UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Electrical and Computer Engineering

ECE 139 **Probability and Statistics** Spring 2019

Homework Assignment #4

(Due on Wednesday 5/1/2019 by 5 pm *in the Homework Box*)

Problem # 1. A smartphone allows a maximum of three unlocking attempts through fingerprint recognition (e.g., Apple's "touch ID") after which a pin or standard log-in is needed. Let p be the probability that the system matches your fingerprint and unlocks the phone (p depends on the quality of the fingerprint sensor). Assume that unlocking attempts are independent. To ensure user satisfaction, the phone manufacturer requires the probability that a user successfully unlocks the phone within the allowed three attempts, to be at least 0.95. What is the minimum value of p necessary to achieve this?

Problem # 2. Random variable X has PMF $P_X(x) = a/x$ for $x = 1, 2, 3, 4$; and $P_X(x) = 0$ otherwise.

a) Find the value of constant a.

b) Find $P[X < 3]$.

c) Find the probability that X is odd.

d) Find $P[1 < X < 4]$.

Problem # 3. A file consisting of X packets is sent over the network to an end user. It is known that X is a random variable with PMF $P_X(x) = cx^2$ for $x = 1, 2, 3, 4$; and $P_X(x) = 0$ otherwise. Each packet is received correctly with probability p, independently of the other packets.

a) Find the constant c.

b) Find $P[X > 2]$.

c) Find the probability that the entire file is received correctly, i.e., all X packets are received correctly.

Problem # 4. A variation on a familiar theme: A packet is received correctly by an end user with probability p. To increase the likelihood that an important message is received, the packet is sent repeatedly N times. Let random variable X be the number of copies received correctly by the end user.

a) Find the PMF $P_X(x)$.

b) If $p = 0.9$, how large should N be to ensure that with probability 0.99 at least one copy of the message is received?

Problem # 5. Yet another variation: A better packet switched network employs the concept of acknowledgment. When the end user's device receives a packet correctly it sends an acknowledgment to the sender. Here too, a packet is received correctly with probability p, but the sender keeps sending copies of a given packet until a copy is correctly received (signaled by acknowledgment). Let random variable N be the number of times the same message packet is sent.

a) Find the PMF $P_N(n)$.

b) To avoid excessive delays (too many repeated transmissions of same packet), it is required that the network maintain $P[N < 4] > 0.95$. Find the minimum value of p that will satisfy this requirement

Problem # 6. Outside the airport terminal, at the shuttle stop, you sometimes wait a long time for the right shuttle that will take you to your hotel. You are quite annoyed to see many other shuttle vans passing. Suppose the total number of shuttle vans N you see passing, until at last you board the right one, is a geometric random variable with parameter $p = 0.02$. What is the probability $P[N > 100]$...?

Problem # 7. The number of packets that arrive at a router node in T milliseconds is a Poisson random variable X with expected value $T/5$. Find:

a) The PMF $P_X(x)$.

b) The probability that in a two millisecond interval, three packets will arrive.

c) The probability that no packets arrive within a given ten millisecond interval.

d) The minimum time interval size that guarantees that at least one packet will arrive, with probability 0.99.