allows us to conclude that this is always the case for real initial conditions ϕ_0^0. Thus we have a one parameter family of real smooth solutions, labeled by the IR parameter ϕ_0^0. With this in mind, we may choose any value of ϕ_0^0 and solve the BPS equations in numerically. In Figure , we plot the solutions obtained for the following choices of initial condition: $\phi_0^0 = \{0.25,\, 0.5,\, 1,\, 1.5,\, 2\}$. In order to get smooth solutions for $u > 0$, we must take $\eta = -1$. It is straighforward to verify that the resulting solutions are completely smooth and have the expected vanishing of e^{2f} at the origin, implying that the spacetime smoothly pinches off. Furthermore, e^{2f}/e^{2u} is seen to asymptote to a constant, which we denote by e^{2f_k}.

0.1 UV asymptotic expansions

As in the holographic Janus solutions in Lorentzian signature , the BPS equations may also be used to obtain the UV asymptotic behavior of the solutions. To do so, we begin by defining an asymptotic coordinate $z = e^{-u}$, where the asymptotic S^5 boundary is reached by taking $u \to \infty$. Consequently, an asymptotic expansion is an expansion around $z = 0$. The coefficients in the UV expansions of the non-zero fields may now be solved for order-by-order using the BPS equations. One finds explicitly that all coefficients are determined in terms of only three independent parameters α, β, and f_k, in accord with the fact that there are three independent first-order differential equations. The first few terms in the expansions are

$$\begin{aligned}
f(z) &= -\log z + f_k - \left(\frac{1}{4}e^{-2f_k} + \frac{1}{16}\alpha^2\right) z^2 + O(z^4) \\
\sigma(z) &= \frac{3}{8}\alpha^2\, z^2 + \frac{1}{4}e^{f_k}\alpha\beta\, z^3 + O(z^4) \\
\phi^0(z) &= \alpha\, z - \left(\frac{5}{4}\alpha\, e^{-2f_k} + \frac{23}{48}\alpha^3\right) z^3 + O(z^4) \\
\phi^3(z) &= e^{-f_k}\alpha z^2 + \beta\, z^3 + O(z^4)
\end{aligned} \tag{1}$$

We have obtained the expansions up to $O(z^8)$, but we display only the first few terms here.

1 Holographic sphere free energy

The goal of this section is to obtain the holographic free energy, i.e. the renormalized on-shell action. We begin by writing the full action,

$$S = S_{\text{6D}} + S_{\text{GH}}$$

$$S_{\text{6D}} = \int du\; d^5x\; \sqrt{G}\,\mathcal{L} \qquad\qquad S_{\text{GH}} = -\frac{1}{2}\int d^5x\sqrt{\gamma}\,\mathcal{K} \tag{1}$$

where S_{6D} is the six-dimensional Euclidean action given in and S_{GH} is the Gibbons-Hawking term. The γ appearing in S_{GH} is the determinant of the induced metric on the boundary