0.1 BPS Equations

We now use the vanishing of the fermionic variations to obtain BPS equations for the warp factor and the three non-zero scalars.

0.1.1 Dilatino equation and projector

We begin by imposing the vanishing of the dilatino variation, $\delta\chi_A = 0$, which implies

$$\frac{1}{2}\gamma^5\sigma'\varepsilon_A = N_0\varepsilon_A + N_3\gamma^7(\sigma^3)^B{}_A\varepsilon_B \tag{1}$$

This equation can be interpreted as a projection condition on the spinors ε_A. Consistency of this projection condition then requires that

$$\sigma' = 2\eta\sqrt{N_0^2 + N_3^2} \tag{2}$$

where $\eta = \pm 1$. Plugging this BPS equation back into then yields a second form of the projection condition,

$$\gamma^5\varepsilon_A = G_0\varepsilon_A - G_3\gamma^7(\sigma^3)^B{}_A\varepsilon_B \tag{3}$$

which is more useful in the derivation of the other BPS equations. In the above, we have defined

$$G_0 = \eta\frac{N_0}{\sqrt{N_0^2 + N_3^2}} \qquad\qquad G_3 = -\eta\frac{N_3}{\sqrt{N_0^2 + N_3^2}} \tag{4}$$

0.1.2 Gravitino equation

The analysis of the gravitino equation $\delta\psi_{A\mu} = 0$ proceeds in exactly the same way as for the Lorentzian case studied in . The procedure gives rise to a first-order equation for the warp factor f and an algebraic constraint. To avoid excessive overlap with that paper, we simply cite the result,

$$f' = 2(G_0S_0 + G_3S_3) \qquad\qquad e^{-2f} = 4(G_0S_0 + G_3S_3)^2 - 4(S_0^2 + S_3^2) \tag{5}$$

0.1.3 Gaugino equations

Finally, we turn toward the gaugino equation $\delta\lambda^I_A = 0$. Again the analysis of this equation proceeds in an exactly analogous manner to the Lorentzian case . The result is

$$\cos\phi^3(\phi^0)' = -(G_0M_0 + G_3M_3) \qquad\qquad (\phi^3)' = i(G_3M_0 - G_0M_3) \tag{6}$$

The right-hand sides of both equations are real, and thus give rise to real solutions when appropriate initial conditions are imposed.