This gives

$$E[Y_1(0,1)|\tau = c] = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Q_{01}(Y_0)|D = 1, M = 1] \\ - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Y_1|D = 0, M = 1], \tag{1}$$

with $p_a = p_{1|0}$, and $p_c + p_a = p_{1|1}$.

0.1 Quantile direct effect under $\mathbf{d = 1}$ on compliers

We show that

$$F_{Y_1(1,1)|\tau=c}(y) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} F_{Y_1|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} F_{Q_{11}(Y_0)|D=0,M=1}(y) \text{ and}$$
$$F_{Y_1(0,1)|\tau=c}(y) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} F_{Q_{01}(Y_0)|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} F_{Y_1|D=0,M=1}(y),$$

which proves that $\theta_1^c(q, 1) = F^{-1}_{Y_1(1,1)|c}(q) - F^{-1}_{Y_1(0,1)|c}(q)$ is identified. In analogy to (), the outcome distribution under $D = 0$ and $M = 0$ equals:

$$F_{Y_1|D=1,M=1}(y) = \frac{p_a}{p_a + p_c} F_{Y_1(1,1)|\tau=a}(y) + \frac{p_c}{p_a + p_c} F_{Y_1(1,1)|\tau=c}(y).$$

Using () and rearranging the equation gives

$$F_{Y_1(1,1)|\tau=c}(y) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} F_{Y_1|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} F_{Q_{11}(Y_0)|D=0,M=1}(y). \tag{2}$$

From (), we have $F_{Y_1(0,1)|D=1,M(1)=1}(y) = F_{Q_{01}(Y_0)|D=1,M=1}(y)$. Applying the law of iterative expectations gives

$$\begin{aligned} F_{Y_1(0,1)|D=1,M(1)=1}(y) &= \frac{p_a}{p_a + p_c} F_{Y_1(0,1)|D=1,M(1)=1,\tau=a}(y) \\ &\quad + \frac{p_c}{p_a + p_c} F_{Y_1(0,1)|D=1,M(1)=1,\tau=c}(y), \\ &\overset{A7}{=} \frac{p_a}{p_a + p_c} F_{Y_1(0,1)|\tau=a}(y) + \frac{p_c}{p_a + p_c} F_{Y_1(0,1)|\tau=c}(y). \end{aligned}$$