in the limit the three punctured sphere we have defined has all C_λ vanishing but the one corresponding to the constant polynomial. The Koornwinder polynomials have higher rank generalizations which should be relevant for higher rank E string theories. In those cases we do not know the three punctured spheres and the relation to Koornwinder polynomials can provide a useful tool to study the indices of these models. The limit we considered does not have a special physical meaning a priori, however the fact that the expressions become simple and the fact that one might generalize the discussion to the higher rank case, make the limit of potential interest.

A Index definitions

We compute the supersymmetric index using the standard definitions of . The index of chiral field charged under flavor U(1) symmetry with charge S and having R-charge $\mathfrak{R}$ is

$$\Gamma_e\big((qp)^{\frac{\mathfrak{R}}{2}} u^S\big).$$

The parameter u is fugacity for the flavor symmetry. We define here

$$\Gamma_e(u) = \prod_{i,j=0}^{\infty} \frac{1 - \frac{1}{u} q^{i+1} p^{j+1}}{1 - u p^i q^j}.$$

We will use the following definitions

$$(s;q) = \prod_{i=1}^{\infty} \big(1 - s q^{i-1}\big), \qquad \theta_r(u) = \prod_{j=1}^{\infty} \big(1 - u r^{j-1}\big)\big(1 - r^j/u\big).$$

Finally we use the condensed conventions

$$f\big(y^{\pm 1}\big) = f(1/y) f(y), \qquad (s_1, \dots, s_k; q) = (s_1;q) \cdots (s_k;q).$$

Contour integrals in the paper are around the unit circle unless we state otherwise.

B Computation of the sphere with two punctures

We give here the derivation of equation. The computation involves calculating several contour integrals over products of elliptic gamma functions