Executive Summary

Since the early twentieth century, the car has changed the way people all around the world live and work. The automobile managed to do what most other inventions have failed to do: to cross class lines. It is no wonder, then, that the car has become such an undeniable staple of international and American culture.

However, several negative consequences—particularly increased traffic, pollution, and oil dependence—have arisen due to the increasingly heavy usage of cars over the course of the past century. With concerns about climate change intensifying, drivers are increasingly looking toward alternative transportation, including car sharing, which carries the benefits of owning a car without paying for many of the expenses. Car sharing companies and several major automakers are now trying to find ways to expand the market beyond cities and college towns by implementing new automobile technologies.

The amount of time using the car and miles driven per day are the two main factors drivers consider when deciding whether to use a car sharing service. By integrating a normalized distribution, the percentage of drivers categorized as low, medium, and high for each of the two factors was found. The percentage of drivers that were low distance and low, medium, and high time was 21.93, 59.11, and 3.42 percent, respectively. Similarly, the percentages for medium distance drivers was 3.21, 8.66, and 0.50 percent, respectively. Those of high distance drivers were 0.83, 2.22, and 0.13 percent, respectively. These numbers are consistent with drivers usually travelling on local roads. The time model was partially sensitive; the distance model was moderately sensitive.

To predict the participation for one-way floating, one-way station, fractional ownership, and round trip car sharing, we developed a model based on the perceived convenience to the client, which compared cost and time differences between the four car sharing service types and a weighted average current methods of commuting. Based on these results, a one-way floating model saves the most money and time for clients. Applying this model to four cities found that Knoxville, TN, will garner the most participation with 3,600 clients, followed by Riverside, CA with 3,054; Richmond, VA with 1,530; and Poughkeepsie, NY with 931.

Just like how car sharing revolutionized travel, the advent of self-driving cars and vehicles that run on alternative or renewable energy will change the industry. The model for participation was adjusted to account for these emerging automobile technologies, which resulted in new rankings of Knoxville, TN with 13,650 clients; Riverside, CA with 13,650; Richmond, VA with 6,267; and Poughkeepsie, NY with 3,615. Time and resource constraints limited the models in each part. In the future, additional factors such as driving age, gender, type of car, and geographic location would further refine our models.

Contents

1 Introduction

1.1 Background

Since the introduction of its mass production in the early 1900s, the car has become an integral part of American lives. It is estimated that approximately 90.7 percent of all American households own at least 1 car [1]. However, consequences such as heavy traffic, increased pollution, and increased oil dependence have arisen. In response to the controversy surrounding heavy automotive use, consumers have turned to alternative options such as car sharing, which allows them to experience the benefits of a private vehicle without the accompanying costs and responsibilities. Fearful of falling profits and decreasing customer bases, major automobile manufacturers such as the General Motors Company and the Ford Motor Company have recently tried to join the growing car sharing phenomenon [2, 3].

1.2 Restatement of the Problem

Our model responds to the need for the expansion of car sharing services into markets beyond those of cities and of college towns and for the implementation of new technologies in these services by addressing the following:

1. What percentage of American drivers fall into each of nine different categories as defined by labels of low, medium, or high daily car usage and low, medium, or high daily mileage?
2. Out of four different car sharing services—one-way floating, one-way station, fractional ownership, and round trip—which one would garner the most participation in the cities of Poughkeepsie, NY; Richmond, VA; Riverside, CA; and Knoxville, TN?
3. Which of the aforementioned four cities would have the highest overall participation with the implementation of all four car sharing services if emerging automotive technologies such as self-driving vehicles and vehicles that operate entirely on alternative fuel or renewable energy are also incorporated?

1.3 Global Assumptions

- *Current domestic and international status.* The economy and politics of the United States and other major world countries will remain stable, as circumstances that would change these factors are too volatile to be included in our model.
- *Automobile usage.* Historical data and trends are accurate. Circumstances that would potentially and significantly increase or decrease automobile usage based upon current trends have been ignored.
- *Legality of car sharing services.* Historical legislation will remain on the books without additional restrictions. Circumstances that would potentially alter public and government perception toward car sharing services have been ignored.

