Proof. See Appendix . **Theorem 5:** Under Assumptions 1-3, 5, 7-8,

a) and Assumptions 4a, 6a, the total average treatment effect on compliers is identified:

$$\Delta_1^c = \frac{p_{1|1}}{p_{1|1}-p_{1|0}} E[Y_1|D=1,M=1] - \frac{p_{1|0}}{p_{1|1}-p_{1|0}} E[Q_{11}(Y_0)|D=0,M=1]$$
$$-\frac{p_{0|0}}{p_{1|1}-p_{1|0}} E[Y_1|D=0,M=0] + \frac{p_{0|1}}{p_{1|1}-p_{1|0}} E[Q_{00}(Y_0)|D=1,M=0].$$

Furthermore, the total quantile treatment effect on compliers $\Delta_1^c(q) = F^{-1}_{Y_1(1,1)|c}(q) - F^{-1}_{Y_1(0,0)|c}(q)$ is identified using the inverse of () and ().

b) and Assumptions 4a, 6b, the average indirect effect under $d=0$ on compliers is identified:

$$\delta_1^c(0) = \frac{p_{1|1}}{p_{1|1}-p_{1|0}} E[Q_{11}(Y_0)|D=1,M=1] - \frac{p_{1|0}}{p_{1|1}-p_{1|0}} E[Y_1|D=0,M=1]$$
$$-\frac{p_{0|0}}{p_{1|1}-p_{1|0}} E[Y_1|D=0,M=0] + \frac{p_{0|1}}{p_{1|1}-p_{1|0}} E[Q_{00}(Y_0)|D=1,M=0].$$

Furthermore, the quantile indirect effect under $d=0$ on compliers $\delta_1^c(q,0) = F^{-1}_{Y_1(0,1)|c}(q) - F^{-1}_{Y_1(0,0)|c}(q)$ is identified using the inverse of () and ().

c) and Assumptions 4b, 6a, the average indirect effect under $d=1$ on compliers is identified:

$$\delta_1^c(1) = \frac{p_{1|1}}{p_{1|1}-p_{1|0}} E[Y_1|D=1,M=1] - \frac{p_{1|0}}{p_{1|1}-p_{1|0}} E[Q_{11}(Y_0)|D=0,M=1]$$
$$-\frac{p_{0|0}}{p_{1|1}-p_{1|0}} E[Q_{00}(Y_0)|D=0,M=0] + \frac{p_{0|1}}{p_{1|1}-p_{1|0}} E[Y_1|D=1,M=0].$$

Furthermore, the quantile indirect effect under $d=1$ on compliers $\delta_1^c(q,1) = F^{-1}_{Y_1(1,1)|c}(q) - F^{-1}_{Y_1(1,0)|c}(q)$ is identified using the inverse of () and ().

Proof. See Appendix .