

Problem # 4 (30) To aid research on the drought, Dr. Emptybottle developed a new probabilistic model for groundwater in Southern California's aquifers. Random variable X denotes the total amount of groundwater at the end of the rainy season. Random variable Y is the total amount of water pumped out during the following dry season. It is believed that Y is uniformly distributed over $[0, x]$, where x denotes the value taken by X . According to Dr. Emptybottle's analysis the PDF of X is

$$f_X(x) = xe^{-x}u(x).$$

- a) Find the joint PDF combining the total groundwater and the amount of water extracted: $f_{X,Y}(x, y)$.
- b) Find the marginal PDF of water extracted: $f_Y(y)$.
- c) Find the conditional PDF $f_{X|Y}(x|y)$, which Dr. Emptybottle plans to use to retroactively adjust her model for total groundwater once it is known how much water was in fact pumped out during the dry season.

(To minimize errors it is advised to carefully identify the (x, y) range for each answer).