1 Lorentzian matter-coupled $F(4)$ gauged supergravity

The theory of matter-coupled $F(4)$ gauged supergravity was first studied in , with some applications and extensions given in . Below we present a short review of this theory, similar to that given in .

1.1 The bosonic Lagrangian

We begin by recalling the field content of the 6-dimensional supergravity multiplet,

$$(e_\mu^a,\, \psi_\mu^A,\, A_\mu^\alpha,\, B_{\mu\nu},\, \chi^A,\, \sigma) \tag{1}$$

The field e_μ^a is the 6-dimensional frame field, with spacetime indices denoted by $\{\mu,\nu\}$ and local Lorentz indices denoted by $\{a,b\}$. The field ψ_μ^A is the gravitino with the index $A,B=1,2$ denoting the fundamental representation of the gauged $SU(2)_R$ group. The supergravity multiplet contains four vectors A_μ^α labelled by the index $\alpha=0,\dots 3$. It will often prove useful to split $\alpha=(0,r)$ with $r=1,\dots,3$ an $SU(2)_R$ adjoint index. Finally, the remaining fields consist of a two-form $B_{\mu\nu}$, a spin-$\frac{1}{2}$ field χ^A, and the dilaton σ. The only allowable matter in the $d=6$, $\mathcal{N}=2$ theory is the vector multiplet, which has the following field content

$$(A_\mu,\, \lambda_A,\, \phi^\alpha)^I \tag{2}$$

where $I=1,\dots,n$ labels the distinct matter multiplets included in the theory. The presence of the n new vector fields A_μ^I allows for the existence of a further gauge group G_+ of dimension $\dim G_+ = n$, in addition to the gauged $SU(2)_R$ R-symmetry. The presence of this new gauge group contributes an additional parameter to the theory, in the form of a coupling constant λ. Throughout this section, we will denote the structure constants of the additional gauge group G_+ by C_{IJK}. However, these will play no role in what follows, since we will be restricting to the case of only a single vector multiplet $n=1$, in which case $G_+=U(1)$. In (half-)maximal supergravity, the dynamics of the $4n$ vector multiplet scalars $\phi^{\alpha I}$ is given by a non-linear sigma model with target space G/K; see e.g. . The group G is the global symmetry group of the theory, while K is the maximal compact subgroup of G. As such, in the Lorentzian case the target space is identified with the following coset space,

$$\mathcal{M} = \frac{SO(4,n)}{SO(4)\times SO(n)} \times SO(1,1) \tag{3}$$

where the second factor corresponds to the scalar σ which is already present in the gauged supergravity without added matter. In the particular case of $n=1$, explored here and in , the first factor is nothing but four-dimensional hyperbolic space $\mathbb{H}_4$. When we analytically continue to the Euclidean case, it will prove very important that we analytically continue the coset space as well, resulting in a dS_4 coset space. This will be discussed more in the following section.