$\mathbf{X} \sim \mathbf{E_p}(\mathbf{h}, \mu, \mathbf{\Sigma})$, if it has a density function given by

$$f_0(\mathbf{x}) \propto |\mathbf{\Sigma^{-1/2}}|\mathbf{h}((\mathbf{x}-\mu)^\top \mathbf{\Sigma^{-1}}(\mathbf{x}-\mu)).$$

where h is a non-negative scalar function, μ is the location parameter and $\mathbf{\Sigma}$ is a $p \times p$ positive definite matrix. Denote by F_0 the corresponding distribution function and by $\Delta_\mathbf{x} = (\mathbf{x}-\mu)^\top \mathbf{\Sigma^{-1}}(\mathbf{x}-\mu)$ the squared Mahalanobis distance of a p-dimensional point $\mathbf{x}$. By Theorem 3.3 of **?** if a depth is affine equivariant () and has maximum at μ () (see Appendix) then a depth is such that $d(\mathbf{x}; \mathbf{F_0}) = \mathbf{g}(\mathbf{\Delta_x})$ for some non increasing function g and we can restrict ourselves without loss of generality, to the case $\mu = \mathbf{0}$ and $\mathbf{\Sigma} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix of dimension p. Under this setting, it is easy to see that the half-space depth of a given point $\mathbf{x}$ is given by $d_{HS}(\mathbf{x}; \mathbf{F_0}) = \mathbf{1} - \mathbf{F_{0,1}}(\sqrt{\mathbf{\Delta_x}})$, where $F_{0,1}$ is a marginal distribution of $\mathbf{X}$. If the function h is such that

$$\frac{\exp(-\frac{1}{2}\Delta)}{h(\Delta)} \to 0, \qquad \Delta \to \infty,$$

then, there exists a Δ^* such that for all $\mathbf{x}$ so that $\Delta_\mathbf{x} > \Delta^*$, $d_{HS}(\mathbf{x}; \mathbf{F_0}) \geq \mathbf{d_{HS}}(\mathbf{x}; \mathbf{\Phi})$, where Φ is the distribution function of the standard normal. Hence,

$$\sup_{\{\mathbf{x}:\mathbf{\Delta_x}>\mathbf{\Delta^*}\}} [d_{HS}(\mathbf{x}; \mathbf{\Phi}) - \mathbf{d_{HS}}(\mathbf{x}; \mathbf{F_0})] < \mathbf{0}$$

and therefore, for all $\beta > 1 - 2F_{0,1}(-\sqrt{(}\Delta^*))$,

$$\sup_{C^\beta(F_0)} [d_{HS}(\mathbf{x}; \mathbf{\Phi}) - \mathbf{d_{HS}}(\mathbf{x}; \mathbf{F_0})] < \mathbf{0}\ .$$

Given an independent and identically distributed sample $\mathbf{X_1}, \ldots, \mathbf{X_n}$, we define the filter in general dimension p introduced previously, where here we use the half-space depth

$$d_n = \sup_{\mathbf{x} \in \mathbf{C^\beta(F)}} \{d_{HS}(\mathbf{x}; \mathbf{\hat{F}_n}) - \mathbf{d_{HS}}(\mathbf{x}; \mathbf{F}(\mathbf{T_{0n}}, \mathbf{C_{0n}}))\}^+,$$

where β is a high order quantile, $\hat{F}_n(\cdot)$ is the empirical distribution function and $F(\mathbf{T_{0n}}, \mathbf{C_{0n}})$ is a chosen reference distribution which depends on a pair of initial location and dispersion estimators, $\mathbf{T_{0n}}$ and $\mathbf{C_{0n}}$. Hereafter, we are going to use the normal distribution $F = N(\mathbf{T_{0n}}, \mathbf{C_{0n}})$. For $\mathbf{T_{0n}}$ and $\mathbf{C_{0n}}$ one might use, e.g., the coordinate-wise median and the coordinate-wise MAD for a univariate filter as in **?**. In order to compute the value d_n, we have to identify the set $C^\beta(F) = \{\mathbf{x} \in \mathbb{R}^\mathbf{p} | \mathbf{d_{HS}}(\mathbf{x}, \mathbf{F}) \leq \mathbf{d_{HS}}(\eta_\beta, \mathbf{F})\}$ where η_β is a large quantile of F. By Corollary 4.3 in and denoting with