Teaching Statement

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I am passionate about teaching mathematics because I believe that it provides an ideal context for students to develop systematic approaches to solving complex problems. I have taught six classes as primary instructor, and have served as a discussion leader nine times. Honest reflection on these experiences has shaped my approach to teaching math, and I will continue to reflect and adapt throughout my teaching career.

Learning Objectives

The mantra: I want each of my students to be equipped with a systematic approach to problem solving. I have found that few students in first and second year math courses have such a system, so I share with them the system which I use in my own work; it can be succinctly stated as ***Try something. Analyze. Try something else.*** and will henceforth be referred to as "*the mantra.*"

When faced with a problem, a student must first **try something**. In my experience, many students get trapped in a state of mental limbo in which they attempt to guess the "correct" thing to try before they have tried anything at all. I believe such ruminations cost considerable time and mental and emotional energy; this is certainly the case in my own work.

Consider the following example from a linear algebra tutorial: I asked the class "*what is an easy example of an invertible matrix?*" One student began to answer "*the ide...*" then trailed off. I said to that student "*it sounded like you were about to give an answer. Can you finish what you were about to say?*" "*I'm not sure... I thought maybe the identity matrix, but I don't know if that's right.*" "*The identity, thank you. Let's check if it's invertible.*" To avoid this state of intellectual purgatory, I advise students to commit to the first thing that comes to their mind by writing down or verbalizing "*I will try to see if X works.*"

Once a student has committed to an approach X, they must **analyze** whether or not X achieved its goal. I encourage the student to first write down a method for checking whether X succeeded. If, for example, a student proposed $F(x)$ as an anti-derivative for the function $f(x) = 2x\sin(x^2)$, then they might write "*I can check my answer $F(x)$ by differentiating it: if I get $f(x)$, then my answer was correct.*" If X was successful, then the student should celebrate. Otherwise, they should try to document why X did not work: "*the function $F(x) = \cos(x^2)$ is not an anti-derivative of $f(x)$ because $F'(x) = -2x\sin(x^2)$, which differs from $f(x)$ by -1.*"

If a student has tried and analyzed X, and judged it to have been unsuccessful, then it is time that they **try something else**, Y. In contrast to the process which produced their initial guess X, the student should allow the results of the analysis of X to guide their choice of Y: "*since $F'(x)$ differed from $f(x)$ by -1, now I will try $G(x) = -\cos(x^2)$.*" From here, the student analyzes Y using the same procedure as before: "*$G'(x) = 2x\sin(x^2) = f(x)$, so $G(x)$ is an anti-derivative of $f(x)$.*" If necessary, "analyze" and "try something else" are iterated until success is achieved, or until the student decides that they need additional help: in the latter situation I ask them to write down their confusion as precisely as they can and then contact me to arrange a meeting.

Effective communication: I also want my students to learn how to communicate their ideas effectively. When I taught calculus in Fall 2019, I gave students examples of good and bad mathematical writing to read and critique, and then we brainstormed a list of traits which exemplify each category. I believe this approach fostered a sense of student ownership over the hallmarks of effective communication, and I plan to repeat this exercise at my next opportunity. As a rough benchmark to keep in mind, I suggest that students write so that anyone in the class would be able to follow their argument.

Of course it is also essential that I "practice what I preach," so I strive to communicate effectively in every student exchange. I have occasionally lost sight of the fact that mindful listening is an equal partner to speaking well, so to improve as a listener, I frequently remind myself to: wait for a student to finish speaking before responding, ask the student to clarify whenever necessary, and respond without judgment or sarcasm.

In My Classroom

Discussion sections: In discussion sections my primary job is to moderate student-driven discourse. I start by posing questions and then give students time to work on them in small groups. While students are working, I float around and try to spend some time with each group. During these interactions, I give hints, and ask leading questions. However, I am very careful to avoid handing students the "correct" answer, and I even avoid checking answers. On this second point there is an important distinction: while I won't say "yes that is correct," I am happy to review a student's analysis, as long as they drive the conversation. Thus if a student were to ask "*is this the correct answer?*" I would respond "*I don't actually know the correct answer. Can you talk me through your analysis?*"

Once I judge that every group has had time to engage with a given problem, I bring everyone back together for class discussion. I ask students to supply their answers which I then record on the board. I have been working to not dismiss "obviously" wrong answers without comment because I believe doing so weakens student buy-in to *the mantra*, and discourages class participation.

When recording a student's answer on the board, I must faithfully translate their response into written form. This can take work, especially when a student (let's call them A) is struggling to express their idea precisely. When this happens, I become a coach: the class is my team, and our goal is to arrive at a precise statement of A's idea. In a linear algebra tutorial, a True/False question purported that some subset S of $\mathbb{R}^3$ was a subspace, and after polling the class, consensus was "False." I asked "*why isn't S a subspace?*" Student A answered "*because you add stuff and go out of it.*" "*Thank you,*" I replied "*can you be a little more precise?*" A didn't respond. "*Can anyone help A out?*" Student B replied "*it's because you can add two vectors in V and get something which is not in V.*" "*Ah, so you're saying that V is not additively closed. Is that what you were trying to say A?*" "*Yes.*"

Having accurately recorded a student's answer, it is time for me to guide the class analysis. Here too I try to act only as scribe and coach so that students drive the discussion. At times though it is necessary that I borrow the spotlight to give short lectures which address the common misconceptions and factual inaccuracies I have observed by visiting with the groups. Whether as a scribe, coach, or lecturer, it is essential that I make every effort to communicate effectively, so I remind myself to: "write everything I say, and say everything I write," and treat the board as a letter written to an absent student.

Lecture courses: When it comes to teaching lecture courses, I try to borrow as much methodology as possible from my discussion sections. For example, I have incorporated occasional problem sessions into my lectures, and I try to model the "Q&A" format of tutorials by beginning new topics with motivating questions which the class then works to answer. I draw lots of pictures to aid in visualization and encourage students to do the same. Finally, I try to cultivate a lighthearted attitude in the classroom (through humor, for example). I admit that I struggled in this area when I taught a class of 200 calculus students; I'm excited for my next chance to teach a large course so that I can improve in this aspect.

Looking forward

There are still many teaching activities which I look forward to trying. Most of my teaching has centered on first-year courses, so I am excited to teach more advanced subjects, especially abstract algebra, and number theory. There are also pedagogical methods that I want to try but have not yet had the chance, such as oral exams, and a "flipped classroom." I want to learn about recruiting students from underrepresented groups to math, and how to make them feel at home once they are here. Above all else though, I look forward to continue teaching people how to solve hard problems; given how many challenges we humans face, I feel at once obligated and privileged to teach math.