STP598sta: Spatiotemporal Analysis

Homework 4

Name: Your name; NetID: Your ID

Due 11:59pm Monday November 30, 2020

Question 1

For a moving average process of the form

$$x_t = w_{t-1} + 2w_t + w_{t+1}$$

where w_t are independent with zero means and variance σ_w^2, determine the autocovariance and autocorrelation functions as a function of lag $h = s - t$ and plot the ACF as a function of h.

Question 2

In this problem, we explore the difference between a random walk and a trend stationary process.

(a) Generate four series that are random walk with drift, $x_t = \delta t + \sum_{i=1}^{t} w_j$, of length $n = 100$ with $\delta = 0.01$ and $\sigma_w = 1$. Call the data x_t for $t = 1, \cdots, 100$. Fit the regression $x_t = \beta t + w_t$ using least squares. Plot the data, the true mean function (i.e. $\mu_t = 0.01t$) and the fitted line, $\hat{x}_t = \hat{\beta} t$, on the same graph. Hint: The following `R` code may be useful.

```
par(mfrow=c(2,2), mar=c(2.5,2.5,0,0)+.5, mgp=c(1.6,.6,0)) # set up
for (i in 1:4){
  x = ts(cumsum(rnorm(100,.01,1)))         # data
  regx = lm(x~0+time(x), na.action=NULL)   # regression
  plot(x, ylab='Random Walk w Drift')      # plots
  abline(a=0, b=.01, col=2, lty=2)         # true mean (red - dashed)
  abline(regx, col=4)                      # fitted line (blue -  solid)
}
```

(b) Generate four series of length $n = 100$ that are linear trend plus noise, say $y_t = 0.01t + w_t$, where t and w_t are as in part (a). Fit the regression $y_t = \beta t + w_t$ using least squares. Plot the data, the true mean function (i.e. $\mu_t = 0.01t$) and the fitted line, $\hat{y}_t = \hat{\beta} t$, on the same graph.

(c) Comment (what did you learn from this assignment).

Question 3

For the AR(2) series $x_t + 1.6x_{t-1} + 0.64x_{t-2} = w_t$, use the difference equation (ref pages 19-25 of lecture 8, or the reustls of Example 3.10 of TSA book) to find the ACF $\rho(h), h = 0, 1, \cdots$; solve for the constants in the ACF using the initial conditions. Then plot the ACF values to lag 10 (use `ARMAacf` as a check on your answers).

Question 4

Generate $n = 100$ observations from each of the three models: (a) AR(1), (b) MAR(1) and (c) ARMA(1,1) with $\phi = 0.6$ and $\theta = 0.9$. Plot the (sample) ACF and PACF for each of the three models and compare with

their theoretic values. What do you find abou their tailing behavior?

Question 5

Simulate $n = 100$ observations from the following state-space model:

$$x_t = 0.8x_{t-1} + w_t, \quad w_t \overset{iid}{\sim} N(0,1)$$
$$y_t = x_t + v_t, \quad v_t \overset{iid}{\sim} N(0,1)$$

where $x_0 \sim N(0, \sigma^2 = 2.5)$. Plot the prediction, filtering, and smoothing using the generated data. Hint: Use the provided demo `prediction_filter_smooth` as a guide.