that $E[Y_1(1,0)|D=0,M(0)=0]=E[Q_{10}(Y_0)|D=0,M=0]$ to finish the proof. First, we use () to evaluate $F_{Y_1(1,0)|D=0,M=0}(y)$ at $h(1,0,1,u)$

$$F_{Y_1(1,0)|D=0,M=0}(h(1,0,1,u))=F_{U|10}(h^{-1}(1,0,1;h(1,0,1,u)))=F_{U|10}(u).$$

Applying $F^{-1}_{Y_1(1,0)|D=0,M=0}(q)$ to both sides, we have

$$h(1,0,1,u)=F^{-1}_{Y_1(1,0)|D=0,M=0}(F_{U|10}(u)). \tag{1}$$

Second, for $F_{Y_0(1,0)|D=0,M=0}(y)$ we have

$$F^{-1}_{U|10}(F_{Y_0(1,0)|D=0,M=0}(y))=h^{-1}(1,0,0;y), \tag{2}$$

using (). Combining () and () yields,

$$h(1,0,1,h^{-1}(1,0,0;y))=F^{-1}_{Y_1(1,0)|D=0,M=0}\circ F_{Y_0(1,0)|D=0,M=0}(y). \tag{3}$$

Note that $h(1,0,1,h^{-1}(1,0,0;y))$ maps the period 1 (potential) outcome of an individual with the outcome y in period 0 under treatment without the mediator. Accordingly, $E[F^{-1}_{Y_1(1,0)|D=0,M=0}\circ F_{Y_0(1,0)|D=0,M=0}(Y_0)|D=0,M=0]=E[Y_1(1,0)|D=1,M=0]$. We can identify $F_{Y_0(1,0)|D=0,M=0}(y)$ under Assumption 2, but we cannot identify $F_{Y_1(1,0)|D=0,M=0}(y)$. However, we show in the following that we can identify the overall quantile-quantile transform $F^{-1}_{Y_1(1,0)|D=0,M=0}\circ F_{Y_0(1,0)|D=0,M=0}(y)$ under the additional Assumption 3a. First, we use () to evaluate $F_{Y_1(1,0)|D=1,M=0}(y)$ at $h(1,0,1,u)$

$$F_{Y_1(1,0)|D=10,M=0}(h(1,0,1,u))=F_{U|10}(h^{-1}(1,0,1;h(1,0,1,u)))=F_{U|10}(u).$$

Applying $F^{-1}_{Y_1(1,0)|D=1,M=0}(q)$ to both sides, we have