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Homework 3: Induction & Recursion

5. A postage of n cents can be formed using just 4-cent and 7-cent stamps, $n \geq 18$.
    a. Base Case:
        i. P(18) can be satisfied with one 4-cent stamp and two 7-cent stamps.
        ii. P(19) can be satisfied with three 4-cent stamps and one 7-cent stamp.
        iii. P(20) can be satisfied with five 4-cent stamps.
        iv. P(21) can be satisfied with three 7-cent stamps.
    b. The inductive hypothesis assumes that P(n) is true.
    c. In the inductive step, we must prove that if P(n) is true, then P(n+1) is true.
    d. Prove P(n+1) for $n \geq 21$
        i. Start with a postage price of n - 3.
        ii. Since $n \geq 21$, $n - 3 \geq 18$. 18 cents is in the range $n \geq 18$, and by the inductive hypothesis, n - 3 cent postage can be purchased.
        iii. When a four-cent postage is added, the total postage becomes $n - 3 + 4 = n + 1$. Therefore, P(n+1) is true.
    e. This step shows that this statement is true whenever $n \geq 18$ because adding more postage to another possible postage amount will still result in a possible postage amount.
6. Every positive integer n can be written as a sum of distinct powers of 2
    a. Base Case:
        i. $P(1) = 2^0 = 1$
    b. Assume that P(k) is true
    c. Prove that if P(k) is true, then P(k+1) is true
        i. Case 1: k+1 is even
            1. If k + 1 is even, then (k+1)/2 is also even, and by the inductive hypothesis, (k-1)/2 can also be written as a sum of distinct powers of 2
            2. Multiply the sum of distinct powers (k-1)/2 by 2. When multiplying a sum of distinct powers, the result is every power increased by one.
            3. Since the original powers were distinct, adding one to every power will also keep it distinct. Therefore, P(k+1) is true.
        ii. Case 2: k+1 is odd
            1. Since k+1 is odd, k is even.
            2. Since $2^0 = 1$ is the only distinct power of 2 that is odd, k must be composed of distinct powers of 2 not including 2^0.
            3. We know that P(k) is a sum of distinct powers of 2 via our inductive hypothesis, and we can further say that P(k) is not composed of 2^0.
            4. Because of this, we can add 2^0, and the result is still a sum of distinct powers of 2. Therefore, P(k+1) is true.

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7. P(n) is a propositional function. Which non-negative integers make P(n) true?
    a. P(0) is true. If P(n) is true, then P(n+2) is true.
        i. Since we do not know if P(1) is true, and adding two to any odd number results in an odd number, n cannot include odd numbers.
        ii. We know P(0) is true. If n = 2, P(2-2) = P(0) = true. If n = 4, P(4-2) = P(2) = true. All even values of n+2 will always reference a value of n such that P(n) is true. This is a pattern of even numbers.
        iii. Therefore, all even non-negative integers make P(n) true.
    b. P(0) is true. If P(n) is true, then P(n+3) is true.
        i. We know P(0) is true. If n = 3, P(3-3) = P(0) = true. If n = 6, P(6-3) = P(3) = true. All even values of n+3 will always reference a value of n such that P(n) is true. This is a pattern of multiples of three.
        ii. Therefore, all non-negative integers that are a multiple of three make P(n) true.
    c. P(0) and P(1) are true. If P(n) and P(n+1) are true, then P(n+2) is true.
        i. We know P(0) and P(1) are true. If n = 2, P(2-2) = P(0) = true. If n = 3, P(3-2) = P(1) = true. Any higher value of n will always reference a value of n such that P(n) is true. This is a pattern of all non-negative integers.
        ii. Therefore, all non-negative integers make P(n) true.
    d. P(0) is true. If P(n) is true, then P(n+2) and P(n+3) are true.
        i. We know P(0) is true, therefore P(0+2) = P(2) and P(0+3) = P(3) is true.
        ii. If n = 4, P(4-2) = P(2) = true, if n = 5, P(5-2) = P(3) = true. This pattern continues to include all non-negative integers except 1.
        iii. Therefore, any non-negative integer n except n = 1 makes P(n) true.