UNIVERSITY OF CALIFORNIA, SANTA BARBARA

Department of Electrical and Computer Engineering

ECE 139 **Probability and Statistics** Spring 2019

Homework Assignment #2

(Due on Wednesday 4/17/2019 by 5 pm *in the Homework Box*)

Problem # 1. A fair die is rolled. Let A_k denote the event that the number of dots facing up is k, and let B_k denote the event that this number is less than k, for $k = 1, \ldots, 6$. Let E and O denote the events that the number is even or odd, respectively. Find the probabilities:

a) $P[O|B_4]$

b) $P[A_2 \cup A_4|B_5]$

c) $P[B_4|O]$

d) $P[B_4|B_5]$

e) $P[E \cap B_5|B_6]$

Problem # 2. Next consider the roll of a pair of fair dice. Find the probabilities of the events:

a) A_k: the *total* number of dots facing up is k, for $k = 2, \ldots, 12$. (There are 11 such events)

b) B : the two dice show different outcomes

Problem # 3. Let the sample space be $S = \{a, b, c\}$. You are informed of the probabilities of two events: $P[\{a, c\}] = 0.6$, and $P[\{b, c\}] = 0.8$. Use the axioms of probability to determine the probability of each of the outcomes a, b and c.

Problem # 4. A statistical survey found that 20% of cars are red, and 10% are electric cars. It was further discovered that of all cars that are either red or electric, 50% are both red and electric.

a) What is the probability $P[R \cap E]$ that a car is both red and electric?

b) What is the conditional probability $P[E|R]$ that a red car is electric?

c) What is the conditional probability $P[R|E]$ that an electric car is red?

Problem # 5. Show that the probability that exactly one of the two events A or B occurs is given by

$$P[A] + P[B] - 2P[A \cap B]$$

Problem # 6. Show that

$$P[A \cap B \cap C] = P[A|B \cap C]\, P[B|C]\, P[C]$$

Problem # 7. A missile can be accidentally launched if two relays A and B both have failed. The probabilities of failure of A and B are known to be 0.01 and 0.03, respectively. It is further known that if A fails then B is more likely to fail (specifically at probability 0.06).

a) What is the probability of an accidental launch?

b) What is the probability that A fails given that B fails?