STP598sta: Spatiotemporal Analysis

Homework 3

Name: Your name; NetID: Your ID

Due 11:59pm Sunday November 1, 2020

Question 1

Compute the coherence (generalized correlation), $\frac{\mathrm{cov}(Y_\ell(\mathbf{s}),Y_{\ell'}(\mathbf{s}+\mathbf{h}))}{\sqrt{\mathrm{cov}(Y_\ell(\mathbf{s}),Y_\ell(\mathbf{s}+\mathbf{h}))\mathrm{cov}(Y_{\ell'}(\mathbf{s}),Y_{\ell'}(\mathbf{s}+\mathbf{h}))}}$:

(a) for the cross-covariance $\Sigma_{\mathbf{Y}(\mathbf{s}),\mathbf{Y}(\mathbf{s}')} = C(\mathbf{s}-\mathbf{s}') = \sum_{j=1}^{p} \rho_j(\mathbf{s}-\mathbf{s}')T_j$.

(b) for the cross-covariance $C(\mathbf{s}-\mathbf{s}') = \sum_{u=1}^{r} \rho_u(\mathbf{s}-\mathbf{s}')T^{(u)}$.

Question 2

Let $Y(\mathbf{s}) = (Y_1(\mathbf{s}), Y_2(\mathbf{s}))^T$ be a bivariate process with a stationary cross-covariance matrix function

$$C(\mathbf{s}-\mathbf{s}') = \begin{pmatrix} c_{11}(\mathbf{s}-\mathbf{s}') & c_{12}(\mathbf{s}-\mathbf{s}') \\ c_{12}(\mathbf{s}'-\mathbf{s}) & c_{22}(\mathbf{s}-\mathbf{s}') \end{pmatrix}$$

and a set of covariates $\mathbf{x}(\mathbf{s})$. Let $\mathbf{y} = (\mathbf{y}_1^T, \mathbf{y}_2^T)^T$ be the $2n \times 1$ data vector, with $\mathbf{y}_1^T = (y_1(\mathbf{s}_1), \cdots, y_1(\mathbf{s}_n))^T$ and $\mathbf{y}_2^T = (y_2(\mathbf{s}_1), \cdots, y_2(\mathbf{s}_n))^T$.

(a) Show that the cokriging predictor has the form

$$\mathrm{E}[Y_1(\mathbf{s}_0)|\mathbf{y}] = \mathbf{x}^T(\mathbf{s}_0)\boldsymbol{\beta} + \boldsymbol{\gamma}^T\Sigma^{-1}(\mathbf{y} - X\boldsymbol{\beta})$$

, with appropriate definitions of $\boldsymbol{\gamma}$ and Σ.

(b) Show further that if $\mathbf{s}_k$ is a site where $y_l(\mathbf{s}_k)$ is observed, then for $l = 1, 2$, $\mathrm{E}[Y_l(\mathbf{s}_k)|\mathbf{y}] = y_l(\mathbf{s}_k)$ if and only if $\tau_l^2 = 0$.

Question 3

In the Case 3: dependent and not identical latent CAR variables for coregionalization MCAR, show that the covariance matrix $(I_{p\times p} \otimes D - B \otimes W)^{-1}$ can be expressed as

$$(I_{p\times p} \otimes D)^{\frac{1}{2}}(I_{pn\times pn} - B \otimes D^{-\frac{1}{2}}WD^{-\frac{1}{2}})(I_{p\times p} \otimes D)^{\frac{1}{2}}$$