From the second term of we get

$$\frac{\theta_p\big(pq^2t^2A^{-2}B^2\big)\theta_p\big((pq)^{-1}q^{-1}t^{-4}A^2B^{-2}\big)}{\theta_p\big(t^{-2}\big)\theta_p\big(q^{-1}t^{-2}B^{-2}\big)\theta_p\big(q^{-1}t^{-2}A^2\big)\theta_p\big(pq^2t^4A^{-2}B^2\big)\theta_p\big(q^{-1}t^{-2}AB^{-1}C^{\pm1}D^{\pm1}\big)}$$
$$\times\oint\frac{\mathrm{d}u}{4\pi iu}\frac{\Gamma_e\big((qp)^{\frac12}t^{-1}A^{\pm1}D^{\pm1}u^{\pm1}\big)\Gamma_e\big((qp)^{\frac12}t^{-1}B^{\pm1}C^{\pm1}u^{\pm1}\big)}{\Gamma\big(u^{\pm2}\big)}$$
$$\times\Gamma_e\big((pq)^{\frac12}tB^{-1}C^{\pm1}u^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}tAD^{\pm1}u^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}tA^{-1}u^{\pm1}D^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}tBu^{\pm1}C^{\pm1}\big)$$
$$\times\Gamma_e\big(AB^{-1}u^{\pm1}z^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}q^{\frac12}tBD^{\pm1}\big(q^{\frac12}z\big)^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}q^{\frac12}tA^{-1}C^{\pm1}\big(q^{\frac12}z\big)^{\pm1}\big)$$
$$\times\oint\frac{\mathrm{d}v}{4\pi iv}\frac{\Gamma_e\big((qp)^{\frac12}t^{-1}A^{\pm1}C^{\pm1}v^{\pm1}\big)\Gamma_e\big((qp)^{\frac12}t^{-1}B^{\pm1}D^{\pm1}v^{\pm1}\big)}{\Gamma\big(v^{\pm2}\big)}$$
$$\times\frac{\theta_p\big((pq)^{\frac12}q^{-1}t^{-3}B^{-1}zD^{\pm1}\big)\theta_p\big((pq)^{\frac12}q^{-1}t^{-3}AzC^{\pm1}\big)}{\theta_p\big(z^2\big)\theta_p\big(t^4z^{-2}\big)}$$
$$\times\Gamma_e\big(A^{-1}Bu^{\pm1}v^{\pm1}\big)\Gamma_e\big((qp)^{\frac12}tB^{-1}D^{\pm1}v^{\pm1}\big)\Gamma_e\big((qp)^{\frac12}tAC^{\pm1}v^{\pm1}\big)T_{\mathfrak{J}_D}(v)+\big\{z\leftrightarrow z^{-1}\big\}.$$

Terms cancel in the integral over u and what is left can be evaluated using the inversion formula as before which sets $v=z$, after some cancelations we get

$$\frac{\Gamma_e\big(\big(AB^{-1}\big)^2\big)\theta_p\big(pq^2t^2A^{-2}B^2\big)\theta_p\big((pq)^{-1}q^{-1}t^{-4}A^2B^{-2}\big)}{\theta_p\big(t^{-2}\big)\theta_p\big(q^{-1}t^{-2}B^{-2}\big)\theta_p\big(q^{-1}t^{-2}A^2\big)\theta_p\big(pq^2t^4A^{-2}B^2\big)\theta_p\big(q^{-1}t^{-2}AB^{-1}C^{\pm1}D^{\pm1}\big)}$$
$$\times\frac{\theta_p\big((qp)^{\frac12}tA^{-1}C^{\pm1}z\big)\theta_p\big((qp)^{\frac12}tBD^{\pm1}z\big)\theta_p\big((pq)^{\frac12}q^{-1}t^{-3}B^{-1}zD^{\pm1}\big)\theta_p\big((pq)^{\frac12}q^{-1}t^{-3}AzC^{\pm1}\big)}{\theta_p\big(z^2\big)\theta_p\big(t^4z^{-2}\big)}$$
$$\times T_{\mathfrak{J}_D}(z)+\big\{z\leftrightarrow z^{-1}\big\}.$$

We compute the contribution from the last term in (**??**)

$$\oint\frac{\mathrm{d}u}{4\pi iu}\frac{\Gamma_e\big((qp)^{\frac12}t^{-1}A^{\pm1}D^{\pm1}u^{\pm1}\big)\Gamma_e\big((qp)^{\frac12}t^{-1}B^{\pm1}C^{\pm1}u^{\pm1}\big)}{\Gamma\big(u^{\pm2}\big)}$$
$$\times\Gamma_e\big((pq)^{\frac12}tB^{-1}C^{\pm1}u^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}tAD^{\pm1}u^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}tA^{-1}u^{\pm1}D^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}tBu^{\pm1}C^{\pm1}\big)$$
$$\times\Gamma_e\big(AB^{-1}u^{\pm1}z^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}tBD^{\pm1}z^{\pm1}\big)\Gamma_e\big((pq)^{\frac12}tA^{-1}C^{\pm1}z^{\pm1}\big)$$
$$\times\oint\frac{\mathrm{d}v}{4\pi iv}\frac{\Gamma_e\big((qp)^{\frac12}t^{-1}A^{\pm1}C^{\pm1}v^{\pm1}\big)\Gamma_e\big((qp)^{\frac12}t^{-1}B^{\pm1}D^{\pm1}v^{\pm1}\big)}{\Gamma\big(v^{\pm2}\big)}$$
$$\times\Gamma_e\big(A^{-1}Bu^{\pm1}v^{\pm1}\big)\Gamma_e\big((qp)^{\frac12}tB^{-1}D^{\pm1}v^{\pm1}\big)\Gamma_e\big((qp)^{\frac12}tAC^{\pm1}v^{\pm1}\big)T_{\mathfrak{J}_D}(v).$$

Integrals can be evaluated using the inversion formula which sets $v=z$ almost everything cancel and we get

$$\Gamma_e\big(\big(AB^{-1}\big)^2\big)T_{\mathfrak{J}_D}(z).$$