

Second, for  $F_{Y_0(0,0)|D=0,M=0}(y)$  we have

$$F_{U|00}^{-1}(F_{Y_0(0,0)|D=0,M=0}(y)) = h^{-1}(0, 0, 0; y). \quad (1)$$

Combining ( ) and ( ) yields,

$$h(0, 0, 1, h^{-1}(0, 0, 0; y)) = F_{Y_1(0,0)|D=0,M=0}^{-1} \circ F_{Y_0(0,0)|D=0,M=0}(y). \quad (2)$$

The left sides of ( ) and ( ) are equal. In contrast to ( ), ( ) contains only distributions that can be identified from observable data. In particular,  $F_{Y_t(0,0)|D=0,M=0}(y) = \Pr(Y_t(0,0) \leq y | D = 0, M = 0) = \Pr(Y_t \leq y | D = 0, M = 0)$ . Accordingly, we can identify  $F_{Y_1(0,0)|D=1,M=0}^{-1} \circ F_{Y_0(0,0)|D=1,M=0}(y)$  by  $Q_{00}(y) \equiv F_{Y_1|D=0,M=0}^{-1} \circ F_{Y_0|D=0,M=0}(y)$ . Parsing  $Y_0$  through  $Q_{00}(\cdot)$  in the treated group without mediator gives

$$\begin{aligned} & E[Q_{00}(Y_0) | D = 1, M = 0] \\ &= E[F_{Y_1|D=0,M=0}^{-1} \circ F_{Y_0|D=0,M=0}(Y_0) | D = 1, M = 0], \\ &= E[F_{Y_1(0,0)|D=0,M=0}^{-1} \circ F_{Y_0(0,0)|D=0,M=0}(Y_0(1,0)) | D = 1, M = 0], \\ &\stackrel{A1, A3b}{=} E[h(0, 0, 1, h^{-1}(0, 0, 0; Y_0(1,0))) | D = 1, M = 0], \\ &\stackrel{A2}{=} E[h(0, 0, 1, h^{-1}(0, 0, 0; Y_0(0,0))) | D = 1, M = 0], \\ &\stackrel{A1, A3a}{=} E[F_{Y_1(0,0)|D=1,M=0}^{-1} \circ F_{Y_0(0,0)|D=1,M=0}(Y_0(0,0)) | D = 1, M = 0], \\ &= E[Y_1(0,0) | D = 1, M = 0] = E[Y_1(0,0) | D = 1, M(1) = 0], \end{aligned} \quad (3)$$

which has data support because of Assumption 4a.