0.1 Estimation

As in Assumption 5.1 of , we assume standard regularity conditions, namely that conditional on $T = t$, $D = d$, and $M = m$, Y is a random draw from that subpopulation defined in terms of $t, d, m \in \{1, 0\}$. Furthermore, the outcome in the subpopulations required for the identification results of interest must have compact support and a density that is bounded from above and below as well as continuously differentiable. Denote by N the total sample size across both periods and all treatment-mediator combinations and by $i \in \{1, ..., N\}$ an index for the sampled subject, such that (Y_i, D_i, M_i, T_i) correspond to sample realizations of the random variables (Y, D, M, T). The total, direct, and indirect effects may be estimated using the sample analogy principle, which replaces population moments with sample moments. For instance, any conditional mediator probability given the treatment, $\Pr(M = m|D = d)$, is to be replaced by an estimate thereof in the sample, $\frac{\sum_{i=1}^{N} I\{M_i=m, D_i=d\}}{\sum_{i=1}^{N} I\{D_i=d\}}$. A crucial step is the estimation of the quantile-quantile transforms. The application of such quantile transformations dates at least back to , see also , , and for recent applications. First, it requires estimating the conditional outcome distribution, $F_{Y_t|D=d,M=m}(y)$, by the conditional empirical distribution $\hat{F}_{Y_t|D=d,M=m}(y) = \frac{1}{\sum_{i=1}^{n} I\{D_i=d, M_i=m, T_i=t\}} \sum_{i:D_i=d,M_i=m,T_i=t} I\{Y_i \leq y\}$. Second, inverting the latter yields the empirical quantile function $\hat{F}^{-1}_{Y_t|D=d,M=m}(q)$. The empirical quantile-quantile transform is then obtained by

$$\hat{Q}_{dm}(y) = \hat{F}^{-1}_{Y_1|D=d,M=m}(\hat{F}_{Y_0|D=d,M=m}(y)).$$

This permits estimating the average and quantile effects of interest. Average effects are estimated by replacing any (conditional) expectations with the corresponding sample averages in which the estimated quantile-quantile transforms enter as plug-