2 Analysis

2.1 Part I: Who's Driving?

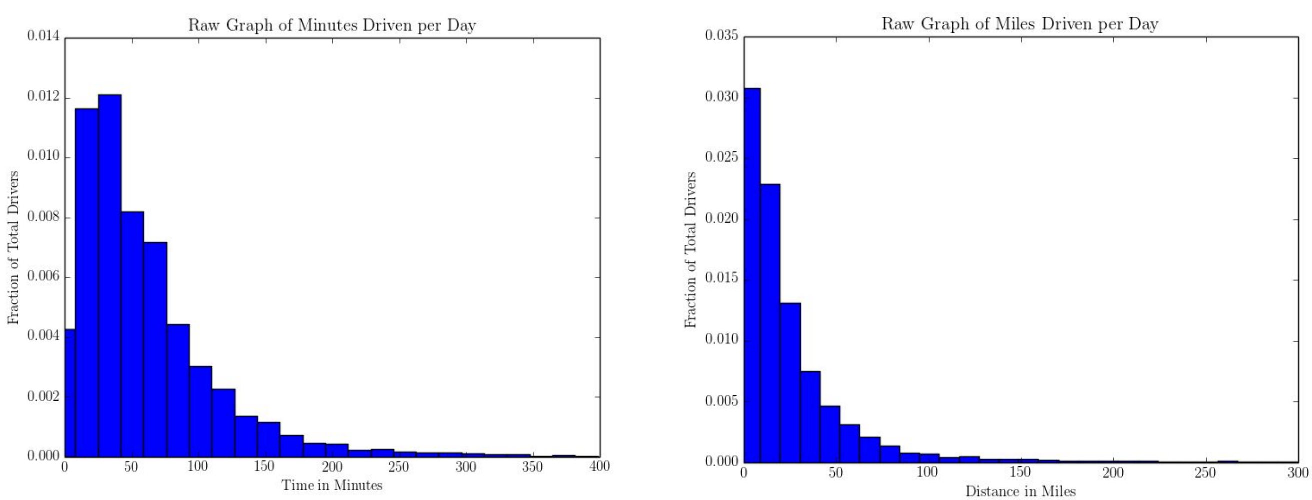
Car sharing companies have identified daily driving time and daily distance travelled as two important factors when making car sharing decisions. A mathematical model was developed to categorize U.S. drivers as low, medium, or high in each of these factors.

2.1.1 Assumptions

- *Age of driver and daily car usage and daily mileage.* The age of the driver does not affect the daily duration of driving trips in minutes or mileage. A chi-square test of independence between age range and daily duration of driving trips in minutes produces a p-value of 0.0528, which is greater than the alpha of 0.05. Similarly, a chi-square test of independence between age range and daily duration of driving trips in minutes produces a p-value of 0.1075. This means that if the data is entirely due to chance, there is a 5.28 percent and 10.75 percent chance respectively of finding a discrepancy between the observed and expected distributions that is at least this extreme.
- *Survey and population representation.* 10,000 data points from a survey are representative of the whole survey and of the population. Due to computational limitations, only 10,000 data points were randomly selected from the National Household Travel Survey dataset. Since the survey included over 1 million unordered data points, this sample can be considered representative of the whole.

2.1.2 Model Development

We began by obtaining representative samples of individual driving data from the 2009 National Household Travel Survey [4]. These data points showed the time and distance of each trip for each person in a household. The time and distance data were then summed for each person per household to get the total time and total distance travelled.



Figures 1a & 1b: Histogram of raw data of minutes and miles drive per day.

By examining Figure 1, we determined the trend to be lognormally distributed with a severe right-skew. To normalize the data to make it more bell-shaped, we took the natural logarithm of each data point and normalized the values so instead of frequency, the histogram displayed the fraction of total drivers. This is so the total area under the curve would equal 1, or the entire population of drivers.

A normal distribution with mean μ and variance σ^2 is defined by

$$N(\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\cdot\sigma^2}}.$$

The standard deviation and mean of each distribution were found, yielding

$$N(\mu_{time}, \sigma_{time}) = \frac{1}{0.9605\sqrt{2\pi}} \cdot e^{\frac{-(x-3.7246)^2}{2\cdot 0.9605^2}}, \text{ and}$$
$$N(\mu_{distance}, \sigma_{distance}) = \frac{1}{1.3675\sqrt{2\pi}} \cdot e^{\frac{-(x-2.6038)^2}{2\cdot 1.3675^2}}$$

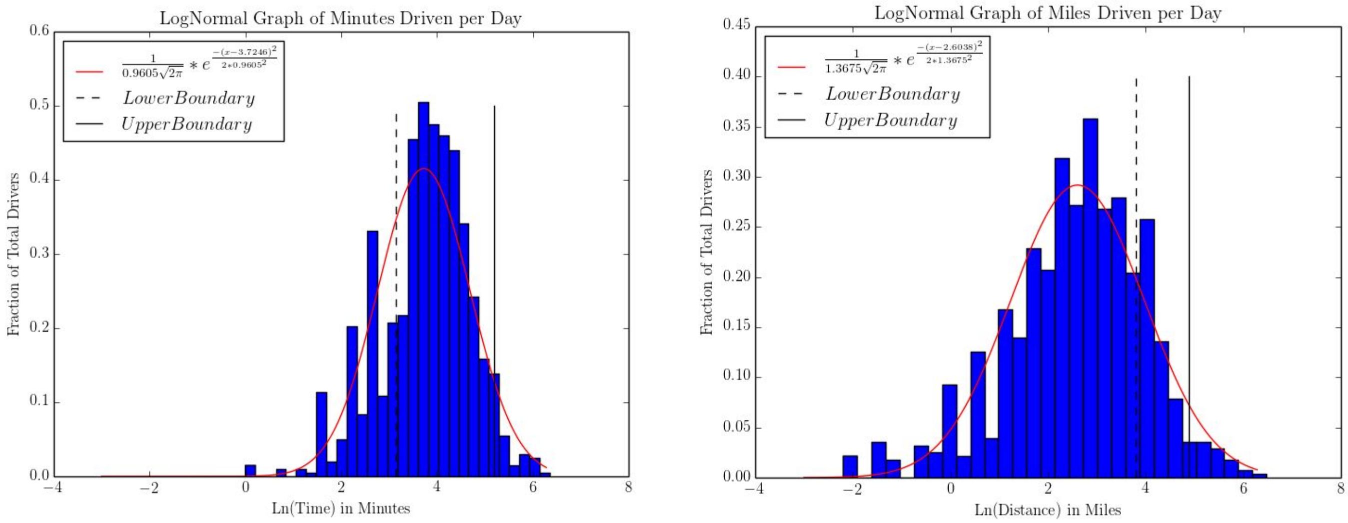
as the models for time and distance per day.

