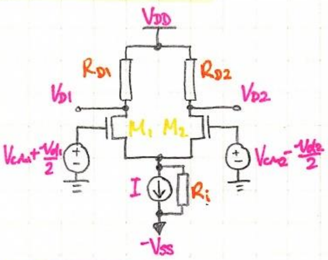
Differential Amplifier



Common-mode rejection ratio (CMRR)

$CMRR = \frac{A_D}{A_C}$

A_D - Difference mode gain

A_C - Common mode gain

Difference mode signal

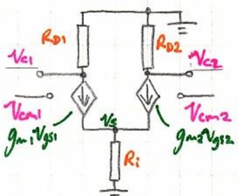
$V_{DO} = V_{D1} - V_{D2}$

Common mode signal

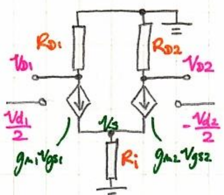
$V_{CO} = \frac{V_{C1} + V_{C2}}{2}$

Small Signal

Common Mode



Difference Mode



If $R_{D1} = R_{D2} = R_D$, $g_{m1} = g_{m2} = g_m$, $V_{cm1} = V_{cm2} = V_{cm}$, $V_{d1} = V_{d2} = V_d$

Common Mode

$g_m V_{gs} + g_m V_{gs} = \frac{V_S}{R_i}$

* $V_{gs} = \frac{1}{2g_m R_i + 1} V_{cm}$

If R_i large, $2g_m R_i \gg 1$

$V_{gs} \approx \frac{1}{2g_m R_i} V_{cm}$

$g_m V_{gs} = \frac{1}{2R_i} V_{cm}$

$V_{C1} = V_{C2} = -\frac{R_D V_{cm}}{2R_i}$

$V_{CO} = V_{C1} - V_{C2} = 0$

$A_C = 0$

Difference Mode

$g_m V_{gs1} + g_m V_{gs2} = \frac{V_S}{R_i}$

$g_m\left(\frac{V_d}{2} - V_S\right) + g_m\left(\frac{-V_d}{2} - V_S\right) = \frac{V_S}{R_i}$

$-2g_m V_S = \frac{V_S}{R_i}$

* g_m independent from R_i so $V_S = 0$

$V_{D1} = -\frac{g_m R_D V_d}{2}$

$V_{D2} = \frac{g_m R_D V_d}{2}$

$V_{DO} = V_{D1} - V_{D2} = -g_m R_D V_d$

$A_D = \frac{V_{DO}}{V_d} = -g_m R_D$

If symmetrical EXCEPT $R_{D1} = R_D$, $R_{D2} = R_D + \Delta R_D$

Common Mode

* $V_{gs} = \frac{1}{2g_m R_i + 1} V_{cm}$

$g_m V_{gs} = \frac{g_m}{2g_m R_i + 1} V_{cm}$

$V_{C1} = \frac{-g_m R_D}{2g_m R_i + 1} V_{cm}$

$V_{C2} = \frac{-g_m (R_D + \Delta R_D)}{2g_m R_i + 1} V_{cm}$

$V_{CO} = V_{C1} - V_{C2}$

$= -\frac{g_m \Delta R_D}{2g_m R_i + 1} V_{cm}$

$A_C = \frac{V_{CO}}{V_{cm}} = -\frac{g_m \Delta R_D}{2g_m R_i + 1}$

Difference Mode

* $V_S = 0$

$V_{D1} = \frac{-g_m V_d R_D}{2}$

$V_{D2} = \frac{g_m V_d (R_D + \Delta R_D)}{2}$

$V_{DO} = V_{D1} - V_{D2}$

$= -g_m V_d \left(R_D + \frac{\Delta R}{2}\right)$

$A_D = \frac{V_{DO}}{V_d} = -g_m\left(R_D + \frac{\Delta R_D}{2}\right)$

$CMRR = \frac{A_D}{A_C}$

$CMRR = \frac{\left(R_D + \frac{\Delta R_D}{2}\right)(2g_m R_i + 1)}{\Delta R_D}$

Operational Amplifiers (OP-amps)

