Problem # 3 (40). Some electronic systems have the annoying tendency to fail catastrophically after the one year warranty period is over, but never before. Let the lifetime (in years) of such a system be a continuous random variable X, with CDF

$$F_X(x) = \begin{cases} 0 & x < 1 \\ 1 - \frac{1}{x} & x \geq 1 \end{cases}$$

a) What is the probability that the system is still working after K years?

b) If the system was indeed still working by the end of K years, what is the probability that it will fail within the next year (i.e., the year $K + 1$)? Is there some truth to the manufacturer's surprising claim that "system reliability improves with age"?

c) On the other hand, if you know the system failed within $2K$ years, what is the probability that it was still working after K years?

d) If you purchased three such systems (which we assume fail independently) and they are all still working after K years, what is the probability that at least one of them is still working by the end of $2K$ years?