To determine what constitutes “low,” “medium,” or “high” mileage, the typical miles travelled per year were calculated. Auto insurance companies often have a low-mileage cutoff between 7,500 and 15,000 miles per year that qualifies drivers for additional discounts [5]. Because each range is different, the mean value of 11,250 miles per year was used as the low-mileage cutoff. The equation to determine miles per day is

$$mi_{year} = 365 \cdot (1 - p_0) \cdot mi_{day},$$

where mi_{year} is the miles travelled per year, p_0 is the probability of not driving on a given day, and mi_{day} is average number of miles driven in a day. p_0 is 0.316, as 31.6 percent of drivers reported they did not drive on a given day [6]. Solving for mi_{day} yields 45.0613 mi/day. Since we normalized the data by taking the natural log of both sides, $\ln mi_{day}$ yields 3.8080 mi/day. The higher bound would be a yearly travel distance of 33,000 mi/year [7]. Using the same equation as before, the daily mileage is 132.1798 miles and $\ln mi_{day}$ is 4.8842.

To determine boundaries for “low,” “medium,” and “high” amounts of time, we analyzed average commute times for working citizens. The average motorist makes two trips a day for a combined 46 minutes, so a “low” driver would only make one trip for a total of 23 minutes [8], or 3.1355 minutes adjusted. Similarly, “mega-commuters” who have well above average commute times drive 180 minutes or more round trip [9]. This threshold is 5.1930 minutes per day adjusted. These are displayed in Figures 2a and 2b.



Figures 2a & 2b: Histogram of lognormal graphs of minutes and miles drive per day. The red curve is the distribution models we found before, while the black vertical lines are the boundaries The graph of distance shows more drivers in the lower third, which is consistent with the raw data, which showed a more significant right tail than time.

2.1.3 Results

Using these bounds, we integrated each function to determine the percentage of drivers who were in each category. Integration was used instead of summation because the values for distance travelled and time spent driving are continuous, since they can take on any value within the range. For example, the percentage of "low" mileage drivers is

$$\int_{-\infty}^{3.8080} N(\mu_{distance}, \sigma_{distance}) = 0.8107,$$

while the percentage of people who used their car for "low" times is

$$\int_{-\infty}^{3.1355} N(\mu_{time}, \sigma_{time}) = 0.2698.$$

Taking the product yields 0.2193, or the value in the top left box. The rest of the table was filled out in a similar fashion.

Table 1: Percentages of people who are low, medium, or high distance and time drivers

		Time Driving		
		Low	Medium	High
Distance	Low	21.93%	59.11%	3.42%
	Medium	3.21%	8.66%	0.50%
	High	0.83%	2.22%	0.13%

The distribution of time and distances in Table 1 found the majority of Americans travel low distances for medium amounts of time. The data is also concentrated with low distances and low to medium travel times, which makes sense, as most travelling is done on local roads to nearby establishments. On the other extreme, high travel time and high distances are usually reserved for long distance travelling on highways, which occur less often.

2.1.4 Sensitivity Analysis

The low/medium and medium/high boundaries for time and distance were changed by +/- 5 percent to show the resulting change in percentage medium drivers. For example, in Table 2, reducing the low/medium boundary by -5 percent for time results in a 7.66 percent increase in medium drivers, while increasing the boundary by 5 percent results in a 9.25 percent decrease in medium drivers.

Table 2: Sensitivity analysis for driving time model

	Time				
Distance		Low		High	
		-5%	+5%	-5%	+5%
	% change in medium drivers	+7.66%	-9.25%	-6.14%	+4.07%

Table 3: Sensitivity analysis for driving distance model

	Distance				
Time		Low		High	
		-5%	+5%	-5%	+5%
	% change in medium drivers	+28.23%	-24.98%	-14.51%	+10.78%

Table 2 shows low sensitivity to changes in distance compared to changes in time, but Table 3 shows moderately large changes in time when distance is changed. This could be due to a variety of factors, including the severe right skew of the raw data, which means many of the data points are concentrated on the lower end, meaning a small change in distance could mean a large change in time.

2.1.5 Strengths and Weaknesses

According to our model, the average time driving per day is 41.45 mins for 13.51 miles. One strength is the accuracy in travel time, as the time traveled is consistent with the real-life average of 46 minutes. However, a weakness is the shorter than expected distance travelled, as the average travel distance is 29.2 miles per day [8]. Many other factors that affected the distance or time travelled were not taken into consideration, including the day of the week, type of car, or geographic location. Given more time, our model would have adjusted for these factors to give a more precise estimate for the percentage of drivers in each category.

2.2 Part II: Zippity Do or Don't?

In order to predict the participation for four different systems of car sharing—one-way floating, one-way station, fractional ownership, and round trip—we developed a model dependent on the perceived convenience to the client. This model was then used to test the viability of implementing the various car sharing systems in four different cities—Poughkeepsie, NY; Richmond, VA; Riverside, CA; and Knoxville, TN.

