expectations gives

$$\begin{aligned}F_{Y_1(1,0)|D=0,M(0)=0}(y) &= \frac{p_n}{p_n+p_c}F_{Y_1(1,0)|D=0,M(0)=0,\tau=n}(y)\\&+\frac{p_c}{p_n+p_c}F_{Y_1(1,0)|D=0,M(0)=0,\tau=c}(y),\\&\overset{A7}{=}\frac{p_n}{p_n+p_c}F_{Y_1(1,0)|\tau=n}(y)+\frac{p_c}{p_n+p_c}F_{Y_1(1,0)|\tau=c}(y).\end{aligned}$$

Using () and rearranging the equation gives,

$$F_{Y_1(1,0)|\tau=c}(y) = \frac{p_{0|0}}{p_{0|0}-p_{0|1}}F_{Q_{10}(Y_0)|D=0,M=0}(y) - \frac{p_{0|1}}{p_{0|0}-p_{0|1}}F_{Y_1|D=1,M=0}(y). \tag{1}$$

In analogy to (), the outcome distribution under $D=0$ and $M=0$ equals:

$$F_{Y_1|D=0,M=0}(y) = \frac{p_n}{p_n+p_c}F_{Y_1(0,0)|\tau=n}(y)+\frac{p_c}{p_n+p_c}F_{Y_1(0,0)|\tau=c}(y).$$

Using () and rearranging the equation gives

$$F_{Y_1(0,0)|\tau=c}(y) = \frac{p_{0|0}}{p_{0|0}-p_{0|1}}F_{Y_1|D=0,M=0}(y) - \frac{p_{0|1}}{p_{0|0}-p_{0|1}}F_{Q_{00}(Y_0)|D=1,M=0}(y). \tag{2}$$

1 Proof of Theorem 4

1.1 Average direct effect on the always-takers

In the following, we show that $\theta_1^a = E[Y_1(1,1)-Y_1(0,1)|\tau=a] = E[Q_{11}(Y_0)-Y_1|D=0,M=1]$. From (), we obtain the first ingredient $E[Y_1(0,1)|a] = E[Y_1|D=0,M=1]$. Furthermore, from () we have $E[Q_{11}(Y_0)|D=0,M=1] = E[Y_1(1,1)|D=0,M(0)=1]$. Under Assumption 7 and 8,