Measuring Capacitance and Inductance

ECE 10BL Lab 6

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Background

The objective of this lab is to analyze the behavior of capacitors and inductors and uses this behavior to build an op-amp circuit that is capable of measuring capacitance and inductance, respectively. This lab shows how we can derive an expression for the capacitance and inductance using the input and output voltages of their respective op-amp circuits.

Procedure

Capacitors are devices that store energy in electric fields, while inductors are devices that store energy in magnetic fields. Unlike resistors or transistors, however, capacitors and inductors change with time. In a capacitor, the charge is proportional to the voltage through the capacitor by a factor C, the capacitance. Since current is defined as the amount of charge that passes a point in a given time, we can derive the following equation for current through a capacitor:

$$i_C(t) = C\frac{dv(t)}{dt} \qquad (1)$$

We can use this relationship to build a circuit to measure capacitance using an op-amp. If we assume the op-amp is ideal, we know that the voltage at both input nodes must be the same. If one input is grounded, this will allow us to perform nodal analysis using current on the input node to solve for capacitance. We have an equation for the capacitance in terms of current, so we can measure current by placing a resistor in the feedback loop. The circuit used to measure capacitance is shown below in Figure 1.

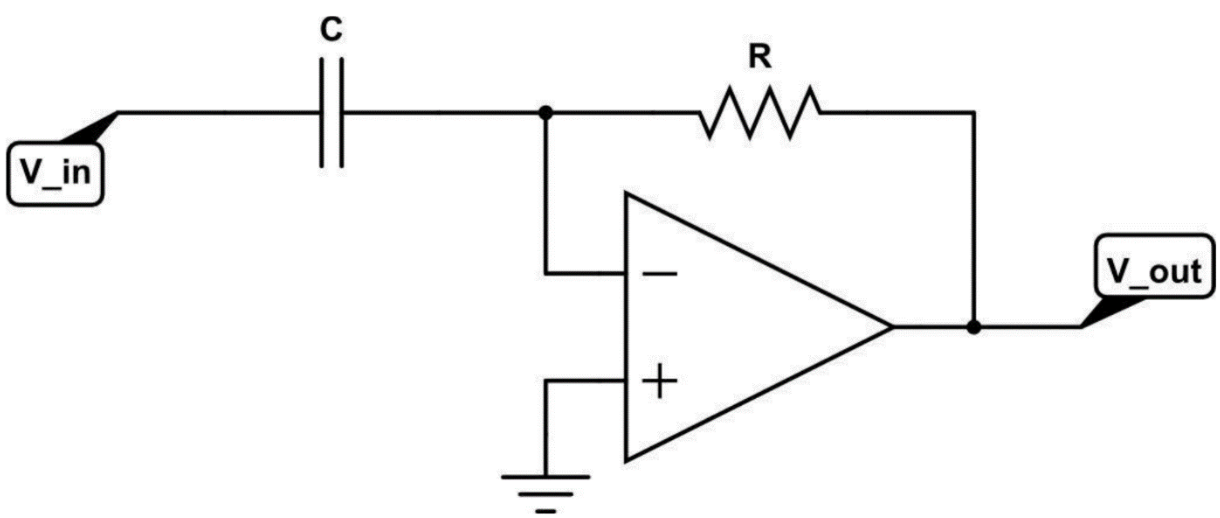

Figure 1: Schematic for Circuit Measuring Capacitance

Inductors also have a relationship similar to equation 1 above. However, instead of current being proportional to the rate of change of voltage, the voltage is proportional to the rate of change in current by a factor L, the inductance:

$$v_L(t) = L\frac{di(t)}{dt} \qquad (2)$$

Again, we can use this relationship to build a circuit to measure inductance using an op-amp. If we assume the op-amp is ideal and one input is grounded, we can use nodal analysis on the input node to determine inductance. We have an equation for the inductance in terms of the voltage, so we can measure the voltage across the inductor by placing it in the feedback loop. The circuit used to measure inductance is shown below in Figure 2.