The four systems in question are as follows:

1. *One-way car sharing floating model.* The client rents a designated shared car on demand and must return the vehicle to a defined area near their destination. Thus, this model necessitates the use of a "jockey" to bring the shared car back to its original location to ensure an even distribution of vehicles among the various defined areas.
2. *One-way car sharing station model.* The client rents a car from a designated car sharing station and must return the vehicle to another station.
3. *Fractional ownership.* Clients jointly purchase and operate a single private vehicle and share the operating and maintenance costs. This car can be parked wherever the clients desire.
4. *Round trip car sharing.* The client rents a car from a designated car sharing station and must return the vehicle to the same station.

2.2.1 Assumptions

For the purposes of modeling prompt, the following assumptions were made:

- *Distribution of shared cars and stations.* Shared cars and car sharing stations will be uniformly distributed across a city. A lack of real-world data prevented the locations and distribution of shared cars and car sharing stations across various cities from being successfully modeled, thus a uniform distribution was utilized.
- *Use of public transportation.* Clients will not use public transportation to arrive at a car sharing station. We cannot predict whether there will be public transportation en route to a car sharing station.
- *Fleet size.* Various car sharing companies will provide the same fleet size for a one-way car sharing floating model as for a one-way car sharing station model or for a round trip car sharing model. The optimal number of designated shared cars for a population has been determined to be approximately 1 car for every 0.33 miles [10]. Thus, for a single city, the number of cars for each type of car sharing service, except for fractional ownership, will be the same.
- *Car insurance.* The price of insurance for a shared car is 3.5 times the cost of regular insurance for a private vehicle. U.S. data shows that insurance for a shared car is typically costs between 3 to 4 times as much as private car insurance, so we used an average value of 3.5 as the insurance cost multiplier for our model [11].
- *Depreciation of cars.* To simplify the model, depreciation was disregarded.

- *Car size.* All fleet cars have 5 seats. Companies like Zipcar offers sedans or 7-seater minivans and SUVs, while smaller companies like car2go only offer 2- and 4-seater subcompact vehicles [12, 13].
- *Use of transportation.* Cars and public transportation are only used for daily commutes. Due to a lack of consistent, real-world data, we disregarded the fact that transportation is used for non-commuter activities.
- *Area of city.* The area of a city can be assumed to be circular. Due to a lack of real-world data on the exact dimensions of the city, we determined that a circular configuration was the most efficient way to perform the double integral.
- *Logistic model for probability of participation.* Probability of participation approaches zero or one, because if more neighbors are participating, a person is more likely to participate as well [14].

2.2.2 Model Development

We used the assumptions above to arrive at a model for the participation in a car sharing program in a specific city. Participation in a car sharing program can be modeled as adding up the population density of drivers between the ages of 25 and 44, inclusive, over the area of the city in question, as car sharing services are mostly used by this particular age group [19]. This provides the function in the form

$$P = \iint_R p(client) \cdot \frac{D}{A} dA,$$

where *P* is the total participation, *p(client)* is the probability of a person being a client of the car sharing program, *D* is the driving population between 25 and 44, and *A* is the area of the city. Multiplying the probability that a citizen of the city will be a client of the car sharing program with the population density of drivers will give the density of clients, and performing a double integral over this value will allow us to arrive at the total number of clients in the area.

The probability that a citizen of the city is a client of the program can be modeled logistically [14]. Under this assumption, the probability function can be modeled in the form

$$\frac{\mathrm{d}p}{\mathrm{d}t} = convenience \cdot p \cdot (1 - p),$$

where *p* is the probability of joining the car sharing program. Solving for *p* yields the following equation

$$p = \frac{1}{1 + Ae^{-convenience}},$$

where *A* is the coefficient relating the convenience to the probability function.

Convenience is dependent on a multitude of factors. We grouped these factors into four categories: time difference, cost difference, vehicles per household, and household per area. Time difference is defined as the time savings by using a car sharing program while cost difference is defined as the monetary savings by using a car sharing program. Negative values for both signifies that instead of saving, one is spending more by using the car sharing program. These four components were modeled together as

$$Convenience = \alpha \cdot \Delta time + \beta \cdot \Delta cost - \gamma \cdot \frac{vehicles}{household} + \delta \cdot \frac{households}{area},$$

where α, β, γ, and δ are the coefficients relating time difference, cost difference, vehicles per household, and households per area to the convenience coefficient. Time difference and cost difference have a positive correlation with convenience because as one saves more money and time, the car sharing program becomes more convenient; if one already owns a vehicle in their household, one will find it less convenient to use a shared car. The final component, households per area, only applies to fractional ownership; the more households nearby, the more likely one is to purchase a joint car with neighbors. However, as the other options do not involve a client sharing a car with a neighboring individual, δ is set to 0.

