1 Monte Carlo results

We performed a Monte Carlo simulation to assess the performance of the proposed filter based on halfspace depth. After the filter flags the outlying observations, the generalized S-estimator is applied to the data with added missing values. Our simulation study is based on the same setup described in **?** to compare significantly the performance of our filter with respect to the filter introduced in their work. We considered samples from a $N_p(\mathbf{0}, \mathbf{\Sigma_0})$, where all values in $diag(\mathbf{\Sigma_0})$ are equal to 1, $p = 10, 20, 30, 40, 50$ and the sample size is $n = 10p$. We consider the following scenarios:

- Clean data: data without changes.
- Cell-Wise contamination: a proportion ϵ of cells in the data is replaced by $X_{ij} \sim N(k, 0.1^2)$, where $k = 1, \ldots, 10$.
- Case-Wise contamination: a proportion ϵ of cases in the data matrix is replaced by $\mathbf{X_i} \sim \mathbf{0.5N(cv, 0.1^2I) + 0.5N(-cv, 0.1^2I)}$, where $c = \sqrt{k(\chi^2_p)^{-1}(0.99)}$, $k = 1, 2, \ldots, 20$ and $\mathbf{v}$ is the eigenvector corresponding to the smallest eigenvalue of $\mathbf{\Sigma_0}$ with length such that $(\mathbf{v} - \mu_\mathbf{0})^\top \mathbf{\Sigma_0^{-1}} (\mathbf{v} - \mu_\mathbf{0}) = \mathbf{1}$.

The proportions of contaminated rows chosen for case-wise contamination are $\epsilon = 0.1, 0.2$, and $\epsilon = 0.02, 0.05$ for cell-wise contamination. The number of replicates in our simulation study is $N = 200$. We measure the performance of a given pair of location and scatter estimators $\hat{\mu}$ and $\hat{\mathbf{\Sigma}}$ using the mean squared error (MSE) and the likelihood ratio test distance (LRT), as in:

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (\hat{\mu}_i - \mu_\mathbf{0})^\top (\hat{\mu}_\mathbf{i} - \mu_\mathbf{0})$$

$$LRT(\hat{\mathbf{\Sigma}}, \mathbf{\Sigma_0}) = \mathbf{\frac{1}{N} \sum_{i=1}^{N} D(\hat{\Sigma}_i, \Sigma_0)}$$

2 Statistical data depth properties

A **depth function** $d(\cdot; F)$ measures the centrality of a point w.r.t. a probability distribution F.

$$d = \mathbb{R}^p \to \mathbb{R}^+ \cup \{0\}, \qquad \mathbf{x \to d(x; F)}$$

A statistical depth function should satisfy the following Properties