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Homework 2: Sets

7. Find f o g and g o f, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, functions from R to R
    a. f o g: $f(g(x)) = f(x+2) = \mathbf{(x+2)^2 + 1}$
    b. g o f: $g(f(x)) = g(x^2 + 1) = \mathbf{x^2 + 3}$
8. f is a function from set A to set B, S and T are subsets of A.
    a. $f(S \cup T) = f(S) \cup f(T)$
        i. Every value in the set $S \cup T$ must be contained within S, T, or both.
        ii. If this is true, then values in the set $f(S \cup T)$ must then be contained with f(S), f(T), or both.
        iii. Rewrite this in logic: values in $f(S \cup T)$ are in f(S) or f(T)
        iv. $f(S \cup T) = f(S) \cup f(T)$
    b. $f(S \cap T) \subseteq f(S) \cap f(T)$
        i. Every value in the set $S \cap T$ must be contained within both S and T.
        ii. If this is true, then values in the set $f(S \cap T)$ must then be contained in both f(S) and f(T)
        iii. Rewrite this in logic: values in $f(S \cap T)$ are in f(S) and f(T)
        iv. $f(S \cap T) = f(S) \cap f(T)$
        v. If $f(S \cap T)$ is equal to $f(S) \cap f(T)$, then it is a subset of $f(S) \cap f(T)$.
        vi. $f(S \cap T) \subseteq f(S) \cap f(T)$
9. If $f: A \rightarrow B$ and $|A| = |B|$ (equal cardinalities), show f is one-to-one if and only if it is onto.
    a. Assume f is not one-to-one but cardinalities are equal.
    b. This can only occur if some B is mapped to by 2+ elements in A. When this occurs, there will be an unused value in the target, and the cardinality of the range will not match that of the target.
    c. Since the range must be equal to the target to be onto, this cannot occur.
    d. Therefore, f must be one-to-one in order to be onto.