2.2.2.1 Time Difference

In order to determine the yearly time difference of utilizing a car sharing system, we factored in those that are currently driving a privately owned car and those that are currently using public transportation. For those that are currently driving a privately owned car, the time difference is the extra time required to walk to the nearest car sharing station or shared car. For those that are currently using public transportation, the time difference is the difference in time spent driving a car and time spent taking public transit. We calculated the overall time difference by using a weighted average of the time differences for privately owned cars and public transportation, given by

$$\Delta time = \frac{ridership_{poc} \cdot \Delta Time_{poc} + ridership_{pt} \cdot \Delta Time_{pt}}{ridership_{poc} + ridership_{pt}},$$

where *poc* is the clients that use privately-owned cars and *pt* is the clients that use public transportation. $\Delta time$ is the overall weighted time difference for the average citizen in hours per day, while $\Delta Time_{poc}$ and $\Delta Time_{pt}$ are the daily time differences for an individual that uses privately-owned cars and public transportation, respectively. This equation is the same for each car sharing business option except that value of $\Delta Time_{poc}$ varies with each business option. Below, we discuss calculating this value for each option.

One-way car sharing floating model: We determined $\Delta Time_{poc}$ for the one-way car sharing model. Studies have shown that a client of a car sharing system will walk at most 0.33 miles to the nearest shared car [10]. This means that each client can be modeled as a point in a circle of radius 0.33 miles, with the center of the circle signifying the location of a shared car. The average distance of a client from the shared car can be modeled as the average distance from the center of the circle, which can be calculated by the formula

$$Distance = \frac{\iint_R dA}{A},$$

where *Distance* is the average distance from the center of the circle and *A* is the area of the circle. This simplifies to

$$Distance = \frac{\int_0^{2\pi} \int_0^r r^2 dr d\Theta}{\pi \cdot r^2},$$

where *r* is the radius of the circle, and finally

$$Distance = \frac{2}{3} r.$$

Since the average distance from the center of a circle is two-thirds the radius, the average distance of any client from the location of a shared car will be (⅔)*0.33 = 0.22 miles. Using a rearranged rate equation, $\Delta Time_{poc}$ can be calculated as

$$\Delta Time_{poc} = 0.22 \text{ miles} \cdot (\frac{1}{walkingspeed} - \frac{1}{drivingspeed}) \frac{\text{hours}}{\text{miles}}.$$

A study by Portland State University estimates the average pedestrian walking speed as 3.34 feet per second, which equates to 2.27 miles per hour [15]. Using this value, the equation simplifies to

$$\Delta Time_{poc} = \frac{0.096916 \cdot (drivingspeed - 2.27)}{drivingspeed}.$$

We next defined the time difference for public transportation, $\Delta Time_{pt}$. A national survey found people spend 24.05 more minutes, or 0.401 hours per day, on public transportation than driving a car [16]. As data on average time saved by driving over public transportation was not available by city, the national average was used for all four cities. However, $ridership_{poc}$ and $ridership_{pt}$ were obtained for each city.

One-way car sharing station model: The difference between this business option and the previous one is that instead of the cars being available on a street, they are only available at an existing station. Therefore, the value that a client will on average walk an additional 0.22 miles must be changed.

We operated under the assumption that the car sharing company will have the same fleet size for each business option. Under the floating model, one car could cover citizens that live in an area of $\pi \cdot 0.33 \text{ miles}^2$ = 0.342 square miles. This means that 5 cars can cover an area of 1.711 square miles. Using the statistic that a car-share station contains an average of 5 shared cars, under the station model, 1 station can cover 1.711 square miles [20]. If this station is centrally located, it can cover

$$\pi \cdot radius^2 = 1.711 \text{miles}^2,$$

and the radius equals 0.7379 miles. Therefore, using a sharing station requires the average client to walk (⅔)*0.737902 = 0.491935 miles to a shared car station, as two-thirds is the average distance to the center of a circle. Using this value for the average distance a client will walk, the equation for $\Delta Time_{poc}$ simplifies to

$$\Delta Time_{poc} = \frac{0.216711 \cdot (drivingspeed - 2.27)}{drivingspeed}.$$

Fractional ownership: For fractional ownership, $\Delta Time_{poc}$ is inherently zero. This is because the clients purchase a private car and are free to park it wherever as if it was privately-owned. However, the rest of the equation still applies.

Round trip car sharing: In this business option, clients are required to return the shared car to a sharing station. This doubles the walking time of the one-way car sharing station business option, as they must both walk to and from the station. Therefore, the time $\Delta Time_{poc}$ is doubled to obtain

$$\Delta Time_{poc} = \frac{0.433423 \cdot (drivingspeed - 2.27)}{drivingspeed}.$$

$\Delta Time_{pt}$ is also doubled to obtain 0.802 hours per day.

2.2.2.2 Cost Difference

Another primary consideration of the likelihood of participation is the cost difference between private car ownership and car sharing, which can be calculated as

$$\Delta cost = cost_{poc} - cost_{pt},$$

where $cost_{poc}$ is the daily cost of private car ownership and $cost_{pt}$ is the cost of public transportation, including car sharing.

A client does not both privately own a car and participate in car sharing. If a client does not already own a car, implementing a car sharing system would not involve the additional cost of buying a car, and for our purposes, we assume that there is no significant learning curve involved in learning to drive a car. If the client already owns a car, he would have already paid for a car, but switching from non-car sharing to car sharing would entail selling the car. Since there is no depreciation in the value of the car, this difference can be ignored.

