where ω_k are

$$\omega_0 = 1, \qquad \omega_1 = -1, \qquad \omega_2 = p^{\frac{1}{2}}, \qquad \omega_3 = -p^{-\frac{1}{2}}.$$

The functions $p_k(h)$ are

$$p_0(h) \equiv \prod_n \theta(p^{\frac{1}{2}}h_n), \qquad p_1(h) \equiv \prod_n \theta\big(-p^{\frac{1}{2}}h_n\big),$$
$$p_2(h) \equiv p\prod_n h_n^{-\frac{1}{2}}\theta(h_n), \qquad p_3(h) \equiv p\prod_n h_n^{\frac{1}{2}}\theta\big(-h_n^{-1}\big),$$

and $\mathcal{E}_k$ is

$$\mathcal{E}_k(\xi; z) \equiv \frac{\theta\big(q^{-\frac{1}{2}}\xi\omega_k^{-1}z\big)\theta\big(q^{-\frac{1}{2}}\xi\omega_k z^{-1}\big)}{\theta\big(q^{-\frac{1}{2}}\omega_k^{-1}z\big)\theta\big(q^{-\frac{1}{2}}\omega_k z^{-1}\big)}.$$

The van Diejen operator and the operator () are the same up to a constant function (independent of z). It's clear that $V(h; z)$ coincides with the corresponding term in (**??**) if we make the identifications

$$h_{1,2,3,4} = t^{-1}A^{\pm1}C^{\pm1}, \qquad h_{5,6,7,8} = t^{-1}B^{\pm1}D^{\pm1}.$$

Since $V_b(h; z)$ is elliptic in z with periods 1 and p and it is easy to check that $W^{\mathfrak{J}_B}_{\mathfrak{J}_D,(1,0;AB^{-1})}(z)$ is also elliptic with the same period, it is enough to show that the two functions have the same poles and residues to prove that they can differ only by a function independent of z. In the fundamental parallelogram V_b has poles at (we assume with no loss of generality that $|p| < |q| \ll |t| < 1$ and the rest of the variables are on unit circle)

$$z = \pm q^{-\frac{1}{2}}p, \qquad z = \pm q^{\frac{1}{2}}, \qquad z = \pm p^{\frac{1}{2}}q^{\pm\frac{1}{2}}.$$

In addition to such poles the operator (**??**) seems to have poles at $z = \pm t^{-2}p, \pm t^2, \pm p^{\frac{1}{2}}t^{\pm2}$ and $z = \pm1, \pm p^{\frac{1}{2}}$, but computation of the residue at these poles yields zero. The computation of the residue at the poles is straightforward, the result is (h is either 1 or -1)

$$\mathrm{Res}_{z\to hq^{\frac{1}{2}}} W^{\mathfrak{J}_B}_{\mathfrak{J}_D,(1,0;AB^{-1})}(z) = -h(p;p)^{-2}\frac{\theta_p\big(hp^{\frac{1}{2}}t^{\pm1}AC^{\pm1}\big)\theta_p\big(hp^{\frac{1}{2}}t^{\pm1}B^{-1}D^{\pm1}\big)}{2q^{-\frac{1}{2}}\theta_p\big(q^{-1}\big)},$$

$$\mathrm{Res}_{z\to hq^{-\frac{1}{2}}} W^{\mathfrak{J}_B}_{\mathfrak{J}_D,(1,0;AB^{-1})}(z) = h(p;p)^{-2}\frac{\theta_p\big(hp^{\frac{1}{2}}t^{\pm1}AC^{\pm1}\big)\theta_p\big(hp^{\frac{1}{2}}t^{\pm1}B^{-1}D^{\pm1}\big)}{2q^{\frac{1}{2}}\theta_p\big(q^{-1}\big)},$$

$$\mathrm{Res}_{z\to hp^{\frac{1}{2}}q^{\frac{1}{2}}} W^{\mathfrak{J}_B}_{\mathfrak{J}_D,(1,0;AB^{-1})}(z) = -h(p;p)^{-2}\frac{A^{-2}B^2\theta_p\big(ht^{\pm1}AC^{\pm1}\big)\theta_p\big(ht^{\pm1}B^{-1}D^{\pm1}\big)}{2p^{-\frac{3}{2}}q^{-\frac{1}{2}}\theta_p\big(q^{-1}\big)},$$

$$\mathrm{Res}_{z\to hp^{\frac{1}{2}}q^{-\frac{1}{2}}} W^{\mathfrak{J}_B}_{\mathfrak{J}_D,(1,0;AB^{-1})}(z) = h(p;p)^{-2}\frac{A^{-2}B^2\theta_p\big(ht^{\pm1}AC^{\pm1}\big)\theta_p\big(ht^{\pm1}B^{-1}D^{\pm1}\big)}{2p^{-\frac{3}{2}}q^{\frac{1}{2}}\theta_p\big(q^{-1}\big)}.$$