

# STP598sta: Spatiotemporal Analysis

## Homework 3

Name: Your name; NetID: Your ID

Due 11:59pm Sunday October 17, 2021

### Question 1

Compute the coherence (generalized correlation),  $\frac{\text{cov}(Y_\ell(\mathbf{s}), Y_{\ell'}(\mathbf{s}+\mathbf{h}))}{\sqrt{\text{cov}(Y_\ell(\mathbf{s}), Y_\ell(\mathbf{s}+\mathbf{h}))\text{cov}(Y_{\ell'}(\mathbf{s}), Y_{\ell'}(\mathbf{s}+\mathbf{h}))}}$ :

- (a) for the cross-covariance  $\Sigma_{\mathbf{Y}(\mathbf{s}), \mathbf{Y}(\mathbf{s}')} = C(\mathbf{s} - \mathbf{s}') = \sum_{j=1}^p \rho_j(\mathbf{s} - \mathbf{s}')T_j$ .
- (b) for the cross-covariance  $C(\mathbf{s} - \mathbf{s}') = \sum_{u=1}^r \rho_u(\mathbf{s} - \mathbf{s}')T^{(u)}$ .

### Question 2

Let  $Y(\mathbf{s}) = (Y_1(\mathbf{s}), Y_2(\mathbf{s}))^T$  be a bivariate process with a stationary cross-covariance matrix function

$$C(\mathbf{s} - \mathbf{s}') = \begin{pmatrix} c_{11}(\mathbf{s} - \mathbf{s}') & c_{12}(\mathbf{s} - \mathbf{s}') \\ c_{12}(\mathbf{s}' - \mathbf{s}) & c_{22}(\mathbf{s} - \mathbf{s}') \end{pmatrix}$$

and a set of covariates  $\mathbf{x}(\mathbf{s})$ . Let  $\mathbf{y} = (\mathbf{y}_1^T, \mathbf{y}_2^T)^T$  be the  $2n \times 1$  data vector, with  $\mathbf{y}_1^T = (y_1(\mathbf{s}_1), \dots, y_1(\mathbf{s}_n))^T$  and  $\mathbf{y}_2^T = (y_2(\mathbf{s}_1), \dots, y_2(\mathbf{s}_n))^T$ .

- (a) Show that the cokriging predictor has the form

$$E[Y_1(\mathbf{s}_0)|\mathbf{y}] = \mathbf{x}^T(\mathbf{s}_0)\boldsymbol{\beta} + \boldsymbol{\gamma}^T \Sigma^{-1}(\mathbf{y} - X\boldsymbol{\beta})$$

, with appropriate definitions of  $\boldsymbol{\gamma}$  and  $\Sigma$ .

- (b) Show further that if  $\mathbf{s}_k$  is a site where  $y_l(\mathbf{s}_k)$  is observed, then for  $l = 1, 2$ ,  $E[Y_l(\mathbf{s}_k)|\mathbf{y}] = y_l(\mathbf{s}_k)$  if and only if  $\tau_l^2 = 0$ .

### Question 3

In the Case 3: dependent and not identical latent CAR variables for coregionalization MCAR, show that the matrix  $(I_{p \times p} \otimes D - B \otimes W)$  can be expressed as

$$(I_{p \times p} \otimes D)^{\frac{1}{2}}(I_{pn \times pn} - B \otimes D^{-\frac{1}{2}}WD^{-\frac{1}{2}})(I_{p \times p} \otimes D)^{\frac{1}{2}}$$