**UNIVERSITY OF CALIFORNIA, SANTA BARBARA**
Department of Electrical and Computer Engineering

ECE 139 **Probability and Statistics** Spring 2019

**Homework Assignment** #5

**(Due on Wednesday 5/8/2019 by 5 pm *in the Homework Box*)**

**Problem # 1**. Random variable $X$ has CDF given by

$$F_X(x) = \begin{cases} 0 & x < -2 \\ 0.3 & -2 \le x < 1 \\ 0.8 & 1 \le x < 5 \\ 1 & x \ge 5 \end{cases}$$

a) Draw a graph for the CDF.

b) Find $P[X > 1]$, $P[X < 1]$, $P[1 < X < 5]$ and $P[1 \le X < 5]$.

c) Find the PMF, $P_X(x)$, (remember it is defined $\forall x \in \mathcal{R}$).

**Problem # 2**. Random variable $X$ has PMF $P_X(x) = ax^2$ for $x = -2, -1, 0, 1, 2$; and $P_X(x) = 0$ otherwise.

a) Draw a graph for the CDF $F_X(x)$ and find the value of constant $a$.

b) Find $P[X > 1]$.

c) Find $P[|X| < 2]$

**Problem # 3**. Show that the CDF of a geometric random variable with parameter $p$ can be expressed as

$$F_X(x) = \begin{cases} 0 & x < 1 \\ 1 - (1-p)^{\lfloor x \rfloor} & x \ge 1 \end{cases}$$

where $\lfloor x \rfloor$ denote the floor function, or the largest integer $k$ such that $k \le x$.

**Problem # 4**. A video server sends a packet containing a video frame to a user. The packet is received correctly with probability $p$, in which case the server receives prompt acknowledgment. If it does not receive acknowledgment within a predefined interval of

time $T$ the server sends it again, and again after another $T$, etc., until acknowledgment is received. However, real-time streaming only allows up to six such attempts before the server must abandon this frame and move to the next one. Let random variable $X$ be the number of attempts made by the server for a given video frame. Find the CDF $F_X(x)$.

**Problem # 5**. The CDF of continuous random variable $X$ is

$$F_X(x) = \begin{cases} 0 & x < -1 \\ c(x+1)^2 & -1 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

a) Find constant $c$.

b) Find $P[X > 1/2]$.

c) Find $P[-3/4 < X \le -1/4]$.

d) Find $P[|X| \le 1/2]$.

e) For what value of $a$ do we get $P[X > a] = 7/16$?

**Problem # 6**. Find the PDF $f_X(x)$ of the random variable $X$ in Problem # 5.

**Problem # 7**. The PDF of continuous random variable $X$ is

$$f_X(x) = \begin{cases} c\cos x & -\pi/2 \le x \le \pi/2 \\ 0 & \text{otherwise} \end{cases}$$

a) Find constant $c$.

b) Find $P[X > \pi/6]$.

c) Find $P[|X| \le \pi/6]$.

d) Find the CDF $F_X(x)$.

**Problem # 8**. Random variable $X$ has PDF

$$f_X(x) = \begin{cases} ae^{-bx} & x > 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $a$ and $b$ are constants. Find the necessary and sufficient conditions on $a$ and $b$ to ensure that $f_X(x)$ is a valid PDF.