Terms cancel in the second integral and we get

$$
\begin{aligned}
&= (q;q)^2(p;p)^2\Gamma_e\big(pq\big(A^{-2}B^2\big)^{\pm 1}\big)\Gamma_e\big((pq)^{\frac{1}{2}}tB^{-1}C^{\pm 1}z^{\pm 1}\big)\Gamma_e\big((pq)^{\frac{1}{2}}tAD^{\pm 1}z^{\pm 1}\big)\\
&\quad\times\oint\frac{\mathrm{d}u}{4\pi i u}\frac{\Gamma_e\big((qp)^{\frac{1}{2}}t^{-1}A^{\pm 1}D^{\pm 1}u^{\pm 1}\big)\Gamma_e\big((qp)^{\frac{1}{2}}t^{-1}B^{\pm 1}C^{\pm 1}u^{\pm 1}\big)}{\Gamma\big(u^{\pm 2}\big)}\\
&\quad\times\Gamma_e\big((pq)^{\frac{1}{2}}tBC^{\pm 1}u^{\pm 1}\big)\Gamma_e\big((pq)^{\frac{1}{2}}tA^{-1}D^{\pm 1}u^{\pm 1}\big)T_{\mathfrak{J}_C}(u)\\
&\quad\times\oint\frac{\mathrm{d}v}{4\pi i v}\frac{1}{\Gamma\big(v^{\pm 2}\big)}\Gamma_e\big(AB^{-1}u^{\pm 1}v^{\pm 1}\big)\Gamma_e\big(A^{-1}Bz^{\pm 1}v^{\pm 1}\big).
\end{aligned}
$$

The integrals can be evaluated using the inversion formula which sets $u = z$:

$$
\begin{aligned}
&= \Gamma_e\big((pq)^{\frac{1}{2}}tB^{-1}C^{\pm 1}z^{\pm 1}\big)\Gamma_e\big((pq)^{\frac{1}{2}}tAD^{\pm 1}z^{\pm 1}\big)\Gamma_e\big((qp)^{\frac{1}{2}}t^{-1}A^{\pm 1}D^{\pm 1}z^{\pm 1}\big)\\
&\times\Gamma_e\big((qp)^{\frac{1}{2}}t^{-1}B^{\pm 1}C^{\pm 1}z^{\pm 1}\big)\Gamma_e\big((pq)^{\frac{1}{2}}tBC^{\pm 1}z^{\pm 1}\big)\Gamma_e\big((pq)^{\frac{1}{2}}tA^{-1}D^{\pm 1}z^{\pm 1}\big)T_{\mathfrak{J}_C}(z).
\end{aligned}
$$

We see that all terms cancel so we get

$$
\begin{aligned}
&T_{\mathfrak{J}_C}(u)\times_u\Big(\big(T_{\mathfrak{J}_B,\mathfrak{J}_C,\mathfrak{J}_D}(w,u,v)\times_w C_{\mathfrak{J}_B}^{(0,0;AB^{-1})}(w)\big)\\
&\quad\times_v\big(T_{\mathfrak{J}_B,\mathfrak{J}_C,\mathfrak{J}_D}(h,z,v)\times_h C_{\mathfrak{J}_B}^{(0,0;A^{-1}B)}(h)\big)\Big) = T_{\mathfrak{J}_C}(z).
\end{aligned}
$$

1 Computation of the sphere with two punctures and a defect

Here we compute the difference operator. The computation is a small twist on the one of the previous section, however it is less straightforward and thus for which we perform another duality operation. After the duality the p vanishing limit is well defined for all fields. Let us write the fields surviving after scaling

$$
\begin{gathered}
(pq)^{\frac{1}{6}}A^{\frac{1}{3}}b^{-\frac{2}{3}}t^{\frac{1}{3}}a^{-\frac{2}{3}}z_2^{-\frac{1}{3}}z_1^{\pm 1}w_2^j,\ (pq)^{\frac{1}{6}}A^{-\frac{2}{3}}b^{\frac{1}{3}}t^{\frac{1}{3}}a^{\frac{1}{3}}z_2^{-\frac{1}{3}}v_1^{\pm 1}w_2^j,\\
(qp)t^{-2},\ (qp)^{\frac{1}{3}}A^{-\frac{1}{3}}t^{-\frac{4}{3}}b^{-\frac{1}{3}}a^{-\frac{1}{3}}z_2^{\frac{1}{3}}v_2^{-1}\big(w_2^j\big)^{-1},\ (qp)^{\frac{1}{3}}A^{-\frac{1}{3}}t^{\frac{2}{3}}b^{-\frac{1}{3}}a^{-\frac{1}{3}}z_2^{\frac{1}{3}}v_2^{-1}\big(w_2^j\big)^{-1},\ ab^{-1}tw_1^{\pm 1},\\
(qp)^{\frac{1}{2}}ab^{-1}v_2^{-1}z_2^{-1},\ (qp)^{\frac{1}{2}}b^{-1}t^{-1}A^{-1}w_1^{\pm 1}a^{-1},\\
(qp)^{\frac{1}{2}}a^{-1}bz_2v_2,\ (qp)^{\frac{1}{2}}z_2^{-1}t^{-1}w_1^{\pm 1}v_2^{-1},\ bta^{-1}w_1^{\pm 1},\ (qp)^{\frac{1}{2}}tv_2Av_1^{\pm 1},\ (qp)^{\frac{1}{2}}abtv_2z_1^{\pm 1},
\end{gathered}
$$

We would like to thank Hee-Cheol Kim, S. Ruijsenaars, Cumrun Vafa, and Gabi Zafrir for relevant discussions. The research was supported by Israel Science Foundation under grant no. 1696/15 and by I-CORE Program of the Planning and Budgeting Committee.