for $C^s_{-(s-n)}$ an arbitrary component of the master field C. Taking $m \to -m$ in the set of equations (), (20), () and $s = m+1$ in (), we can iteratively determine the dependence of the C^{m+1}_m on the C^1_0. From the equation () we obtain

$$\partial_z C^{m+1}_{-m} + \frac{e^\rho}{2} g_3^{2(m+2)}(1,-m-1) C^{m+2}_{-m-1} = 0 \qquad (1)$$

taking into consideration that for certain components C^s_n it is required $|n| \leq s-1$ this iteratively leads to relation of C^{m+1}_m and C^1_0, and from the () analogously for C^{m+1}_{-m} and C^1_0. The general form of the $C^s_\pm$ is then given in terms of $C^{m+1}_{\pm m}$ and coefficients $g^{ts}_u(m,n)$. Knowing $C^s_\pm$ and $C^{m+1}_{\pm m}$ allows to obtain

$$(\delta C)^1_0 = \sum_{n=1}^{s} f^{s,n}_\pm(\lambda) \partial_z^{n-1} \Lambda^{(s)} \partial_z^{s-n} \phi \qquad (2)$$

for $\phi \equiv C^1_0$ and $f^{s,n}_\pm(\lambda)$ expressed in terms of coefficients $g^{st}_u(m,n)$ Using the replacement $\partial_\rho \to -(1 \pm \lambda)$ and writing explicitly first few n values for $f^{s,n}_\pm(\lambda)$, allows to determine its general expression

$$f^{s,n}_\pm(\lambda) = (-1)^s \frac{\Gamma(s+\lambda)}{\Gamma(s-n+1\pm\lambda)} \frac{1}{2^{n-1}(2(\frac{n}{2}-1))!!\left(\frac{n-1}{2}\right)!} \times \prod_{j=1}^{\frac{n-1}{2}} \frac{s+1-n}{2s-2j-1}. \qquad (3)$$

Substituting () in () one obtains the variation of the scalar field

$$(\delta C)^1_0 = \sum_{n=1}^{s} (-1)^s \frac{\Gamma(s\pm\lambda)}{\Gamma(s-n+1\pm\lambda)} \frac{1}{2^{n-1}\left(2\left(\frac{n}{2}\right)-1\right)!!\left(\frac{n-1}{2}\right)!} \times \prod_{j=1}^{\left(\frac{n-1}{2}\right)} \frac{s+j-n}{2s-2j-1} \partial_z^{n-1} \Lambda^{(s)} \partial_z^{s-n} C^1_0. \qquad (4)$$

To consider the coefficient in front, we focus on the term with the lowest number of ∂_z derivatives on the gauge field $\Lambda^{(s)}$, obtained for n=1. Then, () becomes

$$(\delta C)^1_0|_{n=1} = (-1)^s \Lambda^{(s)} \partial^{s-1} C^1_0. \qquad (5)$$

To obtain the linearised equation of motion for the scalar field we act on () with KG operator (). This can be written as

$$\Box_{KG} \tilde{C}^1_0 = \Box_{KG} C^1_0 + \Box_{KG} \delta C^1_0. \qquad (6)$$

Taking $\partial_\rho \to (1 \pm \lambda)$ in $f^{s,n}_\pm(\lambda)$ we have taken and considering the term with highest number of derivatives on C^1_0 leads to

$$\Box_{KG}|_{\text{highest number of derivatives}} (\delta C)^1_0 = \qquad (7)$$
$$= (-1)^s 4e^{-2\rho} \partial(\bar{\partial}\Lambda^{(s)} \partial^{(s-1)} C^1_0) \qquad (8)$$
$$= (-1)^s 4e^{-2\rho} [\partial\bar{\partial}\bar{\Lambda}^{(s)} \partial^{(s-1)} C^1_0 + \bar{\partial}\Lambda^{(s)} \partial^s C^1_0 + \partial\Lambda^{(s)} \bar{\partial}\partial^{(s-1)} C^1_0 + \Lambda^{(s)} \bar{\partial}\partial^s C^1_0]. \qquad (9)$$

The term in () that is of further interest is the one multiplying $4e^{-2\rho}\partial\bar{\partial}$ acting on δC^1_0 which is convenient to compute in the metric formulation.

