 0], \\ &\stackrel{A1, A3a}{=} E[h(1, 0, 1, h^{-1}(1, 0, 0; Y_0(0,0))) | D = 0, M = 0], \\ &\stackrel{A2}{=} E[h(1, 0, 1, h^{-1}(1, 0, 0; Y_0(1,0))) | D = 1, M = 0], \\ &\stackrel{A1, A3b}{=} E[F_{Y_1(1,0)|D=0,M=0}^{-1} \circ F_{Y_0(1,0)|D=0,M=0}(Y_0(1,0)) | D = 0, M = 0], \\ &= E[Y_1(1,0) | D = 0, M = 0] = E[Y_1(1,0) | D = 0, M(0) = 0], \end{aligned} \quad (4)$$

which has data support because of Assumption 4b.