U is constant across T for each individual i. For example, Assumption 3 is satisfied in the fixed effect model $U = \eta + v_t$, with η being a time-invariant individual-specific unobservable (fixed effect) and v_t an idiosyncratic time-varying unobservable with the same distribution in both time periods. and impose time invariance conditional on the treatment status, $UT|D = d$, to identify the average treatment effect on the treated, $\varphi_1 = E[Y_1(1, M(1)) - Y_1(0, M(0))|D = 1]$ or local average treatment effect, $\varphi_1 = E[Y_1(1, M(1)) - Y_1(0, M(0))|\tau = c]$, respectively. We additionally condition on the mediator status to identify direct and indirect effects. For our next assumption, we introduce some further notation. Let $F_{U|d,m}(u)) = \Pr(U \leq u|D = d, M = m)$ be the conditional distribution of U with support $\mathbb{U}_{dm}$.

Assumption 4: Common support given $M = 0$.

(a) $\mathbb{U}_{10} \subseteq \mathbb{U}_{00}$,

(b) $\mathbb{U}_{00} \subseteq \mathbb{U}_{10}$.

Assumption 4a is a common support assumption, implying that any possible value of U in the population with $D = 1, M = 0$ is also contained in the population with $D = 0, M = 0$. Assumption 4b imposes that any value of U conditional on $D = 0, M = 0$ also exists conditional on $D = 1, M = 0$. Both assumptions together imply that the support of U is the same in both populations, albeit the distributions may generally differ. Assumptions 1 to 3 permit identifying direct effects on mixed populations of never-takers and defiers as well as never-takers and compliers, respectively, as formally stated in Theorem 1.

Theorem 1: Under Assumptions 1–3,

(a) and Assumption 4a, the average and quantile direct effects under $d = 1$ conditional on $D = 1$ and $M(1) = 0$ are identified:

$$\theta_1^{1,0}(1) = E[Y_1 - Q_{00}(Y_0)|D = 1, M = 0],$$

$$\theta_1^{1,0}(q, 1) = F^{-1}_{Y_1|D=1,M=0}(q) - F^{-1}_{Q_{00}(Y_0)|D=1,M=0}(q).$$

(b) and Assumption 4b, the average and quantile direct effects under $d = 0$