I. METRIC FORMULATION

In the metric formulation we can express the higher spin field of arbitrary spin s with

$$\phi_{\mu_1....\mu_s} = tr\left(\tilde{e}_{(\mu_1} ... \tilde{e}_{\mu_{s-1}} \tilde{E}_{\mu_s)}\right) \qquad (10)$$

where $\tilde{E}_{\mu s} = \tilde{A}_\mu - \tilde{\bar{A}}_\mu$ and $\tilde{A}_\mu$ and $\tilde{\bar{A}}_\mu$ we define below. The dreibein is determined from the background AdS metric ()

$$e_z = \frac{1}{2} e^\rho (L_1 + L_{-1}) = \frac{1}{2} e^\rho (V^2_1 + V^2_{-1}) \qquad (11)$$
$$e_{\bar{z}} = \frac{1}{2} e^\rho (L_1 - L_{-1}) = \frac{1}{2} e^\rho (V^2_1 - V^2_{-1}) \qquad (12)$$
$$e_\rho = L_0 = V^2_0. \qquad (13)$$

The invariance of the equation () under the gauge transformation for $hs[\lambda] \oplus hs[\lambda]$ for the fields A means

$$A \to A + d\Lambda + [A, \Lambda]_\star \equiv \tilde{A} \qquad (14)$$
$$\bar{A} \to \bar{A} + d\bar{\Lambda} + \left[\bar{A}, \bar{\Lambda}\right]_\star \equiv \tilde{\bar{A}}. \qquad (15)$$

Since Λ parameter is chiral it means $\bar{\Lambda} = 0$ and the field $\tilde{\bar{A}}$ is essentially unchanged. The field $\tilde{A}_\mu$ is then

$$\tilde{A} = A_{AdS} + d\Lambda + [A_{AdS}, \Lambda]_\star. \qquad (16)$$

$d\Lambda$ reads

$$d\Lambda = \sum_{n=1}^{2s-1} \frac{1}{(n-1)!} V^s_{s-n} e^{(s-n)\rho} [(-\partial)^{n-1} \partial\Lambda^{(s)}(z,\bar{z}) dz \qquad (17)$$
$$+ (-\partial)^{n-1} \bar{\partial}\Lambda^{(s)}(z,\bar{z}) d\bar{z} + (-\partial)^{n-1} \Lambda^{(s)}(z,\bar{z})(s-n) d\rho] \qquad (18)$$

and

$$[A_{AdS}, \Lambda]_\star = [e^\rho V^2_1 dz + V^2_0 d\rho, \sum_{n=1}^{2s-1} \frac{1}{(n-1)!} (-\partial)^{n-1} \Lambda^{(s)}(z,\bar{z}) e^{(s-n)\rho} V^s_{s-n}] \qquad (19)$$

To read out the coupling we focus on $\bar{z}....\bar{z}$ component of the field C^1_0 with lowest number of derivatives on gauge field $\Lambda^{(s)}$. The $\star$ multiplication of the dreibeins in () in that case contributes only with first $g^{st}_u(m,n;\lambda)$ coefficient with the each following dreibein that is being multiplied. More explicitly

$$e_{\bar{z}} \star e_{\bar{z}} = \frac{1}{2^2} e^{2\rho} \left(V^2_1 - V^2_{-1}\right) \star (V^2_1 - V^2_{-1}) \qquad (20)$$

From () we notice that the lowest number of derivatives on Λ will appear for lowest n, i.e. for $n = 1$ in summation (). Knowing the relation for the trace of higher spin generators, the required generator V^s_{s-n} will than