result given by

$$Z = \frac{1}{|\mathcal{W}|} \int_{\text{Cartan}} [d\sigma]\; e^{-\frac{8\pi^3 r}{g_{YM}^2}\mathrm{Tr}(\sigma^2)} \det{}_{\mathrm{Ad}} \left(\sin(\mathrm{i}\pi\sigma)\mathrm{e}^{\frac{1}{2}\mathrm{f}(\mathrm{i}\sigma)}\right)$$
$$\times \prod_I \det{}_{\mathrm{R_I}} \left((\cos(\mathrm{i}\pi\sigma))^{\frac{1}{4}} \mathrm{e}^{-\frac{1}{4}\mathrm{f}(\frac{1}{2}-\mathrm{i}\sigma)-\frac{1}{4}\mathrm{f}(\frac{1}{2}+\mathrm{i}\sigma)}\right) + \mathrm{O}\left(\mathrm{e}^{\frac{-16\pi^3 \mathrm{r}}{\mathrm{g_{YM}^2}}}\right) \tag{1}$$

where r is the radius of S^5, σ is a dimensionless matrix, and f is defined as

$$f(y) = \frac{i\pi y^3}{3} + y^2 \log\left(1 - e^{-2\pi i y}\right) + \frac{iy}{\pi}\mathrm{Li}_2\left(e^{-2\pi i y}\right) + \frac{1}{2\pi^2}\mathrm{Li}_3\left(e^{-2\pi i y}\right) - \frac{\zeta(3)}{2\pi^2} \tag{2}$$

The quotient by the Weyl group in amounts to division by a simple numerical factor $|\mathcal{W}| = 2^N N!$. The integral over σ is not restricted to a Weyl chamber. Though this localization result was obtained in the IR theory, it is expected to hold in the UV due to the assumed Q-exactness of the irrelevant UV completion terms. One may rewrite the partition function in terms of the free energy as

$$Z = \frac{1}{|\mathcal{W}|} \int_{\text{Cartan}} [d\sigma]\, e^{-F(\sigma)} + O\left(e^{\frac{-16\pi^3 r}{g_{YM}^2}}\right)$$
$$F(\sigma) = \frac{4\pi^3 r}{g_{YM}^2}\mathrm{Tr}\,\sigma^2 + \mathrm{Tr}_{\mathrm{Ad}} F_V(\sigma) + \sum_I \mathrm{Tr}_{R_I} F_H(\sigma) \tag{3}$$

The definitions of $F_V(\sigma)$ and $F_H(\sigma)$ follow simply from , and using one may obtain the following large argument expansions

$$F_V(\sigma) \approx \frac{\pi}{6}|\sigma|^3 - \pi|\sigma| \qquad\qquad F_H(\sigma) \approx -\frac{\pi}{6}|\sigma|^3 - \frac{\pi}{8}|\sigma| \tag{4}$$

It was argued in that in the large N limit, the perturbative Yang-Mills term - i.e. the first term in the expression for $F(\sigma)$ in - can be neglected, as can be the instanton contributions. Thus in our evaluation of the free energy, we will only concern ourselves with the contributions coming from $F_V(\sigma)$ and $F_H(\sigma)$. The first step in the evaluation of is recasting the matrix integral in a simpler form. The integral over σ in is an integration over the Coulomb branch, which is parameterized by the non-zero vevs of σ. One may write

$$\sigma = \mathrm{diag}\{\lambda_1, \ldots, \lambda_N, -\lambda_1, \ldots, -\lambda_N\} \tag{5}$$

since $USp(2N)$ has N elements in its Cartan. The integration variables are these N λ_i. Normalizing the weights of the fundamental representation of $USp(2N)$ to be $\pm e_i$ with e_i forming a basis of unit vectors for $\mathbb{R}^N$, it follows that the adjoint representation has weights $\pm 2e_i$ and $e_i \pm e_j$ for all $i \neq j$, whereas the anti-symmetric representation has only weights