Baseline comparison: Without the car sharing system, transportation is conducted through public transportation, private vehicles, and walking. Thus the daily cost of transportation without the car sharing system can be calculated as a weighted average

$$cost_{poc} = percent_d \times cost_d + percent_{pt} \times cost_{pt} + percent_w \times cost_w,$$

where $percent_d$ is the percent of the population that drives, $cost_d$ is the daily cost of driving, $percent_{pt}$ is the percent of the population that takes public transportation, $cost_{pt}$ is the daily cost of taking public transportation, $percent_w$ is the percent of the population that walks, and $cost_w$ is the daily cost of walking. Each percentage can be calculated from

U.S. census data [16]. The cost of driving is a function of gas costs, maintenance costs, and insurance costs, which can be modeled as

$$cost_d = cost_{gas} + cost_{maintenance} + cost_{insurance},$$

where the cost of gas is (cost of gas per mile) * (miles traveled daily) / (average mileage in mi/gal). The cost of gas varies by the city, and the average mileage of a five-person car is 20 miles per gallon [17].

Since cars are used for round-trip daily commutes, miles traveled daily can be found through 2 * (daily one-way commute in miles), where the daily commute is city-specific.

Maintenance costs are $5.06 per mile, on average, [18] so $cost_{maintenance}$ is ($5.06 per mile) * (miles traveled daily). The cost of insurance $cost_{insurance}$ varies by city, and $cost_{walking}$ is 0 as walking is free.

One-way car sharing floating model: For car sharing models, the cost is

$$cost_{floating} = cost_{gas} + cost_{rent} + cost_{insurance}.$$

In the floating model, cars are available on the street, within walking distance, and they are parked in the same places as privately owned vehicles would be. Thus, gas costs would be the same as gas costs for privately owned vehicles. With this business model, individual clients are not responsible for car maintenance, but they are responsible for paying insurance on average 3.5 times that of privately owned cars [11].

Rent for the floating model is based on time. Zipcar's ONE>WAY pilot for the floating model charges $6 for every 30 minutes, on top of a $60 yearly membership [21]. There is often a need for a "jockey" to reposition cars, but this is included in the cost of the plan. Thus, the cost of rent $cost_{rent}$ for the one-way sharing floating model is $60 + (30 minutes) * (1 hour / 60 minutes) * (miles driven daily) / (average driving speed in miles per hour).

One-way car sharing station model: Like with the floating model, the cost is

$$cost_{station} = cost_{gas} + cost_{rent} + cost_{insurance}.$$

Although exactly the same driving distances as the floating model are unreasonable, we assume that they will be the same due to difficulties in modeling the actual distance under time constraints. Thus, gas costs would be the same as gas costs for privately owned vehicles and the floating model. Insurance costs are also the same as the floating model.

Rent for the station model is also based on time. If we assume the pricing system is the same as that of the floating model, cost of rent $cost_{rent}$ for the one-way car sharing station model is $60 + (30 minutes) * (1 hour / 60 minutes) * (miles driven per year) / (average driving speed in miles per hour).

Fractional ownership: In fractional ownership, two to five individuals jointly purchase a car for a length of time. Similarly,

$$cost_{fractional} = cost_{gas} + cost_{rent} + cost_{insurance}.$$

The miles traveled stays the same as individuals carpool to the same places as before, but the gas costs are divided among all purchasing parties. Assuming five parties jointly purchase a five-person car, the cost is $208 per person per month, so $cost_{gas}$ = (cost of gas for private cars) / 5, and $cost_{rent}$ is ($208 / month) * (12 month) / (365 days) = $6.84 / day. The cost of insurance $cost_{insurance}$ remains the same.

Round trip car sharing: In this business option, clients return rented cars to their starting positions. Again,

$$cost_{roundtrip} = cost_{gas} + cost_{rent} + cost_{insurance}.$$

In Zipcar plans, the costs of gas and insurance are already factored into the costs of the plans [22]. Thus, we need only to find the cost of rent.

After a flat annual membership of $60, there are three ways to calculate the cost of rent: by day, by hour, and by mile. Assuming daily usage of cars for commutes, a discussion of each pricing method follows, with rates from Zipcar.

By day: The daily cost of rent is ($60 / year) * (1 year / 365 days) + $84 / day = $84.16.

By hour: Clients drive to work in the morning and back in the afternoon. Assuming an eight-hour workday, the rent cost would be $60 + (8 hours + (miles driven per day) / (average driving speed in miles per hour)) * $10 / hour.

By mile: The rent cost would be $60 + (miles driven per year) * ($0.46 / mile).

2.2.3 Results

In order to obtain the total participation of each city, the probability function must be properly defined, which means the convenience coefficient and its corresponding factors must be determined from historical data. To determine the corresponding factors, historical data of 6 cities were used to figure out the coefficients: Phoenix, AZ; San Antonio, TX; New Orleans, LA; Trenton, New Jersey; Los Angeles, CA; and Seattle, WA [16, 23, 24, 25].

By combining the convenience coefficient and the probability logistic equations from Section 2.2.2 and rearranging the variables, we obtain the following equation:

$$e^{-Convenience} = \frac{\frac{1}{p} - 1}{A}, \text{ where}$$

$$Convenience = \alpha \cdot \Delta time + \beta \cdot \Delta cost - \gamma \cdot \frac{vehicles}{household} + \delta \cdot \frac{households}{area}.$$

Plugging in the equations with the historical data of the 6 cities, we solved for p, the probability that a citizen will become a client of the car sharing program a given city. By averaging the cities by model, we obtained that one-way car sharing floating model was the best to the client, as the $\Delta time$ was negative and low in magnitude and the $\Delta cost$ was positive and high in magnitude. This means clients do not spend a significant amount of time walking to the shared car relative to the savings they get for renting a shared car.

