Dividing by this factor $\Gamma_e\big(\big(AB^{-1}\big)^2\big)$ and adding all contributions we get

$$\frac{\theta_p\big(pq^2t^2A^{-2}B^2\big)\theta_p\big((pq)^{-1}q^{-1}t^{-4}A^2B^{-2}\big)}{\theta_p\big(t^{-2}\big)\theta_p\big(q^{-1}t^{-2}B^{-2}\big)\theta_p\big(q^{-1}t^{-2}A^2\big)\theta_p\big(q^{-1}t^{-2}AB^{-1}C^{\pm1}D^{\pm1}\big)\theta_p\big(q^{-1}A^2B^{-2}\big)}$$
$$\times\frac{\theta_p\big((pq)^{\frac12}t^{-1}B^{\pm1}D^{\pm1}z\big)\theta_p\big((pq)^{\frac12}t^{-1}A^{\pm1}C^{\pm1}z\big)}{\theta_p\big(qz^2\big)\theta\big(z^2\big)}T_{\mathfrak{J}_D}(qz)$$
$$+\frac{\theta_p\big(pq^2t^2A^{-2}B^2\big)\theta_p\big((pq)^{-1}q^{-1}t^{-4}A^2B^{-2}\big)}{\theta_p\big(t^{-2}\big)\theta_p\big(q^{-1}t^{-2}B^{-2}\big)\theta_p\big(q^{-1}t^{-2}A^2\big)\theta_p\big(pq^2t^4A^{-2}B^2\big)\theta_p\big(q^{-1}t^{-2}AB^{-1}C^{\pm1}D^{\pm1}\big)}$$
$$\times\frac{\theta_p\big((pq)^{\frac12}t^{\pm1}BD^{\pm1}z\big)\theta_p\big((pq)^{\frac12}t^{\pm1}A^{-1}C^{\pm1}z\big)\theta_p\big(q^{-1}t^{-4}\big)\theta_p\big(q^{-1}t^{-4}A^2B^{-2}z^2\big)}{\theta_p\big(q^{-1}A^2B^{-2}\big)\theta_p\big(t^{-4}z^2\big)\theta_p\big(z^2\big)\theta_p\big(q^{-1}z^2\big)}T_{\mathfrak{J}_D}(z)$$
$$+\frac{\theta_p\big(pq^2t^2A^{-2}B^2\big)\theta_p\big((pq)^{-1}q^{-1}t^{-4}A^2B^{-2}\big)}{\theta_p\big(t^{-2}\big)\theta_p\big(q^{-1}t^{-2}B^{-2}\big)\theta_p\big(q^{-1}t^{-2}A^2\big)\theta_p\big(pq^2t^4A^{-2}B^2\big)\theta_p\big(q^{-1}t^{-2}AB^{-1}C^{\pm1}D^{\pm1}\big)}$$
$$\times\frac{\theta_p\big((qp)^{\frac12}tA^{-1}C^{\pm1}z\big)\theta_p\big((qp)^{\frac12}tBD^{\pm1}z\big)\theta_p\big((pq)^{\frac12}t^3BD^{\pm1}z^{-1}\big)\theta_p\big((pq)^{\frac12}t^3A^{-1}C^{\pm1}z^{-1}\big)}{\theta_p\big(z^2\big)\theta_p\big(t^4z^{-2}\big)}$$
$$\times T_{\mathfrak{J}_D}(z)+\big\{z\leftrightarrow z^{-1}\big\}+T_{\mathfrak{J}_D}(z).$$

Taking away overall factor of

$$\frac{\theta_p\big(pq^2t^2A^{-2}B^2\big)\theta_p\big((pq)^{-1}q^{-1}t^{-4}A^2B^{-2}\big)}{\theta_p\big(t^{-2}\big)\theta_p\big(q^{-1}t^{-2}B^{-2}\big)\theta_p\big(q^{-1}t^{-2}A^2\big)\theta_p\big(q^{-1}t^{-2}AB^{-1}C^{\pm1}D^{\pm1}\big)\theta_p\big(q^{-1}A^2B^{-2}\big)},$$

we get

$$\frac{\theta_p\big((pq)^{\frac12}t^{-1}B^{\pm1}D^{\pm1}z\big)\theta_p\big((pq)^{\frac12}t^{-1}A^{\pm1}C^{\pm1}z\big)}{\theta_p\big(qz^2\big)\theta\big(z^2\big)}T_{\mathfrak{J}_D}(qz)$$
$$+\frac{\theta_p\big(q^{-1}t^{-4}\big)\theta_p\big(q^{-1}t^{-4}A^2B^{-2}z^2\big)\theta_p\big((pq)^{\frac12}t^{\pm1}BD^{\pm1}z\big)\theta_p\big((pq)^{\frac12}t^{\pm1}A^{-1}C^{\pm1}z\big)}{\theta_p\big(q^{-2}t^{-4}A^2B^{-2}\big)\theta_p\big(t^{-4}z^2\big)\theta_p\big(z^2\big)\theta_p\big(q^{-1}z^2\big)}T_{\mathfrak{J}_D}(z)$$
$$+\frac{\theta_p\big(q^{-1}A^2B^{-2}\big)\theta_p\big((pq)^{\frac12}t^2BD^{\pm1}(t^{-1}z)^{\pm1}\big)\theta_p\big((pq)^{\frac12}t^2A^{-1}C^{\pm1}(t^{-1}z)^{\pm1}\big)}{\theta_p\big(q^{-2}t^{-4}A^2B^{-2}\big)\theta_p\big(z^2\big)\theta_p\big(t^4z^{-2}\big)}T_{\mathfrak{J}_D}(z)$$
$$+\big\{z\leftrightarrow z^{-1}\big\}$$
$$+\frac{\theta_p\big(t^{-2}\big)\theta_p\big(q^{-1}t^{-2}B^{-2}\big)\theta_p\big(q^{-1}t^{-2}A^2\big)\theta_p\big(q^{-1}t^{-2}AB^{-1}C^{\pm1}D^{\pm1}\big)\theta_p\big(q^{-1}A^2B^{-2}\big)}{\theta_p\big(pq^2t^2A^{-2}B^2\big)\theta_p\big((pq)^{-1}q^{-1}t^{-4}A^2B^{-2}\big)}T_{\mathfrak{J}_D}(z).$$

To summarize, we have shown that

$$T_{\mathfrak{J}_D}(v)\times_v\big(\big(T_{\mathfrak{J}_B,\mathfrak{J}_C,\mathfrak{J}_D}(w,u,v)\times_w C_{\mathfrak{J}_B}^{(0,0;A^{-1}B)}(w)\big)$$
$$\times_u\big(T_{\mathfrak{J}_B,\mathfrak{J}_C,\mathfrak{J}_D}(h,u,z)\times_h C_{\mathfrak{J}_B}^{(1,0;AB^{-1})}(h)\big)\big),$$