0.1 Estimation

-in estimates. Taking $\theta_1^{1,0}$ (see Theorem 1) as an example, an estimate thereof is

$$\hat{\theta}_1^{1,0}(1) = \frac{1}{\sum_{i=1}^{n} I\{D_i = 1, M_i = 0, T_i = 1\}} \sum_{i:D_i=1,M_i=0,T_i=1} Y_i$$
$$- \frac{1}{\sum_{i=1}^{n} I\{D_i = 1, M_i = 0, T_i = 0\}} \sum_{i:D_i=1,M_i=0,T_i=0} \hat{Q}_{00}(Y_i).$$

Likewhise, quantile effects are estimated based on the empirical quantiles. For the estimation of total ATE and QTE, show that the resulting estimators are $\sqrt{N}$-consistent and asymptotically normal, see their Theorems 5.1 and 5.3. These properties also apply to our context when splitting the sample into subgroups based on the values of a binary treatment and mediator (rather than the treatment only). For instance, the implications of Theorem 1 in when considering subsamples with $D = 1$ and $D = 0$ carry over to considering subsamples with $D = 1, M = 0$ and $D = 0, M = 0$ for estimating the average direct effect on never-takers. In contrast to , however, some of our identification results include the conditional mediator probabilities $\Pr(M = m|D = d)$. As the latter are estimated with $\sqrt{N}$-consistency, too, it follows that the resulting effect estimators are again $\sqrt{N}$-consistent and asymptotically normal. We use a non-parametric bootstrap approach to calculate the standard errors. show the validity of the bootstrap approach for such kind of estimators, which follows from their asymptotic normality. For the case that identifying assumptions to only hold conditional on observed covariates, denoted by X, estimation must be adapted to allow for control variables. Following a suggestion by in their Section 5.1, basing estimation on outcome residuals in which the association of X and Y has been purged by means of a regression is consistent under the additional assumption that the effects of D and M are homogeneous across covariates. As an alternative, propose a flexible semiparametric estimator that does not impose such a homogeneity-in-covariates assumption and show $\sqrt{N}$-consistency and asymptotic normality.