Let $\mathbf{X}$ be a $\mathbb{R}^p$-valued random variable with distribution function F. For a point $\mathbf{x} \in \mathbb{R}^{\mathbf{p}}$, we consider the statistical data depth of $\mathbf{x}$ with respect to F be $d(\mathbf{x}; \mathbf{F})$ such that d satisfies the four properties given in **?** and **?** and reported in Appendix of the Supplementary Material. Given an independent and identically distributed sample $\mathbf{X_1}, \ldots, \mathbf{X_n}$ of size n, we denote $\hat{F}_n(\cdot)$ its empirical distribution function and by $d(\mathbf{x}; \hat{\mathbf{F}}_\mathbf{n})$ the sample depth. We assume that, $d(\mathbf{x}; \hat{\mathbf{F}}_\mathbf{n})$ is a uniform consistent estimator of $d(\mathbf{x}; \mathbf{F})$, that is,

$$\sup_{\mathbf{x}} |d(\mathbf{x}; \hat{\mathbf{F}}_\mathbf{n}) - \mathbf{d}(\mathbf{x}; \mathbf{F})| \stackrel{\mathbf{a.s.}}{\to} \mathbf{0} \qquad \mathbf{n} \to \infty,$$

a property enjoined by many statistical data depth functions, e.g., among others simplicial depth (**?**), half-space depth (**?**). One important feature of the depth functions is the α-depth trimmed region given by $R_\alpha(F) = \{\mathbf{x} \in \mathbb{R}^{\mathbf{p}} : \mathbf{d}(\mathbf{x}; \mathbf{F}) \geq \alpha\}$; for any $\beta \in [0, 1]$, we will denote $R^\beta(F)$ the smallest region $R_\alpha(F)$ that has probability larger that or equal to β according to F. Throughout, subscripts and superscripts for depth regions are used for depth levels and probability contents, respectively. Let $C^\beta(F)$ be the complement in $\mathbb{R}^p$ of the set $R^\beta(F)$. Let $m = \max_{\mathbf{x}} d(\mathbf{x}; \mathbf{F})$, be the maximum of the depth, for simplicial depth $m \leq 2^{-p}$, for half-space depth $m \leq 1/2$. Given a high order quantile β, we define a filter of dimension p based on

$$d_n = \sup_{\mathbf{x} \in \mathbf{C}^\beta(\mathbf{F})} \{d(\mathbf{x}; \hat{\mathbf{F}}_\mathbf{n}) - \mathbf{d}(\mathbf{x}; \mathbf{F})\}^+, \tag{1}$$

where $\{a\}^+$ represents the positive part of a, and we mark as outliers all the $\lfloor nd_n/m \rfloor$ observations with the smallest population depth (where $\lfloor a \rfloor$ is the largest integer less then or equal to a). This define a filter in the general dimension p. We have the following result, with obvious proof. If $\sup_{\mathbf{x}} |d(\mathbf{x}; \hat{\mathbf{F}}_\mathbf{n}) - \mathbf{d}(\mathbf{x}; \mathbf{F})| = \mathbf{o}(\mathbf{n})$ (a.s.) then $nd_n \to 0$ as $n \to \infty$. If the above result holds, then the filter would be consistent. In the next subsection we are going to illustrate this approach using the half-space depth.

0.1 Filters based on Half-space Depth

Let $\mathbf{X}$ be a $\mathbb{R}^p$-valued random variable with distribution function F. For a point $\mathbf{x} \in \mathbb{R}^{\mathbf{p}}$, the half-space depth of $\mathbf{x}$ with respect to F is defined as the minimum probability of all closed half-spaces including $\mathbf{x}$:

$$d_{HS}(\mathbf{x}; \mathbf{F}) = \min_{\mathbf{H} \in \mathcal{H}(\mathbf{x})} \mathbf{P_F}(\mathbf{X} \in \mathbf{H}).$$

where $\mathcal{H}(\mathbf{x})$ indicates the set of all half-spaces in $\mathbb{R}^p$ containing $\mathbf{x} \in \mathbb{R}^{\mathbf{p}}$. A random vector $\mathbf{X} \in \mathbb{R}^{\mathbf{p}}$ is said elliptically symmetric distributed, denoted by