Homework 2

Your submission must be uploaded to crowdmark by **11:59pm on December 5, 2019**. No exceptions.

Problem 1

For each of the following functions, determine whether the function is even, odd, or neither. Justify your answer by demonstrating that the function satisfies the defining property of being even or odd, or by producing concrete counterexamples which show the function is neither.

(a) $f(x) = 2x^5 - 3x^2 + 2$;

(b) $f(x) = x^3 - x^7$;

(c) $f(x) = e^{-x^2}$;

(d) $f(x) = 1 + \sin x$.

Problem 2

What is the domain for each of the following functions? Justify your answers.

(a) $f(x) = \frac{1 - e^{x^2}}{1 - e^{1-x^2}}$;

(b) $g(t) = \sqrt{10^t - 100}$.

Problem 3

Find all values a and b which make the function $f(x)$ continuous everywhere:

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{for } x < 2; \\ ax^2 - bx + 3, & \text{for } 2 \le x < 3; \\ 2x - a + b, & \text{for } x \ge 3. \end{cases}$$

Justify your answer.

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Problem 4

Consider the function

$$f(x) = \ln(x^2 + 9).$$

Find the intervals where f is increasing and decreasing. Find the intervals where f is concave up and concave down. Find all local minimum and maximum values for f. Find all inflection points for f. Justify your answers.

Problem 5

Let $g(x)$ be a differentiable function which satisfies

$$g(x) + x\sin(g(x)) = x^2$$

What is $g'(0)$?

Problem 6

Find each of the limits:

(a) $\lim_{x\to\infty} \frac{\ln\sqrt{x}}{x^2}$;

(b) $\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1-4x}}{x}$;

(c) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\tan^{-1}(x)}\right)$;

(d) $\lim_{x\to\infty} x^{e^{-x}}$.

Problem 7

A right circular cylinder is inscribed in a right cone with height H and radius R. What is the largest possible volume of such a cylinder?

Problem 8

At noon ship A is 150 km due west of ship B. Ship A sails east at 35 km/h and ship B sails north at 25 km/h. How fast is the distance between the two ships changing at $4:00$pm?

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Problem 9

(a) Express

$$\int_2^5 \left(x^2 + \frac{1}{x} \right) dx$$

as a limit of left Riemann sums.

(b) Find a function $g(x)$ and numbers a and b so that

$$\lim_{n \to \infty} \left(\sum_{i=1}^n \frac{3}{n} \sqrt{1 + \frac{3i}{n}} \right) = \int_a^b g(x) dx.$$

Problem 10

Compute

$$\int_0^{\pi/4} \frac{\sin x}{1 + \cos^2(x)} dx.$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$