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Homework 5: Relations

1. R and S are reflexive relations on a set A
   a. $R \cup S$ is reflexive - true
      i. Let x be an element in the set A ($x \in A$).
      ii. If R is reflexive, then all $(x,x) \in R$ will also be contained in $R \cup S$. The same applies for S.
      iii. Therefore, $R \cup S$ is reflexive.
   b. $R \cap S$ is reflexive - true
      i. Let x be an element in the set A ($x \in A$).
      ii. If R and S are reflexive, then $(x,x) \in R$ and $(x,x) \in S$ for every element x in the set A. As a result, (x,x) will be contained in $R \cap S$.
      iii. Therefore, $R \cap S$ is reflexive.
   c. R - S is reflexive - false
      i. Counterexample:
      ii. Let the sets R and S be equal (R = S).
      iii. $R - S = Ø$, which is not reflexive, since there are no elements.
      iv. Therefore, R - S is not reflexive.
   d. R o S is reflexive - true
      i. Let x be an element in the set A ($x \in A$).
      ii. $(a, c) \in R \circ S$ if there exists a $b \in A$ such that $(a, b) \in S$ and $(b, c) \in R$
      iii. If R and S are reflexive, then a = b = c = x satisfy $(x, x) \in S$ and $(x, x) \in R$.
      iv. Therefore, $(x, x) \in R \circ S$, and R o S is reflexive.
2. The set R on a set A is reflexive if and only if R^{-1} is reflexive.
   a. Let x be an element in the set A ($x \in A$).
   b. If R is reflexive, then $(x,x) \in R$ for every element x in A. Taking the inverse of $(x,x) \in R$ yields $(x,x) \in R^{-1}$. The elements of R match R^{-1}, so R^{-1} is reflexive.
   c. If R^{-1} is reflexive, then $(x,x) \in R^{-1}$ for every element x in A. Taking the inverse of $(x,x) \in R^{-1}$ yields $(x,x) \in R$. The elements of R^{-1} match R, so R is reflexive.
   d. Therefore, the set R on a set A is reflexive if and only if R^{-1} is reflexive.
3. Use the directed graph for R to obtain the directed graph for $\bar{R} = \{ (a, b) \in A \times A \mid (a, b) \notin R \}$
   a. You can obtain the directed graph of $\bar{R}$ by adding all missing edges and removing all original edges from the directed graph of R.

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4. Give a poset that has:
    a. a minimal element but no maximal element
        i. All positive integers
        ii. (Z^+, ≤)
    b. a maximal element but no minimal element
        i. All negative integers
        ii. (Z^-, ≤)
    c. neither a minimal element nor a maximal element
        i. All integers
        ii. (Z, ≤)
5. Must a finite nonempty poset have a maximal element?
    a. True.
    b. Let R be a finite nonempty poset and let x be an element in set R. ($x \in R$)
    c. If x is the only element in set R, then x must be the maximum (and minimum) element.
    d. If there is more than one element, suppose there is an element y in set R, and assume that x is the maximal element.
    e. If $x > y$, then x remains the maximal element. If $y > x$, then we set the value for y to be the new maximal element. We can repeat this process for every element in the poset R. Since R is finite, there will eventually be a distinct value for x, which is your maximal element.
    f. Therefore, there will always be a maximal element in a finite nonempty poset.