since I_0 is $O(z^6)$ and hence vanishes in the $\epsilon \to 0$ limit. However, some of the one-point functions *will* depend on Ω. It may be the case that only certain choices of Ω correspond to supersymmetric schemes, but since the final free energy will be independent of Ω we will not worry about this choice. While in principle gives us the free energy, its evaluation on our numerical solutions is complicated by the integration over u in S_{6D}. As such, we will take a slightly roundabout approach to the calculation of the free energy, first calculating its derivative $dF/d\alpha$ and then integrating over the UV parameter α. This will allow us to circumvent the integration over u. In order to get $dF/d\alpha$, it will first be necessary to calculate the one-point functions of the dual field theory operators. This is the topic of the following subsection.

0.0.1 One-point functions

By the usual AdS/CFT dictionary, the one-point functions of the operators dual to the three scalar fields and the metric are given by

$$\begin{aligned} \langle \mathcal{O}_\sigma \rangle &= \lim_{\epsilon\to 0} \frac{1}{\epsilon^3}\frac{1}{\sqrt{\gamma}}\frac{\delta S_{ren}}{\delta\sigma} & \langle \mathcal{O}_{\phi^0} \rangle &= \lim_{\epsilon\to 0} \frac{1}{\epsilon^4}\frac{1}{\sqrt{\gamma}}\frac{\delta S_{ren}}{\delta\phi^0} \\ \langle \mathcal{O}_{\phi^3} \rangle &= \lim_{\epsilon\to 0} \frac{1}{\epsilon^3}\frac{1}{\sqrt{\gamma}}\frac{\delta S_{ren}}{\delta\phi^3} & \langle T^i{}_j \rangle &= \lim_{\epsilon\to 0} \frac{1}{\epsilon^5}\frac{1}{\sqrt{\gamma}}\gamma_{jk}\frac{\delta S_{ren}}{\delta\gamma_{ik}} \end{aligned} \tag{1}$$

We may obtain the explicit values of these vacuum expectation values by varying the on-shell action . The variation of the counterterm action S_{ct} is straightforward. The variation of S_{6D} gives rise to one piece which vanishes on the equations of motion, as well as a boundary term which must be accounted for. We find

$$\begin{aligned} \langle \mathcal{O}_\sigma \rangle &= \lim_{\epsilon\to 0} \frac{1}{\epsilon^3}\left[-2z\partial_z\sigma + 6\sigma - \frac{3}{4}(\varphi^0)^2 + \Omega\left(10\sigma - \frac{15}{4}\left(\phi^0\right)^2\right)\right] \\ \langle \mathcal{O}_{\phi^0} \rangle &= \lim_{\epsilon\to 0} \frac{1}{\epsilon^4}\Bigg[-\frac{1}{2}\cos^2\phi^3 z\partial_z\phi^0 + \frac{1}{2}\phi^0 + \frac{1}{12}\left(\phi^0\right)^3 - \frac{3}{2}\phi^0\sigma - \frac{1}{16}R\phi^0 \\ &\qquad + \Omega\left(\frac{45}{16}\left(\phi^0\right)^3 - \frac{15}{2}\phi^0\sigma\right)\Bigg] \\ \langle \mathcal{O}_{\phi^3} \rangle &= \lim_{\epsilon\to 0} \frac{1}{\epsilon^3}\left[\frac{1}{2}z\partial_z\phi^3 - \phi^3\right] \\ \langle T^i{}_j \rangle &= \lim_{\epsilon\to 0} \frac{1}{\epsilon^5}\left[\frac{1}{2}\left(\mathcal{K}\gamma^{\rangle|} - \mathcal{K}^{\rangle|}\right) + \frac{2}{\sqrt{\gamma}}\frac{\delta S_{ct}}{\delta\gamma_{ij}}\right] \end{aligned} \tag{2}$$

Evaluating the limits, we get the following one-point functions

$$\begin{aligned} \langle \mathcal{O}_\sigma \rangle &= \frac{5}{2}e^{f_k}\alpha\beta\,\Omega & \langle \mathcal{O}_{\phi^0} \rangle &= \frac{3}{2}e^{-f_k}\beta - \frac{15}{8}e^{f_k}\alpha^2\beta\,\Omega \\ \langle \mathcal{O}_{\phi^3} \rangle &= \frac{1}{2}\beta & \langle T^i{}_i \rangle &= -\frac{5}{2}e^{-f_k}\alpha\beta \end{aligned} \tag{3}$$