

**MTH 510**

**Inverse Problems and Data Assimilation**

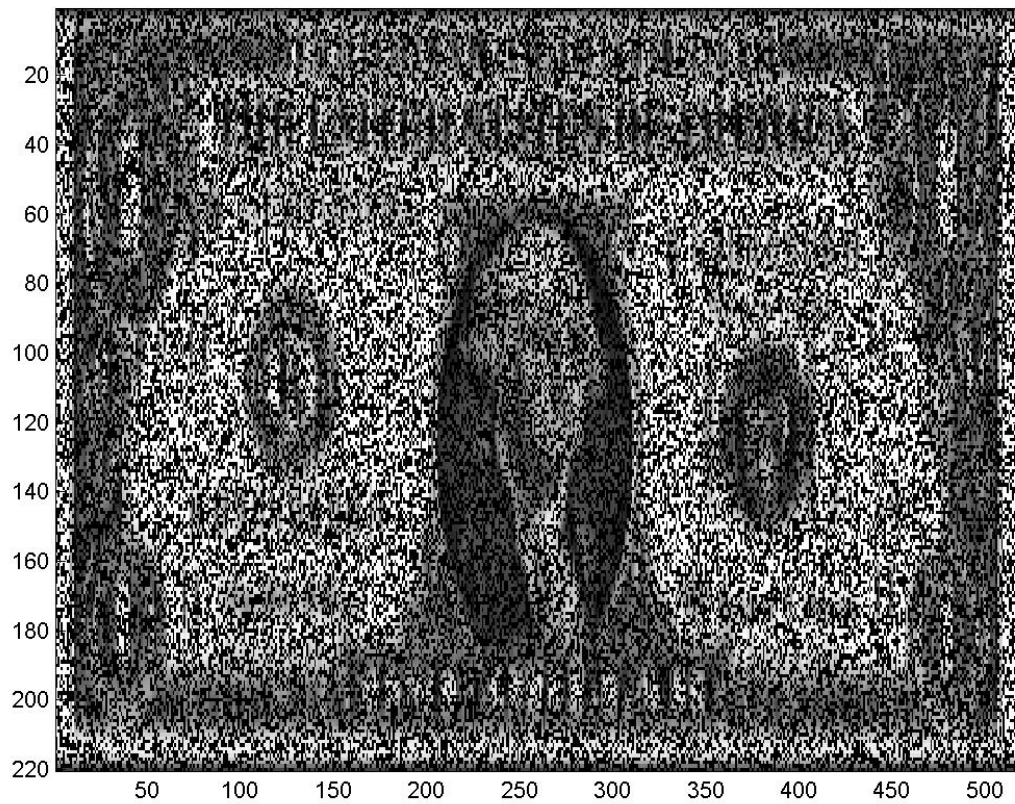
**Final Project**

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**Given image:**



*Figure 1: Blurred, gappy data*

**Implementing TSVD Regularization with truncation index = 87:**



*Figure 2: Reconstructed image*

**Serial number:** K03040506H

***Code used to perform TSVD reconstruction to generate the above image:***

```
# -*- coding: utf-8 -*-
"""
Created on Sat Dec 7 16:20:14 2019

@author: Andrew
"""

from PIL import Image
import numpy as np
import pandas as pd
from scipy.sparse import diags

D_hat = pd.read_csv("prdata.txt",header=None,delim_whitespace=True) ###
Blurred, gappy data ###
D_hat = pd.DataFrame.to_numpy(D_hat)
M =
pd.DataFrame.to_numpy(pd.read_csv("mask.txt",header=None,delim_whitespace=True)) ### Mask ###
n = D_hat.shape[0]
m = D_hat.shape[1]
B = np.zeros((n,m))
s = 0.45
power = 10
B = diags([s,(1-(2*s)),s],[-1,0,1],shape=(n,n)).toarray()
A = np.linalg.matrix_power(B,power) ### Blurring operator ###

### TSVD Reconstruction ###

p = 87 ### Truncation index ###
X_hat_tsvd = np.zeros((n,m))
for j in range(0,m):
    column = M[:,j]
    rows = np.nonzero(column)
    a = A[rows][:]
    d_hat = D_hat[:,j]
    d_hat = d_hat[rows][:].reshape((a.shape[0],1))
    U,S,V = np.linalg.svd(a,full_matrices=True)
    S = S.reshape(a.shape[0],1)
    x_hat = np.zeros((A.shape[0],1))
    for i in range(0,p):
        sigma_i = S[i][0]
        u_i = U[:,i].reshape(U.shape[0],1)
        v_i = V[i,:].reshape(V.shape[0],1)
        x_hat = x_hat+(((np.transpose(u_i)@d_hat)/sigma_i)*v_i)
```

```
x_hat_tsvd = x_hat
X_hat_tsvd[:,j] = x_hat_tsvd[:,0]  ### Reconstructed approximation of
true image ###
reconstructed_image_tsvd = Image.fromarray(X_hat_tsvd)
reconstructed_image_tsvd.show()
```