To determine the rankings of the cities, we must obtain $\Delta time$ and $\Delta cost$ of the 4 cities in question. We found the public transportation ridership of the four cities in question: Poughkeepsie, NY [26]; Richmond, VA [16]; Riverside, CA [16]; and Knoxville, TN [27], as well as their driving speeds [28, 29, 30]. The time differences were obtained and are compiled in the table below:

Table 4: Summative table for $\Delta time$

	Floating	Station	Fractional	Round Trip
Poughkeepsie, NY	-0.0665	-0.1721	0.0188	-0.3441
Richmond, VA	-0.0783	-0.1851	0.0080	-0.3702
Riverside, CA	-0.0793	-0.1863	0.0072	-0.3725
Knoxville, TN	-0.0876	-0.1979	0.0016	-0.3958

To read Table 4, find the city in the first row and corresponding business option in the first column. The intersection of the two gives $\Delta time$ for the business option in the city in terms of hours per day. For instance, if the company implements a one-way floating model in Poughkeepsie, New York, the average citizen will save -0.0665 hours, or spend 0.0665 more hours by switching to the car-share system. This comes out to a client spending roughly 4 minutes to walk to the shared car than to drive directly to their destination.

We found the necessary variables to calculate change in cost: the cost of gas [31], the length of a daily commute [32], and the cost of insurance [33] and organized results in Table 5.

Table 5: Summative table for $\Delta cost$

	Floating	Station	Fractional	Round trip, by day	Round trip, by hour	Round trip, by mile
Poughkeepsie, NY	\$99.12	\$99.12	\$93.41	\$30.33	\$26.37	\$103.34
Richmond, VA	\$75.44	\$75.44	\$69.55	\$2.28	-\$0.96	\$77.96
Riverside, CA	\$101.28	\$101.28	\$95.67	\$32.25	\$26.52	\$105.06
Knoxville, TN	\$80.52	\$80.52	\$74.64	\$9.36	\$6.86	\$84.39

We organized the values of the of one-way car sharing floating model for the four cities as well as the overall statistics of the cities, and plugged them into the convenience, probability, and participation models. We obtained the following ranking for the four cities in question:

1. Knoxville, TN: 3,600 clients
2. Riverside, CA: 3,054 clients
3. Richmond, VA: 1,530 clients
4. Poughkeepsie, NY: 931 clients

2.2.4 Sensitivity Analysis

To determine the weakest parts of the model, we performed a sensitivity analysis on the variables in the model for $\Delta time$. Due to space and time constraints, the sensitivity analysis of other model variables were left out, but a similar sensitivity analysis to the one below could be constructed for each model variable.

Table 6: Sensitivity analysis for $\Delta time$

	$\Delta Time_{poc}$		$\Delta Time_{pt}$		$ridership_{pt}$	
	-5%	+5%	-5%	+5%	-5%	+5%
% change in Poughkeepsie	-1.73%	+1.73%	+6.42%	-6.42%	-1.42%	+1.42%
% change in Richmond	-0.62%	+0.62%	+5.51%	-5.51%	-0.51%	+0.51%
% change in Riverside	-0.55%	+0.55%	+5.45%	-5.45%	-0.45%	+0.45%
% change in Knoxville	-0.11%	+0.11%	+5.09%	-5.09%	-0.09%	+0.09%

Table 6 shows the sensitivity analysis of $\Delta time$, the overall time difference, if the the three variables listed are changed. is the time difference for privately-owned cars, $\Delta Time_{pt}$ for public transportat $\Delta Time_{poc}$ ion, and $ridership_{pt}$ is the public transportation ridership. To read the table, find the intersection of the percent changes with the corresponding percent changes in the 4 cities in question. For instance, if $\Delta Time_{poc}$ is increased by 5 percent, the magnitude $\Delta time$ in Poughkeepsie will increase by 1.73 percent. This is expected because if the magnitude of $\Delta Time_{poc}$ increases, citizens will spend more time on car sharing options. This increases the magnitude of $\Delta time$, as people overall are spending more time on the car sharing option; because this is undesirable, the box is highlighted in red. Looking at the result, it is clear that $\Delta Time_{pt}$ is the most influential variable in the determination of $\Delta time$, meaning that in future data collection, collecting accurate data regarding the times spent on public transportation is most important.

2.2.5 Strengths and Weaknesses

A particular strength of our model was that we were able to factor in the density of individuals that may participate in such car sharing systems and integrate instead of doing a direct proportion. The model for the convenience coefficient is also very inclusive. Even so, the model was not all-inclusive. We assumed uniform population and car sharing station distributions, but this is not generally true to reality. We also only consider the cost of the business model to the end consumer, the city public; we do not take into account the cost to the city for its implementation, or the cost or benefit to the environment. Finally, transportation usage was modeled as only a daily commute, which is unrealistic and disregards all the other uses for transportation (i.e. grocery shopping).

