Andrew Lu
6088157

Homework 3: Induction & Recursion

1. 1/(1*2) + 1/(2*3) + … + 1/(n*(n+1))
    a. Find a formula for small values of n
        i. If n = 1, value is 1/2
        ii. If n = 2, value is 1/2 + 1/3 = 4/6 = 2/3
        iii. If n = 3, value is 2/3 + 1/12 = 9/12 = 3/4
        iv. If n = 4, value is 3/4 + 1/20 = 16/20 = 4/5
        v. If n = 5, value is 4/5 + 1/30 = 25/30 = 5/6
        vi. A formula for the expression is n/(n+1).
    b. Prove that 1/(1*2) + 1/(2*3) + … + 1/(n*(n+1)) = n/(n+1)
        i. f(1) = 1/(1*(1+1)) = (1)/(2)
        ii. Assume f(n) = n/(n+1)
        iii. Show f(n+1) = (n+1)/(n+2)
            1. f(n+1) = /(1*2) + 1/(2*3) + … + 1/(n*(n+1)) + 1/((n+1)*(n+2))
            2. = f(n) + 1/((n+1)*(n+2))
            3. = n/(n+1) + 1/((n+1)*(n+2))
            4. = (n*(n+2)+1)/((n+1)*(n+2)
            5. = (n^2+2n+1)/((n+1)*(n+2))
            6. = $(n+1)^2$/((n+1)*(n+2))
            7. = (n+1)/(n+2)
2. P(n): n! < n^n, where n is an integer > 1
    a. P(2): 2! < 2^2
    b. 2*1 < 2*2 → 2 < 4, true
    c. The inductive hypothesis assumes P(n) is true
    d. In the inductive step, we must prove that if P(n) is true, then P(n+1) is true
    e. P(n+1): (n+1)! < $(n+1)^{(n+1)}$
        i. (n+1)(n!) < $(n+1)^n$ * (n+1)
        ii. n! < $(n+1)^n$
        iii. If n is an integer > 1, n^n < $(n+1)^n$, so statement is true
    f. This inequality holds true for integers > 1 because P(2), the case where n = 2, and our base case, is true, and P(n+1) is true whenever P(n).

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3. n people in a group, each know a scandal that no one else knows. In a conversation, the two people in conversation share all scandals they know about. G(n) is the minimum calls required for all n people to learn about scandals. Prove $G(n) \leq 2n - 4$ for $n \geq 4$
    a. (Not part of proof) Verify the pattern by hand
        i. If 4 people, 4 calls are required. G(4) = 4.
        ii. If 5 people, 6 calls are required. G(5) = 6.
        iii. If 6 people, 8 calls are required. G(6) = 8.
        iv. All are equal to n + (n-4) = 2n - 4
    b. Base Case: n = 4
        i. Four people (A, B, C, D), four scandals (1, 2, 3, 4).
            1. A knows 1, B knows 2, C knows 3, D knows 4
        ii. A calls B. A and B both know 1 and 2.
        iii. C calls D. C and D both know 3 and 4.
        iv. A calls D. A and D know all four.
        v. B calls C. B and C know all four.
    c. Assume that $G(n) \leq 2n - 4$
    d. Prove that $G(n+1) \leq 2(n+1) - 4$
        i. Let x be person n+1, and (A, B, C, …) be all people up to n.
            1. A knows 1, B knows 2, …, x knows n
        ii. x calls A. A and x both know scandal 1 and n
        iii. A performs routine in case G(n), spreading the scandal to every person (except x) and learning every scandal in 2n-4 steps
        iv. x calls A again. Since A knows everything, x knows everything.
        v. Total of 2n - 4 + 2 = 2n - 2 steps
            1. Equal to 2(n+1) - 4 = 2n - 2 steps