(located at some cutoff distance $u = \Lambda$), while $\mathcal{K}$ is the trace of the extrinsic curvature $\mathcal{K}_{ij}$ of the radial S^5 slices. The latter is defined as

$$\mathcal{K}_i = \frac{d}{du}\gamma_i \tag{1}$$

In general, the on-shell action is divergent and requires renormalization. The addition of infinite counterterms is standard in holographic renormalization , but in the current case we must also add finite counterterms in order to preserve supersymmetry . We will begin our exploration of counterterms in this section by first considering the finite counterterms in the limit of a flat domain wall, after which we move onto infinite counterterms in the more general case of a curved domain wall. Finally, appropriate curved space finite counterterms will be fixed by demanding finiteness of the one-point functions of the dual operators.

0.1 Finite counterterms

In order to obtain finite counterterms, we will make use of the Bogomolnyi trick . To do so, we will first need to identify a superpotential W. Though we will find that no exact superpotential can be found for our solutions - in the sense that there is no superpotential which can recast all of the BPS equations in gradient flow form - we will be able to identify an *approximate* superpotential. By "approximate" here, we mean that it does yield gradient flow equations up to terms of order $O(z^5)$, where the asymptotic coordinate z was defined earlier as $z = e^{-u}$. This is useful since, as we will see later, we will only need terms up to $O(z^5)$ to obtain all divergent and finite counterterms. Terms of higher order will all vanish in the $\epsilon \to 0$ limit, i.e. when the UV cutoff is removed. Thus the approximate superpotential will yield all finite counterterms.

0.1.1 Approximate superpotential

In order to identify a candidate superpotential, we begin by recalling the form of the scalar potential V. With the choice of coset representative and consistent truncation outlined in Section , one finds that

$$V(\sigma, \phi^i) = -9m^2 e^{2\sigma} - 12m^2 e^{-2\sigma}\cosh\phi^0\cos\phi^3 + m^2 e^{-6\sigma}\cosh^2\phi^0 + m^2 e^{-6\sigma}\cos 2\phi^3 \sinh^2\phi^0$$

This scalar potential can in fact be rewritten as

$$V = 4(N_0^2 + N_3^2) + \frac{1}{4}(M_0^2 + M_3^2) - 20(S_0^2 + S_3^2) \tag{2}$$

Then for BPS solutions, implies that

$$V = (\sigma')^2 + \frac{1}{4}\left(-(\phi^{3\prime})^2 + \cos^2\phi^3(\phi^{0\prime})^2\right) - 20(S_0^2 + S_3^2) \tag{3}$$