the law of iterative expectations, gives

$$\begin{aligned} E[Y_1(1,0)|D=0,M=0] &= \frac{p_n}{p_n+p_c}E[Y_1(1,0)|D=0,M=0,\tau=n] \\ &\quad + \frac{p_c}{p_n+p_c}E[Y_1(1,0)|D=0,M=0,\tau=c], \\ &\overset{A7}{=} \frac{p_n}{p_n+p_c}E[Y_1(1,0)|\tau=n] + \frac{p_c}{p_n+p_c}E[Y_1(1,0)|\tau=c]. \end{aligned}$$

After some rearrangements and using (), we obtain

$$E[Y_1(1,0)|\tau=c] = \frac{p_n+p_c}{p_c}E[Q_{10}(Y_0)|D=0,M=0] - \frac{p_n}{p_c}E[Y_1|D=1,M=0].$$

This gives

$$\begin{aligned} E[Y_1(1,0)|\tau=c] &= \frac{p_{0|0}}{p_{0|0}-p_{0|1}}E[Q_{10}(Y_0)|D=0,M=0] \\ &\quad - \frac{p_{0|1}}{p_{0|0}-p_{0|1}}E[Y_1|D=1,M=0], \end{aligned} \tag{1}$$

using $p_n = p_{0|1}$, and $p_c + p_n = p_{0|0}$.

0.1 Quantile direct effect under d = 0 on compliers

We show that

$$F_{Y_1(1,0)|\tau=c}(y) = \frac{p_{0|0}}{p_{0|0}-p_{0|1}}F_{Q_{10}(Y_0)|D=0,M=0}(y) - \frac{p_{0|1}}{p_{0|0}-p_{0|1}c}F_{Y_1|D=1,M=0}(y) \text{ and}$$

$$F_{Y_1(0,0)|\tau=c}(y) = \frac{p_{0|0}}{p_{0|0}-p_{0|1}}F_{Y_1|D=0,M=0}(y) - \frac{p_{0|1}}{p_{0|0}-p_{0|1}}F_{Q_{00}(Y_0)|D=1,M=0}(y),$$

which proves that $\theta_1^c(q,0) = F^{-1}_{Y_1(1,0)|c}(q) - F^{-1}_{Y_1(0,0)|c}(q)$ is identified. From (), we have $F_{Y_1(1,0)|D=0,M(0)=0}(y) = F_{Q_{10}(Y_0)|D=0,M=0}(y)$. Applying the law of iterative gives