$V_o = A(V^+ - V^-)$

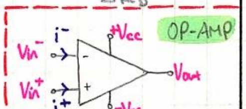
ideal: $A = \infty$ $\quad r_{out} = 0$

$i^- = i^+ = 0$ $\quad r_{in} = \infty$

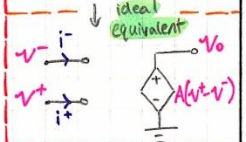
$V^+ - V^- = 0$ (virtual short)

Negative feedback → output signal fed back into input

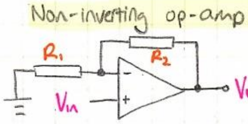
OP-AMP



↓ ideal equivalent



Non-inverting op-amp



$V_{out} = \frac{R_1 + R_2}{R_1} V_{in}$

$G = \frac{R_1 + R_2}{R_1}$

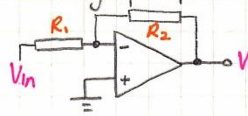
Gain

A = open loop gain

G = closed loop gain

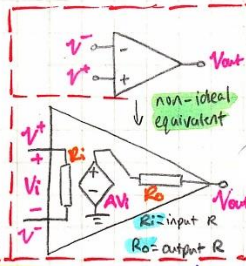
$= \frac{V_{out}}{V_{in}}$

Inverting op-amp



$V_{out} = -\frac{R_2}{R_1} V_{in}$

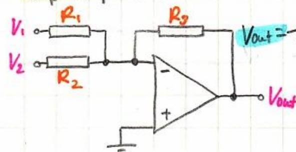
$G = -\frac{R_2}{R_1}$



R_i = input R

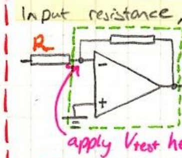
R_o = output R

Op-amp adder



$V_{out} = -R_3\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$

Input resistance, R_i

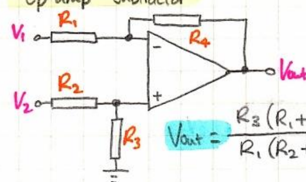


Use non-ideal equivalent

Input R, R_i'

$R_i = R_i' + R_-$ (series)

Op-amp subtractor



$V_{out} = \frac{R_3(R_1 + R_4)}{R_1(R_2 + R_3)} V_2 - \frac{R_4}{R_1} V_1$

Op amp saturation

+ve: $A(V^+ - V^-) > +V_{cc}$

-ve: $A(V^+ - V^-) < -V_{cc}$

Energy Storage Elements:

Capacitors (Electrical Energy)

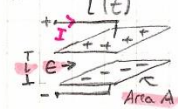
$q(t) = C\,v(t)$

$i(t) = C\frac{dv(t)}{dt}$ $\quad v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$

stored energy $= w_E(t) = \frac{q^2(t)}{2C} = \frac{Cv(t)^2}{2}$

Parallel Plate capacitor

$C(t) = \frac{\epsilon A(t)}{l(t)}$



Inductors (Magnetic Energy)

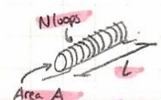
$\lambda(t) = L\,i(t)$ [Flux linkage]

$v(t) = L\frac{di(t)}{dt}$ $\quad i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt$

stored energy $= w_M(t) = \frac{\lambda^2(t)}{2L} = \frac{Li(t)^2}{2}$

Inductor

$L(t) = \frac{\mu N^2 A(t)}{l(t)}$



Combinations

Capacitors		Inductors	
Series	Parallel	Series	Parallel
C_1, C_2	C_1, C_2	L_1, L_2	L_1, L_2
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = L_1 + L_2$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$