2.3 Part III: Road Map to the Future

As technology advances and cars become automated and greener, people may be more inclined to participate in car sharing systems. This will influence a person's desire to turn to car sharing systems; as a result, car sharing systems may continue to increase in popularity. As the population ages, a different population is transitioned into the situation of requiring daily transportation. To model participation for the future, we take into account current city demographics and how they will affect participation for the future.

2.3.1 Assumptions

– *Fuel efficiency.* Over time, cars have been increasing in fuel efficiency. We assume the fuel economy of all cars used in our model of the future will be the constant projected value of 56.2 mpg [34].

2.3.2 Model Development

Going back to the equation to determine the convenience coefficient,

$$Convenience = \alpha \cdot \Delta time + \beta \cdot \Delta cost - \gamma \cdot \frac{vehicles}{household} + \delta \cdot \frac{households}{area},$$

we determined that as a result of automation and greener cars, both $\Delta time$ and $\Delta cost$ are affected. Clients will no longer spend time walking to the nearest shared car or car sharing station. Additionally, the fuel efficiency of cars will increase, as the vehicle that arrives at their doorstep will be environmentally friendly and use less fuel.

Based on projected data, by 2035, the fuel economy for new passenger cars in the U.S. will 56.2 mpg, changing the fuel efficiency in the cost model from above to 56.2 mpg from 25 mpg [34]. The time difference for a privately owned car goes to 0, as the cars are self-driving and come to you.

In order to account for the shift in demographics, we integrated over the population of 0- to 18-year-olds: those who will be of driving age in the future.

The following tables reflect the changes in $\Delta time$ and $\Delta cost$.

Table 6: Summative table for $\Delta time$ in hours for advanced car technologies

	floating	station	fractional	round trip
Poughkeepsie, NY	-0.0188	-0.0188	-0.0188	-0.0377
Richmond, VA	-0.0080	-0.0080	-0.0080	-0.0160
Riverside, CA	-0.0072	-0.0072	-0.0072	-0.0144
Knoxville, TN	-0.0016	-0.0016	-0.0016	-0.0032

Table 7: Summative table for $\Delta cost$ for advanced car technologies

	floating	station	fractional	round trip, day	round trip, hour	round trip, mile
Poughkeepsie, NY	$99.16	$99.16	$93.09	$29.92	$25.95	$102.92
Richmond, VA	$75.47	$75.47	$69.31	$1.96	-$1.27	$77.64
Riverside, CA	$101.33	$101.33	$95.34	$31.83	$26.10	$104.64
Knoxville, TN	$80.56	$80.56	$74.37	$9.03	$6.53	$84.06

2.3.3 Results

We organized the values of the of one-way car sharing floating model for the four cities as well as the overall statistics of the cities, and plugged them into the convenience, probability, and participation models. We obtained the following ranking for the four cities in question:

1. Riverside, CA: 16,029 clients
2. Knoxville, TN: 13,650 clients
3. Richmond, VA: 6,267 clients
4. Poughkeepsie, NY: 3,615 clients

2.3.4 Strengths and Weaknesses

Our model for car sharing participation that takes into account emerging automobile technologies has several strengths and weaknesses. Our model is easily adapted to reflect changes in fuel efficiency and demographics, and we obtain a new ranking of cities that accounts for changes in the future. However, we do not take into account the difference in attitudes between the different generations and its effect on participation in car sharing. We do not provide a projection of participation or population for years into the future; rather, we only provide one data point.

3 Conclusion

3.1 Further Studies

Our further studies to improve our models would depend mostly upon the extension of computational time and the availability of real-world data. For Part I, factors such as the age of drivers, type of car, time of day, day of the week, and geographic location (i.e., regional data) were not included in the model. While data for these these elements had been found, time constraints prevented us from including them into our final model. As for Parts II and III, our models could be improved mostly with the availability of real-world data. Such data would allow us to disregard many of our assumptions, such as uniform population density, circular area of a city, and mutually exclusive factors. We would have also liked to model and compare the cost of the four different car sharing services to the city in question rather than to potential clients, as well as determine whether the various car sharing services could be affordable for low-income clients.

3.2 Summation

Overall, we consider our model(s) for each part of the problem to be successful. In Part I, we integrated a normalized distribution of 10,000 data points to categorize drivers into nine different categories, with 59.11 percent of them being low distance, medium time drivers. This model is consistent with local road driving data.

For Part II, a logistic probability distribution for participation formed the basis of a linear combination for perceived convenience of different car sharing services to potential climates. Based on the model, a one-way car sharing floating model saves the most money and time for clients. Applying this model to four cities found that Knoxville, TN, will garner the most participation with 3,600 clients, followed by Riverside, CA with 3,054; Richmond, VA with 1,530; and Poughkeepsie, NY with 931.

For Part III, the previous participation model was adjusted to account for emerging automobile technologies such as self-driving cars and vehicles that run on alternative or renewable energy. Again, the model suggested that the one-way car sharing floating model would save the most money and time for clients. The new rankings for the cities were as follows: Knoxville, TN with 13,650 clients; Riverside, CA with 13,650; Richmond, VA with 6,267; and Poughkeepsie, NY with 3,615.

Based on our models, we would recommend car sharing companies and automakers to implement a one-way car sharing floating model as a way to expand existing markets and to create new ones.

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