Useful Formulas and Definitions

1. Events and Probabilities:
   (a) Conditional Probability: $P[A|B] = \frac{P[AB]}{P[B]}$
   (b) Bayes' Theorem: $P[B|A] = \frac{P[A|B]P[B]}{P[A]}$
   (c) Independent events: $P[AB] = P[A]P[B]$
2. Probability Distribution and Density:
   (a) CDF: $F_X(x) = P[X \leq x]$
   (b) CDF and PDF: $F_X(x) = \int_{-\infty}^{x} f_X(t)dt$
   (c) Marginal PDF: $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$
   (d) Conditional PDF:
   $$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$
   (e) Independent random variables: $f_{X,Y}(x,y) = f_X(x)f_Y(y)$
   (f) Region probability:
   $$P[(X,Y) \in R] = \iint_R f_{X,Y}(x,y)dxdy$$
   (g) Derived random variable: Let $Y = g(X)$, then
      i. General $g(\cdot)$:
      $$f_Y(y) = \sum_{x_n : g(x_n)=y} \left. \frac{f_X(x)}{|\frac{dy}{dx}|} \right|_{x=x_n}$$
      ii. Monotone $g(\cdot)$:
      $$f_Y(y) = \left. \frac{f_X(x)}{|\frac{dy}{dx}|} \right|_{x=g^{-1}(y)}$$
3. Expectation and Moments:
   (a) Expectation: $E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f_{X,Y}(x,y)dx\,dy$
   (b) Moments about origin: $E[X^n]$, joint moments: $E[X^nY^k]$
   (c) Central moments: $E[(X-\mu_X)^n]$, central joint moments: $E[(X-\mu_X)^n(Y-\mu_Y)^k]$, where $\mu_X = E[X]$ and $\mu_Y = E[Y]$
   (d) Correlation: $r_{X,Y} = E[XY]$, covariance: $Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$
   (e) Uncorrelated: $E[XY] = E[X]E[Y]$, orthogonal: $E[XY] = 0$

(f) Second moment property: $E[X^2] = \mu_X^2 + \sigma_X^2$,
where $\sigma_X^2 = E[(X - \mu_X)^2]$

(g) Second joint moment property: $E[XY] = \mu_X \mu_Y + Cov(X, Y)$

(h) Correlation coefficient:

$$\rho_{X,Y} = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

4. "Famous" Random Variables:

(a) Bernoulli (p):

$$P_X(x) = \begin{cases} 1-p & x = 0 \\ p & x = 1 \\ 0 & otherwise. \end{cases}$$

(b) Binomial (n, p) :

$$P_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, ..., n \\ 0 & otherwise. \end{cases}$$

(c) Geometric (p) :

$$P_X(x) = \begin{cases} p(1-p)^{x-1} & x = 1, 2, ... \\ 0 & otherwise. \end{cases}$$

(d) Uniform (a, b):

$$f_X(x) = \begin{cases} 1/(b-a) & a \leq x \leq b \\ 0 & otherwise. \end{cases}$$

(e) Exponential (λ):

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & otherwise. \end{cases}$$

(f) Gaussian (μ, σ):

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

5. General Purpose Math:

(a)

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

(b) Arithmetic series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

(c) Geometric series

$$\sum_{i=0}^{n-1} r^i = \frac{1-r^n}{1-r}$$

(d) Integration in parts:

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

(e) Some common integrals

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C, \qquad a \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int \sin(x) dx = -\cos(x) + C$$

(f) Some trigonometry

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\cos(x) + \cos(y) = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\tan^2(x) + 1 = \frac{1}{\cos^2(x)}$$

$$\sin(x) = \cos(\pi/2 - x) = -\sin(-x)$$

$$\cos(\pi/4) = \sqrt{2}/2, \quad \cos(\pi/3) = 1/2, \quad \cos(\pi/6) = \sqrt{3}/2$$