Capacitor

↳ discharged → short

↳ fully charged → open

↳ $v(t)$ continuous

Inductor

↳ discharged → open

↳ fully charged → short

↳ $i(t)$ continuous

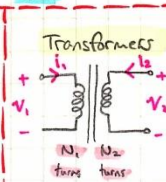
MOSFET Gate Capacitance

$C_{GS} = \frac{\epsilon_{ox} LW}{d}$

$C_{ox} = \frac{\epsilon_{ox}}{d}$

$C_{GS} = C_{ox} LW$

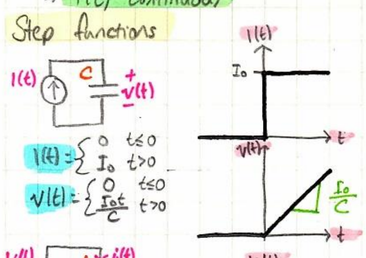
Transformers



$\frac{V_1(t)}{N_1} = \frac{V_2(t)}{N_2}$

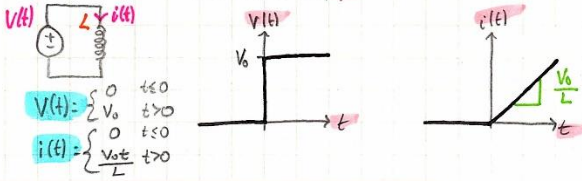
$N_1 i_1(t) = -N_2 i_2(t)$

Step functions



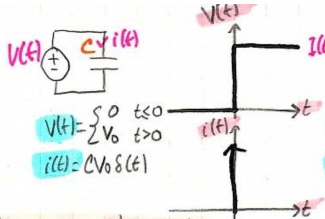
$I(t) = \begin{cases} 0 & t \le 0 \\ I_0 & t > 0 \end{cases}$

$v(t) = \begin{cases} 0 & t \le 0 \\ \frac{I_0 t}{C} & t > 0 \end{cases}$



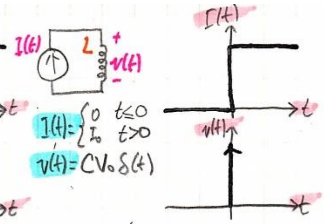
$V(t) = \begin{cases} 0 & t \le 0 \\ V_0 & t > 0 \end{cases}$

$i(t) = \begin{cases} 0 & t \le 0 \\ \frac{V_0 t}{L} & t > 0 \end{cases}$



$V(t) = \begin{cases} 0 & t \le 0 \\ V_0 & t > 0 \end{cases}$

$i(t) = CV_0\delta(t)$



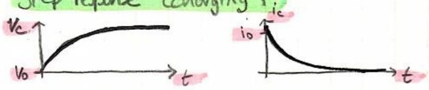
$I(t) = \begin{cases} 0 & t \le 0 \\ I_0 & t > 0 \end{cases}$

$V(t) = CV_0\delta(t)$

First order transients

Capacitor (RC circuits)

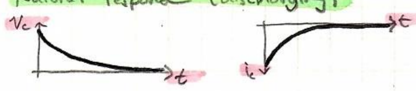
Step response (charging)



Form: $V_c(t) = A(1 - e^{-t/\tau})$

$i_c(t) = Be^{-t/\tau}$

Natural response (discharging)

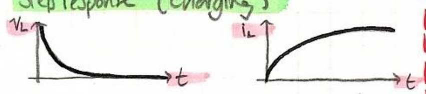


Form: $V_c(t) = Ae^{-t/\tau}$

$i_c(t) = -Be^{-t/\tau}$

Inductor (RL circuits)

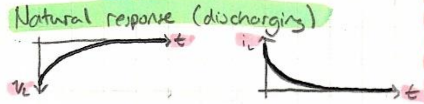
Step response (charging)



Form: $V_L(t) = Ae^{-t/\tau}$

$i_L(t) = B(1 - e^{-t/\tau})$

Natural response (discharging)



Form: $V_L(t) = -Ae^{-t/\tau}$

$i_L(t) = Be^{-t/\tau}$

RC circuits
$\tau = RC$

RL circuits
$\tau = \frac{L}{R}$

Typically (dis)charges $\frac{1}{e} = 36.8\%$ in 1τ

≈ Fully (dis)charges in 5τ

Zero input response
↳ non-zero initial conditions
↳ input drive is zero

Zero state response
↳ zero initial conditions
↳ non-zero input drive

Total response
ZIR + ZSR

How to solve First order transients

1) Find homogeneous solution (natural response)
2) Find particular solution (forced / driven response)
3) Solution = homogeneous sltn + particular sltn.
4) Apply initial conditions

↳ Apply Kirchoffs to get differential eq.
↳ Set constants = 0 → homogeneous sltn.
↳ Sub homogeneous sltn into diff eq. w/ constants
→ Method of undetermined coefficients → add unknown K
↳ Sub in initial conditions

