Let $\Delta_1^\tau = E[Y_1(1, M(1)) - Y_1(0, M(0))|\tau]$ denote the ATE conditional on $\tau \in \{a, c, de, n\}$; $\theta_1^\tau(d)$ and $\delta_1^\tau(d)$ denote the corresponding direct and indirect effects. Because $M(1) = M(0) = 0$ for any never-taker, the indirect effect for this group is by definition zero ($\delta_1^n(d) = E[Y_1(d, 0) - Y_1(d, 0)|\tau = n] = 0$) and $\Delta_1^n = E[Y_1(1, 0) - Y_1(0, 0)|\tau = n] = \theta_1^n(1) = \theta_1^n(0) = \theta_1^n$ equals the direct effect for never-takers. Correspondingly, because $M(1) = M(0) = 1$ for any always-taker, the indirect effect for this group is by definition zero ($\delta_1^a(d) = E[Y_1(d, 1) - Y_1(d, 1)|\tau = a] = 0$) and $\Delta_1^a = E[Y_1(1, 1) - Y_1(0, 1)|\tau = a] = \theta_1^a(1) = \theta_1^a(0) = \theta_1^a$ equals the direct effect for always-takers. For the compliers, both direct and indirect effects may exist. Note that $M(d) = d$ due to the definition of compliers. Accordingly, $\theta_1^c(d) = E[Y_1(1, d) - Y_1(0, d)|\tau = c]$ equals the direct effect for compliers, $\delta_1^c(d) = E[Y_1(d, 1) - Y_1(d, 0)|\tau = c]$ equals the indirect effect for compliers, and $\Delta_1^c = E[Y_1(1, 1) - Y_1(0, 0)|\tau = c]$ equals the total effect for compliers. In the absence of any direct effect, the indirect effects on the compliers are homogeneous, $\delta_1^c(1) = \delta_1^c(0) = \delta_1^c$, and correspond to the local average treatment effect. Analogous results hold for the defiers. As already mentioned, we will also consider direct effects conditional on specific values $D = d$ and mediator states $M = M(d) = m$, which are denoted by $\theta_1^{d,m}(d) = E[Y_1(1, m) - Y_1(0, m)|D = d, M(d) = m]$. These parameters are identified under weaker assumptions than strata-specific effects, but are also less straightforward to interpret, as they refer to mixtures of two strata. For instance, $\theta_1^{1,0}(1) = E[Y_1(1, 0) - Y_1(0, 0)|D = 1, M(1) = 0]$ is the effect on a mixture of never-takers and defiers, as these two groups satisfy $M(1) = 0$. Likewise, $\theta_1^{0,0}(0)$ refers to never-takers and compliers satisfying $M(0) = 0$, $\theta_1^{0,1}(0)$ to always-takers and defiers satisfying $M(0) = 1$, and $\theta_1^{1,1}(1)$ to always-takers and compliers satisfying $M(1) = 1$.

## 0.1 Quantile effects

We denote by $F_{Y_t(d,m)}(y) = \Pr(Y_t(d, m) \leq y)$ the cumulative distribution function of $Y_t(d, m)$ at outcome level $y$. Its inverse, $F^{-1}_{Y_t(d,m)}(q) = \inf\{y : F_{Y_t(d,m)}(y) \geq q\}$, is the quantile function of $Y_t(d, m)$ at rank $q$. The total QTE are denoted by