

UNIVERSITY OF CALIFORNIA, SANTA BARBARA

Department of Electrical and Computer Engineering

ECE 139

Probability and Statistics

Spring 2019

Homework Assignment #6

(Due on Wednesday 5/22/2019 by 5 pm *in the Homework Box*)

Problem # 1. The following random variables X, Y, Z, W are known to be Gaussian.

- a) X has standard deviation $\sigma_X = 10$ and $P[X \leq 10] = 0.933$. Find its mean μ_X .
- b) Y has standard deviation $\sigma_Y = 10$ and $P[Y \leq 0] = 0.067$. Find its mean μ_Y .
- c) It is known that $P[Z \leq 10] = 0.977$. Express its mean μ_Z in terms of its standard deviation σ_Z .
- d) It is known that $P[W > 12] = 1/2$, but its standard deviation is unknown. Can you find μ_W ?

Problem # 2. The peak temperature in summer in Antarctica is (or used to be until recently...) a Gaussian random variable T with mean $\mu = -75$ degrees Fahrenheit. It has been further established that $P[T < -80] = 0.1587$. Find the variance of T .

Problem # 3. Let X be a random variable with CDF:

$$F_X(x) = \begin{cases} 0 & x < -1 \\ x/4 + 1/2 & -1 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

Sketch the CDF and then find the following:

- a) $P[X < 1]$ and $P[X \leq 1]$.
- b) The PDF $f_X(x)$.
- c) The mean $E[X]$.
- d) The variance σ_X^2 .

Problem # 4. An attempt to establish a video call via some social media app may fail with probability 0.1. If connection is established and if no connection failure occurs thereafter, then the duration of a typical video call in minutes is an exponential random variable X with $E[X] = 3$. However, due to an unfortunate bug in the app all calls are disconnected after 6 minutes. Let random variable Y denote the overall call duration (i.e., $Y = 0$ in case of failure to connect, $Y = 6$ when a call gets disconnected due to the bug, and $Y = X$ otherwise.) Find:

- a) $F_Y(y)$
- b) $f_Y(y)$
- c) $E[Y]$
- d) σ_Y^2

Problem # 5. Random variable X is uniform in $(-10, 10)$. Find:

- a) $P[X < 3]$
- b) $E[X]$
- c) $E[X^5]$
- d) $E[e^X]$

Problem # 6. Uniform random variable Y was shown to have mean $E[Y] = 100$, and further that $P[Y > 125] = 1/4$. What is $P[Y < 80]$?

Problem # 7. X is uniform in the interval $(-c, c)$. Find $P[|X| \leq \sigma_X^2]$.

Problem # 8. Let $Y = X^2$ where X is the random variable of Problem 7. Find the mean $E[Y]$ and variance σ_Y^2 .

Problem # 9. Let X be a random variable with PDF:

$$f_X(x) = \begin{cases} x/2 & 0 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $E[X]$, σ_X^2 , and $E[X^3]$.

Problem # 10. Let X be a random variable with CDF:

$$F_X(x) = \begin{cases} 0 & x < -5 \\ (x+5)^2/144 & -5 \leq x < 7 \\ 1 & 7 \leq x \end{cases}$$

Find $E[X]$, σ_X^2 , and $E[X^3]$.