

0.1 Average indirect effect under $d = 0$ on compliers

In () and (), we show that

$$\begin{aligned}\delta_1^c(0) &= E[Y_1(0, 1) - Y_1(0, 0)|\tau = c], \\ &= \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Q_{11}(Y_0)|D = 1, M = 1] - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Y_1|D = 0, M = 1] \\ &\quad - \frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Y_1|D = 0, M = 0] + \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Q_{00}(Y_0)|D = 1, M = 0].\end{aligned}$$

0.2 Quantile indirect effect under $d = 0$ on compliers

In () and (), we show that $F_{Y_1(0,1)|c}(y)$ and $F_{Y_1(0,0)|c}(y)$ are identified. Accordingly,

$\delta_1^c(q, 0) = F_{Y_1(0,1)|c}^{-1}(q) - F_{Y_1(0,0)|c}^{-1}(q)$ is identified.

0.3 Average indirect effect under $d = 1$ on compliers

In () and (), we show that

$$\begin{aligned}\delta_1^c(1) &= E[Y_1(1, 1) - Y_1(1, 0)|\tau = c], \\ &= \frac{p_{1|1}}{p_{1|1} - p_{1|0}} E[Y_1|D = 1, M = 1] - \frac{p_{1|0}}{p_{1|1} - p_{1|0}} E[Q_{11}(Y_0)|D = 0, M = 1] \\ &\quad - \frac{p_{0|0}}{p_{0|0} - p_{0|1}} E[Q_{00}(Y_0)|D = 0, M = 0] + \frac{p_{0|1}}{p_{0|0} - p_{0|1}} E[Y_1|D = 1, M = 0].\end{aligned}$$

0.4 Quantile indirect effect under $d = 1$ on compliers

In () and (), we show that $F_{Y_1(1,1)|c}(y)$ and $F_{Y_1(1,0)|c}(y)$ are identified. Accordingly,

$\delta_1^c(q, 1) = F_{Y_1(1,1)|c}^{-1}(q) - F_{Y_1(1,0)|c}^{-1}(q)$ is identified.