UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Electrical and Computer Engineering

ECE 139 **Probability and Statistics** Spring 2019

Homework Assignment #3

(Due on Wednesday 4/24/2019 by 5 pm *in the Homework Box*)

Problem # 1. You have in your pocket two coins, one bent (comes up heads with probability 3/4) and one fair (comes up heads with probability 1/2). Not knowing which is which, you choose one at random and toss it. If it comes up heads you guess that it is the biased coin (reasoning that this is the more likely explanation of the observation), and otherwise you guess it is the fair coin. Find the probability that your guess is wrong.

Problem # 2. You still have the same two coins in your pocket, but this time you decide to choose one at random, toss it, and then toss *the other coin*. Let H_1 be the event that the first toss outcome is heads, and H_2 the event that the second toss outcome is heads.

a) Find the probability $P[H_1 \cap H_2]$.

b) Find $P[H_1]$ and $P[H_2]$.

c) Are H_1 and H_2 independent events? (Also explain why this result should be expected).

Problem # 3. A pair of packets is sent over the network to an end user. It is known that the first packet will be lost with probability 1/3. If the first packet is received, then the second packet is lost with probability 1/6. If the first packet is lost then the second packet is lost with probability 2/3. Find the probabilities of the events:

a) Exactly one packet is received by the end user.

b) Both packets are lost.

Problem # 4. A variation on a familiar theme: You have two coins in your pocket. The first coin produces heads with probability p_1 when tossed, and the second coin produces heads with probability p_2. You select a coin at random and toss it N times. Express:

a) The probability that exactly k outcomes are heads.

b) Given that k outcomes are heads, the probability that the coin being tossed is in fact the first coin.

Problem # 5. A student is known to arrive late for a class 40% of the time. If the class meets 5 times each week find:

a) The probability that the student arrives late for at least 3 classes in a given week.

b) The probability the student will not be late at all in a given week.

Problem # 6. Consider independent events A and B.

a) Show that events A and B^c are also independent. (Justify intuitively for partial credit; prove rigorously for full credit.) Hint: One straightforward way is to show that $P[B^c|A] = P[B^c]$.

b) How about the events A^c and B^c? (OK to assume part (a) even if you were not able to prove it).

c) Given known probabilities $P[A] = p_a$ and $P[B] = p_b$, express the following probabilities: $P[A \cap B]$, $P[A \cap B^c]$, and $P[A^c \cap B^c]$.

Problem # 7. (A challenge problem?):

a) A and B are mutually exclusive events with probabilities p_a and p_b, respectively. Independent trials of the experiment are performed to see which one of the two occurs first. What is the probability that event A occurs before event B?

b) Next solve a similar problem for the case where A and B are independent events (and not mutually exclusive). What is the probability that B does not occur before A? (Note that now we must allow for the possibility that they occur simultaneously).

Note: Answers to this problem should only be in terms of p_a *and* p_b.