Homework 1

Your submission must be uploaded to crowdmark by **11:59pm on October 4, 2019**. No exceptions.

Problem 1

Find all values x in the interval $[0, 2\pi]$ for which the equation

$$2\cos x + \sin 2x = 0$$

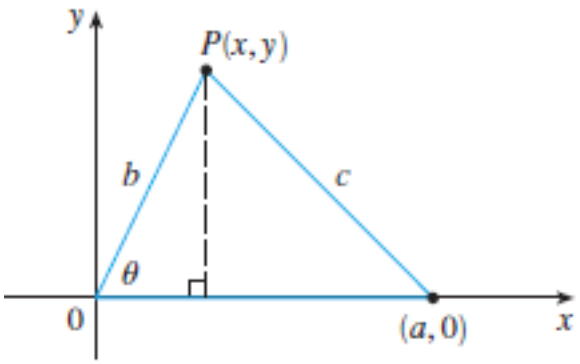
is satisfied.

Problem 2

Prove the **Law of Cosines:** if a triangle has side lengths a, b, c, and θ is the angle in between the sides of length a and b, then

$$c^2 = a^2 + b^2 - 2ab\cos\theta.$$

[*Hint:* Consult the figure below. Write x and y in terms of θ and use the distance formula to find c].



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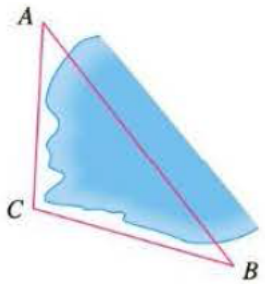
Problem 3

You need to calculate the distance $|AB|$ across an inlet depicted as in the figure:



By walking along the inlet, you were able to measure that $|AC| = 820$ m, $|BC| = 910$ m, and that the angle formed at C measures 103°. Use the law of cosines from the previous problem to find $|AB|$.

Problem 4

Suppose two sides of a triangle T are measured to have lengths a and b. Let θ denote the angle between these two sides. Show that the area A of T is

$$A = \frac{ab \sin \theta}{2}$$

Problem 5

A function f is said to be **odd** if

$$f(-x) = -f(x)$$

for every x in the domain of f. Prove that the function

$$f(x) = \frac{1 - e^{1/x}}{1 + e^{1/x}}$$

is odd.

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Problem 6

Drake has offered to hire you as a sound engineer on his next project. He's only willing to employ you for a maximum length of time of one month. He proposes two possible methods of payment:

(I) You receive $\$1,000,000$, but only if you work every day for the entire month (this particular month has 31 days);

(II) You receive 2 cents on the first day, 4 cents on the second, 8 cents on the third, and generically, 2^n cents on day n.

(a) Assume that you work the full month: is option (I) or (II) a better payment option? How much better is the better option?

(b) Now suppose that your goal is to make $\$1,000,000$, but you're rather lazy: you'd prefer to work as few days as possible. Assuming you get to choose either (I) or (II), what is the minimum number of days you can work and still make $\$1,000,000$?

Problem 7

Suppose the graph of $y = \log_2(x)$ is drawn on a coordinate grid where the unit of measurement is 1 cm. How many km to the right must you go before the height of the graph reaches 1 m?

Problem 8

Evaluate the limit

$$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}.$$

Problem 9

Suppose that

$$\lim_{t\to 1} \frac{f(t) - e^\pi}{t-1} = 16.$$

What is $\lim_{t\to 1} f(t)$?

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Problem 10

Use the Intermediate Value Theorem to show that there is a real number x which is exactly one more than its cube. Find an interval I of length at most 0.5 containing x and use the Intermediate value Theorem to prove that I has this property. You should not use a graphing calculator/computer program to find the root for you.