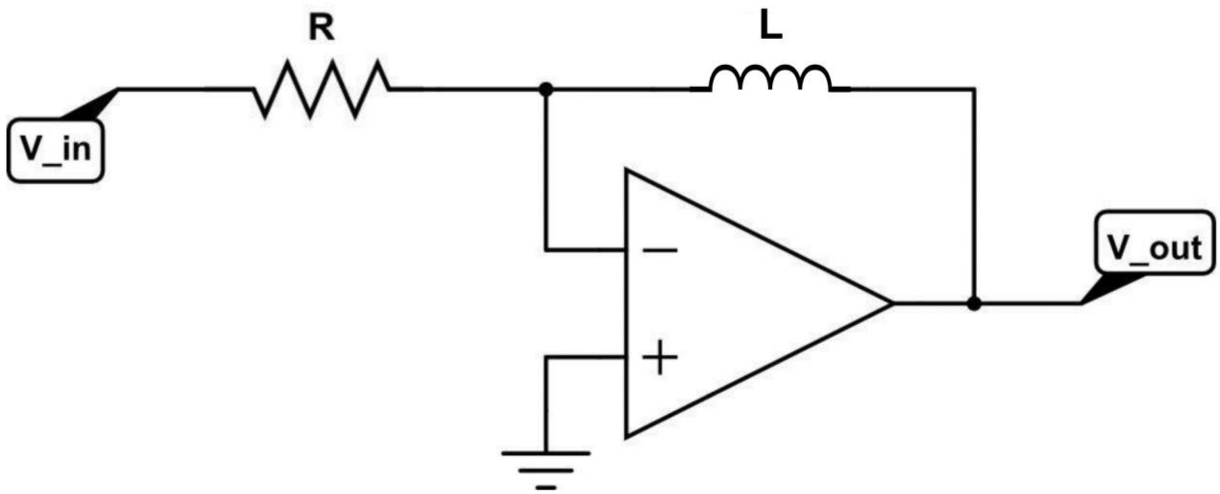

Figure 2: Schematic for Circuit Measuring Inductance

Analysis

We began the lab by building the circuit in Figure 1. An input sine wave with an amplitude of 100mV was used to drive the input, and the input and output voltages were measured to determine the capacitance.

We can begin to derive a formula for capacitance in terms of the known values. As shown in equation 1, we can relate current to the change in voltage of the capacitor. Since the voltage across the capacitor is equal to the input voltage minus the negative input node of the op-amp which is 0V, the change in voltage across the capacitor is equal to the change in the input voltage. By using nodal analysis via currents, we can solve for V_{OUT} in terms of capacitance and our known values as shown:

$$v_{out}(t) = -RC\frac{dV_{in}(t)}{dt} \qquad (3)$$

We know that the input is driven by a sine wave. We can model the sine wave and find its derivative as follows:

$$v_{in}(t) = Asin(2\pi ft) \tag{4}$$

$$\frac{dV_{in}(t)}{dt} = 2\pi fAcos(2\pi ft) \tag{5}$$

Substituting equation 5 into equation 3, we can obtain:

$$v_{out}(t) = -RCA(2\pi f)cos(2\pi ft) \tag{6}$$

We know that the cosine function can only output values from 0 to 1. As a result, the amplitude of the output voltage must equal the amplitude of the cosine function. Using this, we can finally solve for the capacitance as follows:

$$C = \frac{A_{out}}{2\pi fRA_{in}} \tag{7}$$

A_{OUT} was approximated by subtracting the minimum output voltage from the maximum output voltage and dividing the number by two. A_{IN} was found in an equivalent manner. The resistor had a fixed value of 1kΩ, and the sine wave featured a frequency of 1kHz.

For the 100nF ceramic capacitor, our calculated capacitance was 231nF. This result is in the same order of magnitude but is largely off from the theoretical value of 100nF. This could have been the result of a few errors, which is detailed in the Conclusion.

For the 10μF electrolytic capacitor, our calculated capacitance was 10.4μF. This result was calculated using the same method as the ceramic capacitor and is much closer to the theoretical value.

We continue the lab by building the circuit in Figure 2. Like the capacitor, we can derive a formula for inductance using nodal analysis. We start by solving for V_{OUT} in terms of inductance and our known values as shown:

$$v_{out}(t) = -\frac{L}{R}\frac{dV_{in}(t)}{dt} \tag{8}$$

The input sine wave is still the same, except the frequency has been changed to 100kHz. Likewise, we can use amplitudes to express our final value for inductance. Solving for inductance yields the following expression:

$$L = \frac{RA_{out}}{2\pi fA_{in}} \tag{9}$$

We then tested this using our 100μH inductor. Constructing the circuit in Figure 2 and measuring the output and input voltages, our calculated inductance was 146μH. This is not as exact as we had hoped but was on the same order of magnitude as the theoretical inductance.

The data collected in this lab is tabulated below:

Table 1: Measured Capacitances and Inductance Values

Part Used	Amplitude A_{OUT} (V)	Amplitude A_{IN} (V)	Resistance R (Ω)	Frequency f (Hz)	Calculated C / L	Percent Error
100nF Ceramic Capacitor	146 mV	101 mV	1 kΩ	1 kHz	231 nF	131%
10μF Electrolytic Capacitor	704 mV	11 mV	1 kΩ	1 kHz	10.4 μF	4%
100μH Inductor	9.55 mV	104 mV	1 kΩ	100 kHz	146 μH	46%

Conclusion

In this lab, we built and tested circuits utilizing op-amps to measure capacitance and inductance. Driving the input with a sine wave, we derived a formula to calculate capacitance and inductance using the amplitudes of the input and output waves. To test this derivation, we measured two capacitors and one inductor. The trials performed in this lab showed that the circuit was built correctly and outputted the correct order of magnitude for all trials but was unsuccessful in getting a close value in some cases.

One issue that we encountered that could have impacted the result was a malfunctioning breadboard. During our initial build, we did not receive the expected output from the circuit. We rebuilt our circuit using a different part of the breadboard in the exact same way, and it worked from that point on.