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Homework 4: Algorithms

1. “A program with a given input prints the digit 1” is unsolvable
    a. There is a program P and an input I. Does program P with input I print 1?
    b. Proof by Contradiction:
        i. Assume there exists a program print1(P, I).
            1. This program will return true if P prints the digit 1 with input I
        ii. Assume there is a program “kryptonite” with the following algorithm:

```
void kryptonite (String p) {
   if(print1(p,p)) {
      return; // Do not print the digit 1
   }
   print(1); // Print the digit 1
   return;
}
```

        iii. Case 1: print1(p,p) is true
            1. If program p prints 1, then the if statement will return without printing 1, creating a contradiction.
        iv. Case 2: print1(p,p) is false
            1. If program p does not print 1, then the if statement will not be executed and 1 will be printed, creating another contradiction.
2. f(n, m) is Ω(g(n, m))
    a. f and g be two functions, and n and m are positive integers.
    b. f(n, m) = Ω(g(n, m)) if there are positive constants c, n_0, and m_0 such that for any $n \geq n_0$ and $m \geq m_0$, $f(n, m) \geq c*g(n, m)$.
3. $f(x, y) = x^5y^3 + x^4y^4 + x^3y^5$. Is $f(x, y) = \Omega(x^3y^3)$?
    a. $f(x, y) = x^5y^3 + x^4y^4 + x^3y^5$ and $g(x, y) = x^3y^3$
    b. Select $c = 1$, $x_0 = 1$, $y_0 = 1$. Show that for any $x \geq 1$ and $y \geq 1$, $f(x, y) \geq g(x, y)$.
        i. $x^5y^3 + x^4y^4 + x^3y^5 \geq x^3y^3$
        ii. $x^3y^3(x^2 + xy + y^2) \geq x^3y^3$
        iii. Prove that $x^2 + xy + y^2 \geq 1$
            1. Since $x \geq 1$ and $y \geq 1$, $x^2 \geq x \geq 1$, $xy \geq 1$, and $y^2 \geq y \geq 1$
            2. Add inequalities $x^2 \geq 1$, $xy \geq 1$, and $y^2 \geq 1$ to get $x^2 + xy + y^2 \geq 1$
        iv. Therefore, for $x \geq 1$ and $y \geq 1$, $f(x, y) \geq g(x, y)$.