are identified:

$$F_{Y_1(1,0)|\tau=c}(y) = \frac{p_{0|0}}{p_{0|0} - p_{0|1}} F_{Q_{10}(Y_0)|D=0,M=0}(y) - \frac{p_{0|1}}{p_{0|0} - p_{0|1}c} F_{Y_1|D=1,M=0}(y), \tag{1}$$

$$F_{Y_1(0,0)|\tau=c}(y) = \frac{p_{0|0}}{p_{0|0} - p_{0|1}} F_{Y_1|D=0,M=0}(y) - \frac{p_{0|1}}{p_{0|0} - p_{0|1}} F_{Q_{00}(Y_0)|D=1,M=0}(y). \tag{2}$$

Therefore, the direct quantile effect under $d = 0$ on compliers, $\theta_1^c(q, 0) = F^{-1}_{Y_1(1,0)|c}(q) - F^{-1}_{Y_1(0,0)|c}(q)$, is identified. **Proof.** See Appendix . **Theorem 4:** Under Assumptions 1–2, 5, 7-8,

a) and Assumption 6a, the average and quantile direct effects on always-takers are identified:

$$\theta_1^a = \theta_1^{0,1}(0) \text{ and } \theta_1^a(q) = \theta_1^{0,1}(q, 0).$$

b) and Assumption 6, the average direct effect under $d = 1$ on compliers is identified:

$$\theta_1^c(1) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} \theta_1^{1,1}(1) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} \theta_1^{0,1}(0).$$

Furthermore, the potential outcome distributions under $d = 1$ for compliers are identified:

$$F_{Y_1(1,1)|\tau=c}(y) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} F_{Y_1|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} F_{Q_{11}(Y_0)|D=0,M=1}(y), \tag{3}$$

$$F_{Y_1(0,1)|\tau=c}(y) = \frac{p_{1|1}}{p_{1|1} - p_{1|0}} F_{Q_{01}(Y_0)|D=1,M=1}(y) - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} F_{Y_1|D=0,M=1}(y). \tag{4}$$

Therefore, the direct quantile effect under $d = 1$ on compliers $\theta_1^c(q, 1) = F^{-1}_{Y_1(1,1)|c}(q) - F^{-1}_{Y_1(0,1)|c}(q)$ is identified.