E storage in equivalent components

If two capacitors, C_1, C_2 make C_{eq}

$V_{C1}(t) + V_{C2}(t) = V_{Ceq}(t)$

$V_{C1}(t) = A_1e^{-t/\tau} + B$

$V_{C2}(t) = A_2e^{-t/\tau} - B$

$V_{Ceq} = (A_1 + A_2)e^{-t/\tau}$

If two inductors L_1, L_2 make L_{eq}

$i_{L1}(t) + i_{L2}(t) = i_{Leq}(t)$

$i_{L1}(t) = A_1e^{-t/\tau} + B$

$i_{L2}(t) = A_2e^{-t/\tau} - B$

$i_{Leq} = (A_1 + A_2)e^{-t/\tau}$

If two storage elements of same type. $X_1, X_2 \to X_{eq}$

X_{eq} fully charges / discharges as $t \to \infty$

$$\underbrace{[E_{x1}(0) + E_{x2}(0)]}_{\text{initial E in both elements}} - \underbrace{E_{xeq}(0)}_{\text{initial E in eq element}} = \underbrace{[E_{x1}(\infty) + E_{x2}(\infty)]}_{\text{E trapped in both elements}}$$

General form of equations:

Capacitor

$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)]e^{-t/\tau}$

$V_c(t) = V_c(0)e^{-t/\tau} + V_c(\infty)(1 - e^{-t/\tau})$

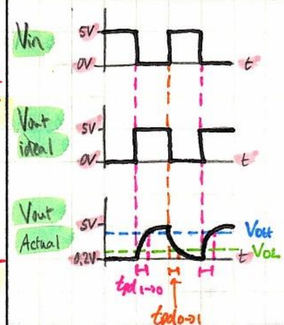
Inductor

$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau}$

$i_L(t) = i_L(0)e^{-t/\tau} + i_L(\infty)(1 - e^{-t/\tau})$

Propagation delay (gate delay)

↳ time for circuit to reliably output desired output



$t_{pd1\to0}$ 1→0 change at INPUT

$t_{pd0\to1}$ 0→1 change at INPUT

Rise time: 0→1 change at OUTPUT

Fall time: 1→0 change at OUTPUT

$t_{pd} = \max(t_{pd1\to0}, t_{pd0\to1})$

for multiple input/output:

input/output pair $ij \to t_{pd}^{ij}$

$t_{pd} = \max_{ij} t_{pd}^{ij}$

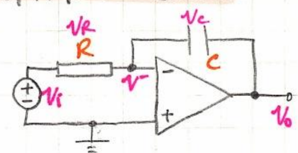
State Variables

$\frac{d}{dt}(\text{state variable}) = f(\text{state variable, input variable})$

If linear:

$\frac{d}{dt}\begin{pmatrix}\text{state} \\ \text{variable}\end{pmatrix} = K_1\begin{pmatrix}\text{state variable} \\ \text{present value}\end{pmatrix} + K_2\begin{pmatrix}\text{input} \\ \text{variable}\end{pmatrix}$

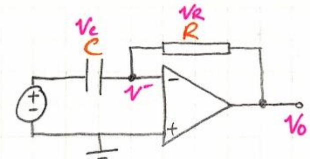
Op Amp RC Circuits



Integrator

$V_c = -\frac{1}{RC}\int V_R\,dt$

$V_o \approx -\frac{1}{RC}\int V_i\,dt$



Differentiator

$V_R = -RC\frac{dV_c}{dt}$

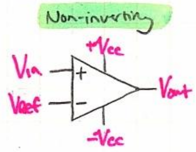
$V_o \approx -RC\frac{dV_i}{dt}$

Op amp positive feedback

↳ turn continuous analog signals into 2 state signals

Comparator

Non-inverting



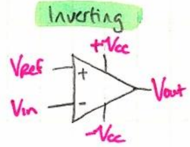
If $V_{in} > V_{ref}$

$V_{out} = +V_{cc}$

If $V_{in} < V_{ref}$

$V_{out} = -V_{cc}$

Inverting



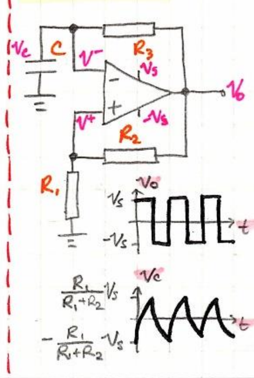
If $V_{in} > V_{ref}$

$V_{out} = -V_{cc}$

If $V_{in} < V_{ref}$

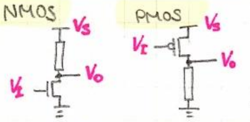
$V_{out} = +V_{cc}$

RC oscillator

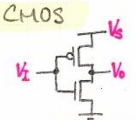


Inverter circuits - triode (MOSFET)

