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Homework 1: Logic

10. Find a counterexample to these universally quantified statements
    a. $\forall x\ \exists y\ (x = 1/y)$
        i. x = 2 is a counterexample
        ii. In order for this statement to be true, y must equal ½, but ½ is not in the domain (integers).
    b. $\forall x\ \exists y\ (y^2 - x < 100)$
        i. x = -101 is a counterexample
        ii. If x = -101, this statement can never be true for any value of y. y^2 will always be positive, and as a result, $y^2 + 101$ will always be greater than 100
    c. $\forall x\ \forall y\ (x^2 \neq y^3)$
        i. x = 1 and y = 1 is a counterexample
        ii. $\forall x\ \forall y$ states that every value of x and every value of y, including instances where x = y, must satisfy the condition in order for the statement to be true.
        iii. When x = 1 and y = 1, $x^2 = y^3$, this making this statement false.
11. $\exists x\ \forall y\ (x \leq y^2)$
    a. Domain consists of all positive integers
        i. True because of x = 1
        ii. If x = 1, all values of y in the domain will yield a number greater than or equal to 1 when put into y^2
    b. Domain consists of all integers
        i. True because of x = 0
        ii. If x = 0, all values of y in the domain will yield a number greater than or equal to 0 when put into y^2
    c. Domain consists of all nonzero real numbers
        i. True because of x = -1
        ii. y^2 will always yield a number greater than zero (zero is not in the domain)
12. If $\forall x\ (P(x) \vee Q(x))$ and $\forall x\ ((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is true

a.	$\forall x\ (P(x) \vee Q(x))$	Hypothesis
b.	c is an arbitrary element in the domain	Hypothesis
c.	$P(c) \vee Q(c)$	Universal Instantiation (a, b)
d.	$\forall x\ ((\neg P(x) \wedge Q(x)) \rightarrow R(x))$	Hypothesis
e.	$(\neg P(c) \wedge Q(c)) \rightarrow R(c)$	Universal Instantiation (b, d)
f.	$\neg(\neg P(c) \wedge Q(c)) \vee R(c)$	Conditional Identity (e)
g.	$P(c) \vee \neg Q(c) \vee R(c)$	De Morgan's Law (f)
h.	$P(c) \vee P(c) \vee R(c)$	Resolution (c, g)
i.	$P(c) \vee R(c)$	Idempotent Law (h)
j.	$R(c) \vee P(c)$	Commutative Law (i)
k.	$\neg R(c) \rightarrow P(c)$	Conditional Identity (j)
l.	$\forall x(\neg R(x) \rightarrow P(x))$	Universal Generalization (k)

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13. "Logic is difficult or not many students like logic." $L \vee \neg S$
"If mathematics is easy, then logic is not difficult." $M \rightarrow \neg L$

Define the following propositional variables:
L indicates that logic is difficult
S indicates that many students like logic
M indicates that mathematics is easy

a. That mathematics is not easy, if many students like logic.
   i. Convert to variables: $S \rightarrow \neg M$. This is our desired conclusion.
   ii. $L \vee \neg S$ Hypothesis
   iii. $M \rightarrow \neg L$ Hypothesis
   iv. $\neg L \rightarrow \neg S$ Conditional Identity
   v. $M \rightarrow \neg S$ Hypothetical Syllogism
   vi. $\neg M \vee \neg S$ Conditional Identity
   vii. $\neg S \vee \neg M$ Commutative Law
   viii. $S \rightarrow \neg M$ Conditional Identity
   ix. This is a valid conclusion
b. That mathematics is not easy or logic is difficult.
   i. Convert to variables: $\neg M \vee L$. This is our desired conclusion.
   ii. $L \vee \neg S$ Hypothesis
   iii. $M \rightarrow \neg L$ Hypothesis
   iv. $\neg M \vee \neg L$ Conditional Identity
   v. This is not a valid conclusion, since $M \rightarrow \neg L$ implies $\neg M \vee \neg L$, which differs from the desired conclusion of $\neg M \vee L$.
c. That if not many students like logic, then either mathematics is not easy or logic is not difficult.
   i. Convert to variables: $\neg S \rightarrow (\neg M \vee \neg L)$. This is our desired conclusion.
   ii. $L \vee \neg S$ Hypothesis
   iii. $M \rightarrow \neg L$ Hypothesis
   iv. $\neg L \rightarrow \neg S$ Conditional Identity
   v. $M \rightarrow \neg S$ Hypothetical Syllogism
   vi. $\neg M \vee \neg S$ Conditional Identity
   vii. $\neg S \vee (L \wedge \neg M)$ Distributive Law
   viii. This is not a valid conclusion (?)

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14. p_1, p_2, p_3, and p_4 equivalent if $p_1 \leftrightarrow p_4$, $p_2 \leftrightarrow p_3$, and $p_1 \leftrightarrow p_3$.
    a. p_1 and p_4 must have the same truth value in order to be true.
    b. Similarly, p_2 and p_3 must have the same truth value in order to be true.
    c. If p_1 and p_3 have the same truth value, then you can guarantee that all propositions are equivalent, since p_1's value matches p_2's value, and p_3's value matches p_4's value.
15. Every positive integer can be written as the sum of the squares of 3 integers
    a. 19 is a counterexample
    b. $0^2 = 0$, $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$
    c. In order for 19 to be true, it must be written as a sum consisting of only three numbers in the set 0, 1, 4, 9, 16.
    d. This is not possible with only three integers - the closest you can get is 1, 1, and 16, which only yields 18.