

we discuss it in detail. We compute

$$\begin{aligned}
& T_{\mathfrak{J}_B, \mathfrak{J}_C, \mathfrak{J}_D}(w, u, v) \times_w C_{\mathfrak{J}_B}^{(1,0;AB^{-1})}(w) \\
&= (q; q)(p; p) \oint \frac{dw}{4\pi iw} \frac{\prod_{j=1}^8 \Gamma_e((qp)^{\frac{1}{2}} \frac{1}{t} a_j^{-1} w^{\pm 1})}{\Gamma(w^{\pm 2})} C_{\mathfrak{J}_B}^{(1,0;AB^{-1})}(w) T_{\mathfrak{J}_B, \mathfrak{J}_C, \mathfrak{J}_D}(w, u, v) \\
&\sim \oint \frac{dw}{4\pi iw} \frac{\prod_{j=1}^8 \Gamma_e((qp)^{\frac{1}{2}} \frac{1}{t} a_j^{-1} w^{\pm 1})}{\Gamma(w^{\pm 2})} \prod_{j=1}^8 \Gamma_e((pq)^{\frac{1}{2}} t a_j w^{\pm 1}) \Gamma_e \left( \frac{(pq)^{\frac{1}{2}} q w^{\pm 1}}{t AB^{-1}} \right) \\
&\times \Gamma_e \left( \frac{AB^{-1} w^{\pm 1}}{(pq)^{\frac{1}{2}} q t^3} \right) \Gamma_e((qp)^{\frac{1}{2}} t (B^{-1} A)^{\pm 1} w^{\pm 1}) \Gamma_e \left( \frac{qp}{t^2} \right) \oint \frac{dy}{4\pi iy} \frac{\Gamma_e \left( \frac{(pq)^{\frac{1}{2}}}{t^2} (AB^{-1})^{\pm 1} y^{\pm 1} \right)}{\Gamma_e(y^{\pm 2})} \\
&\times \Gamma_e(t y^{\pm 1} w^{\pm 1}) \oint \frac{dw_1}{4\pi iw_1} \oint \frac{dw_2}{4\pi iw_2} \frac{\Gamma_e \left( \frac{(pq)^{\frac{1}{2}}}{t^2} w_1^{\pm 1} w_2^{\pm 1} \right)}{\Gamma_e(w_2^{\pm 2}) \Gamma_e(w_1^{\pm 2})} \Gamma_e((qp)^{\frac{1}{4}} t A^{\frac{1}{2}} B^{-\frac{1}{2}} y^{\frac{1}{2}} w_1^{\pm 1} u^{\pm 1}) \\
&\times \Gamma_e((qp)^{\frac{1}{4}} A^{\frac{1}{2}} B^{\frac{1}{2}} y^{-\frac{1}{2}} w_1^{\pm 1} D^{\pm 1}) \Gamma_e((qp)^{\frac{1}{4}} t A^{-\frac{1}{2}} B^{\frac{1}{2}} y^{-\frac{1}{2}} w_2^{\pm 1} u^{\pm 1}) \\
&\times \Gamma_e((qp)^{\frac{1}{4}} A^{-\frac{1}{2}} B^{-\frac{1}{2}} y^{\frac{1}{2}} D^{\pm 1} w_2^{\pm 1}) \Gamma_e((qp)^{\frac{1}{4}} t A^{-\frac{1}{2}} B^{\frac{1}{2}} y^{\frac{1}{2}} w_1^{\pm 1} v^{\pm 1}) \\
&\times \Gamma_e((qp)^{\frac{1}{4}} A^{-\frac{1}{2}} B^{-\frac{1}{2}} y^{-\frac{1}{2}} C^{\pm 1} w_1^{\pm 1}) \Gamma_e((qp)^{\frac{1}{4}} t A^{\frac{1}{2}} B^{-\frac{1}{2}} y^{-\frac{1}{2}} w_2^{\pm 1} v^{\pm 1}) \\
&\times \Gamma_e((qp)^{\frac{1}{4}} A^{\frac{1}{2}} B^{\frac{1}{2}} y^{\frac{1}{2}} w_2^{\pm 1} C^{\pm 1}),
\end{aligned}$$

where  $\sim$  means equality up to overall factors independent of  $w, u, v$ . Using the identity  $\Gamma_e\left(\frac{pq}{z}\right)\Gamma_e(z) = 1$  and the elliptic beta integral formula we evaluate the integral over  $w$  and up to overall factors we get

$$\begin{aligned}
& \Gamma_e(pq^2) \oint \frac{dy}{4\pi iy} \frac{\Gamma_e((pq)^{\frac{1}{2}} q A^{-1} B y^{\pm 1}) \Gamma_e((pq)^{-\frac{1}{2}} q^{-1} t^{-2} AB^{-1} y^{\pm 1})}{\Gamma_e(y^{\pm 2})} \oint \frac{dw_1}{4\pi iw_1} \oint \frac{dw_2}{4\pi iw_2} \\
&\times \frac{\Gamma_e \left( \frac{(pq)^{\frac{1}{2}}}{t^2} w_1^{\pm 1} w_2^{\pm 1} \right)}{\Gamma_e(w_2^{\pm 2}) \Gamma_e(w_1^{\pm 2})} \Gamma_e((qp)^{\frac{1}{4}} t A^{\frac{1}{2}} B^{-\frac{1}{2}} y^{\frac{1}{2}} w_1^{\pm 1} u^{\pm 1}) \Gamma_e((qp)^{\frac{1}{4}} A^{\frac{1}{2}} B^{\frac{1}{2}} y^{-\frac{1}{2}} w_1^{\pm 1} D^{\pm 1}) \\
&\times \Gamma_e((qp)^{\frac{1}{4}} t A^{-\frac{1}{2}} B^{\frac{1}{2}} y^{-\frac{1}{2}} w_2^{\pm 1} u^{\pm 1}) \Gamma_e((qp)^{\frac{1}{4}} A^{-\frac{1}{2}} B^{-\frac{1}{2}} y^{\frac{1}{2}} D^{\pm 1} w_2^{\pm 1}) \\
&\times \Gamma_e((qp)^{\frac{1}{4}} t A^{-\frac{1}{2}} B^{\frac{1}{2}} y^{\frac{1}{2}} w_1^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{4}} A^{-\frac{1}{2}} B^{-\frac{1}{2}} y^{-\frac{1}{2}} C^{\pm 1} w_1^{\pm 1}) \\
&\times \Gamma_e((qp)^{\frac{1}{4}} t A^{\frac{1}{2}} B^{-\frac{1}{2}} y^{-\frac{1}{2}} w_2^{\pm 1} v^{\pm 1}) \Gamma_e((qp)^{\frac{1}{4}} A^{\frac{1}{2}} B^{\frac{1}{2}} y^{\frac{1}{2}} w_2^{\pm 1} C^{\pm 1}). \quad (1)
\end{aligned}$$

We got a zero multiplying the integral, but we will see next that some of the integrals are pinched giving finite result. We start by evaluating the integral over  $y$  using the residue theorem and we get that the integral over  $w_1$  is pinched due to  $\Gamma_e((qp)^{\frac{1}{4}} t A^{-\frac{1}{2}} B^{\frac{1}{2}} y^{\frac{1}{2}} w_1^{\pm 1} v^{\pm 1})$  term for the poles