NMOS PMOS

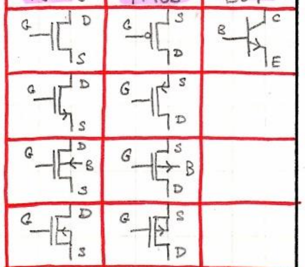


CMOS



V_I	NMOS	PMOS	V_O
0	OFF	ON	1
1	ON	OFF	0

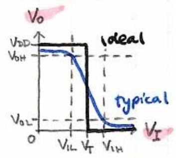
NMOS	PMOS	BJT



G-gate D-drain
S-source B-base

B-base
C-Collector
E-Emitter

Transfer Curve



$$V_T = \frac{V_{TN} + \sqrt{\frac{K_P}{K_N}}\,(V_S + V_{TP})}{1 + \sqrt{\frac{K_P}{K_N}}}$$

When $I_{DN} = -I_{DP}$

Unified Model

NMOS $V_T > 0$

$$i_{DS} = \begin{cases} 0 & V_{GS} < V_T & \text{cutoff} \\ K\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right] & V_{GS} \geq V_T \quad V_{DS} < V_{GS}-V_T & \text{triode} \\ \frac{K}{2}(V_{GS}-V_T)^2 & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS}-V_T & \text{saturation} \end{cases}$$

PMOS $V_T < 0$

$$i_{SD} = \begin{cases} 0 & V_{SG} < |V_T| & \text{cutoff} \\ K\left[(V_{SG}-|V_T|)V_{SD} - \frac{V_{SD}^2}{2}\right] & V_{SG} \geq |V_T| \quad V_{SD} < V_{SG}-|V_T| & \text{triode} \\ \frac{K}{2}(V_{SG}-|V_T|)^2 & V_{SG} \geq |V_T| \quad V_{SD} \geq V_{SG}-|V_T| & \text{saturation} \end{cases}$$

$K = \mu C_{ox} \frac{W}{L}$

$K' = \mu C_{ox}$

set $\begin{cases} \mu = \text{mobility} \\ C_{ox} = \text{capacitance per area of gate oxide} \end{cases}$

designed $\begin{cases} W = \text{MOSFET width} \\ L = \text{MOSFET length} \end{cases}$

SR Model

$i_D = \frac{V_{DS}}{R_{on}}$ $R_{on} = R_n \frac{L}{W}$

under triode conditions

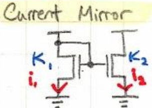
N MOSFETs in parallel
↳ Act as 1 MOSFET w/ dimensions K_N
↳ $K_N = N K_0$
K_0 - dimensions of 1
↳ allows for greater I

Diode connected MOSFET

Always in saturation: $V_{DS} = V_{GS}$

Small signal: $r_{DS} = \frac{v_{gs}}{g_m v_{gs}} = \frac{1}{g_m}$

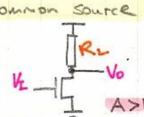
Current Mirror



$\frac{i_2}{i_1} = \frac{K_2}{K_1}$

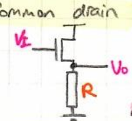
Amplifiers - saturation (MOSFET)

Common source



A>1

Common drain

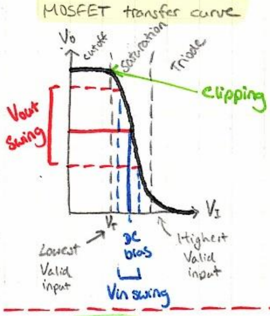


A≈1

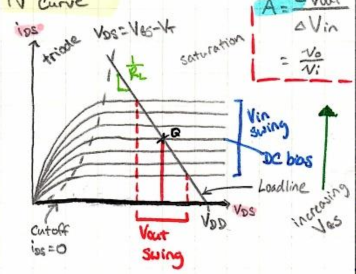
SS Gain

$A = \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{v_o}{v_i}$

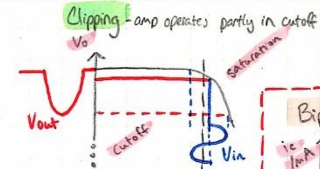
MOSFET transfer curve



IV Curve



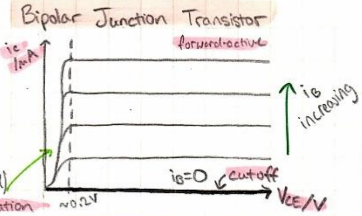
Clipping - amp operates partly in cutoff

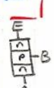


Highest Valid Input

$V_{in\,max} = \frac{-1 + \sqrt{1 + 2V_{DD}R_L K}}{R_L K}$

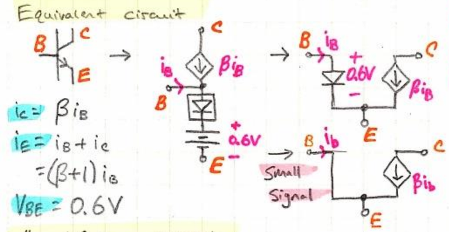
Bipolar Junction Transistor





Mode	E-B Junction	C-B Junction
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

Equivalent circuit



$i_C = \beta i_B$

$i_E = i_B + i_C$
$= (\beta+1)\, i_B$

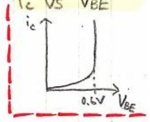
$V_{BE} = 0.6V$

Forward Active

$i_B > 0$

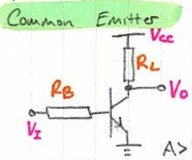
$V_{CE} > 0.2V$

i_C vs V_{BE}



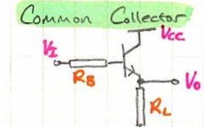
Amplifiers (BJT)

Common Emitter



A>1

Common Collector



A≈1

$i_B = \frac{V_I - 0.6}{R_B}$

$i_C = \beta i_B$

$i_E = (\beta+1)\, i_B$

$V_O = V_{CC} - i_C R_L$
$= V_{CC} - \beta \frac{R_L}{R_B}(V_I - 0.6)$

Transfer Curve of Common Emitter

Operating point Q

Small Signal Analysis

MOSFET

$v_{GS} = V_{GS} + v_{gs}$

$\underbrace{i_D}_{\text{total}} = \underbrace{I_D}_{\text{DC}} + \underbrace{i_d}_{\text{AC/SS}}$

Linear Taylor series expansion

$y = f(x_0) + \left.\frac{df}{dx}\right|_{x_0}(x - x_0)$

$\underbrace{i_D}_{\text{total } i_D} = \underbrace{\frac{K}{2}(V_{GS}-V_T)^2}_{\text{DC } I_D} + \underbrace{K(V_{GS}-V_T)v_{gs}}_{\text{AC/SS } i_d}$

$i_d = K(V_{GS}-V_T)v_i = g_m v_i$

g_m = transconductance $= K(V_{GS}-V_T)$

Input/Output Resistance (SS)

↳ set independent V, I sources = 0

↳ Use SS model → apply v_{test} at input/output

↳ $r = \frac{v_{test}}{i_{test}}$

MOSFET amplifier - common source

$r_i \approx \infty$ $r_o = R_L$

MOSFET amplifier - common drain

$r_i \approx \infty$ $r_o \approx \frac{1}{g_m}$

BJT amplifier - common emitter

$r_i = R_B$ $r_o = R_L$

BJT amplifier - common collector

$r_i = R_B + (\beta+1)R_E$ $r_o \approx \frac{R_B}{\beta+1}$

Common Source

$V_{OUT} = V_{OUT} + v_{out}$
$= \underbrace{V_{DD} - I_D R_L}_{\text{DC}} - \underbrace{i_d R_L}_{\text{AC SS}}$

$v_{out} = -i_d R_L$
$= -g_m R_L v_{gs}$

$A = \frac{v_{out}}{v_{in}} = -g_m R_L$

Current gain

Current gain $= \frac{i_o}{i_i}$

↳ $\frac{\text{incremental } i \text{ output}}{\text{incremental } i \text{ input}}$

Power gain

Power gain $= \frac{v_o i_o}{v_i i_i}$
$= v_{gain} \times i_{gain}$

Equivalent Models