

**MTH 510**

**Inverse Problems and Data Assimilation**

**Homework #2**

**Submitted by Andrew Huffman**

**October 25, 2019**



- **Regularization Function**

```
def regularize(A,D_hat,method,p):
    global X_hat
    global S
    if method == "tsvd":
        U,S,V = np.linalg.svd(A,full_matrices=True)
        S = S.reshape(A.shape[0],1)
        X_hat = np.zeros((D_hat.shape[0],D_hat.shape[1]))
        for j in range(0,D_hat.shape[1]):
            x_hat = np.zeros((D_hat.shape[0],1))
            d_hat = D_hat[:,j].reshape(D_hat.shape[0],1)
            for i in range(0,p):
                sigma_i = S[i][0]
                u_i = U[:,i].reshape(U.shape[0],1)
                v_i = V[i,:].reshape(V.shape[0],1)
                x_hat = x_hat+(((np.transpose(u_i)@d_hat)/sigma_i)*v_i)
            X_hat[:,j] = x_hat[:,0]
        reconstructed_image = Image.fromarray(X_hat)

    elif method == "tikhonov":
        U,S,V = np.linalg.svd(A,full_matrices=True)
        S = S.reshape(A.shape[0],1)
        X_hat = np.zeros((D_hat.shape[0],D_hat.shape[1]))
        for j in range(0,D_hat.shape[1]):
            x_hat = np.zeros((D_hat.shape[0],1))
            d_hat = D_hat[:,j].reshape(D_hat.shape[0],1)
            for i in range(0,D_hat.shape[0]):
                sigma_i = S[i][0]
                f_i = (sigma_i**2)/((sigma_i**2)+p**2)
                u_i = U[:,i].reshape(U.shape[0],1)
                v_i = V[i,:].reshape(V.shape[0],1)
                x_hat = x_hat+(f_i*(((np.transpose(u_i)@d_hat)/sigma_i)*v_i))
            X_hat[:,j] = x_hat[:,0]
        reconstructed_image = Image.fromarray(X_hat)
```

- Reconstructed image from TSVD with truncation index  $k = 97$



Figure 1: Reconstructed image using TSVD with  $k = 97$

Year: 1984

Inscriptions: IN GOD WE TRUST, LIBERTY

- TSVD L-curve

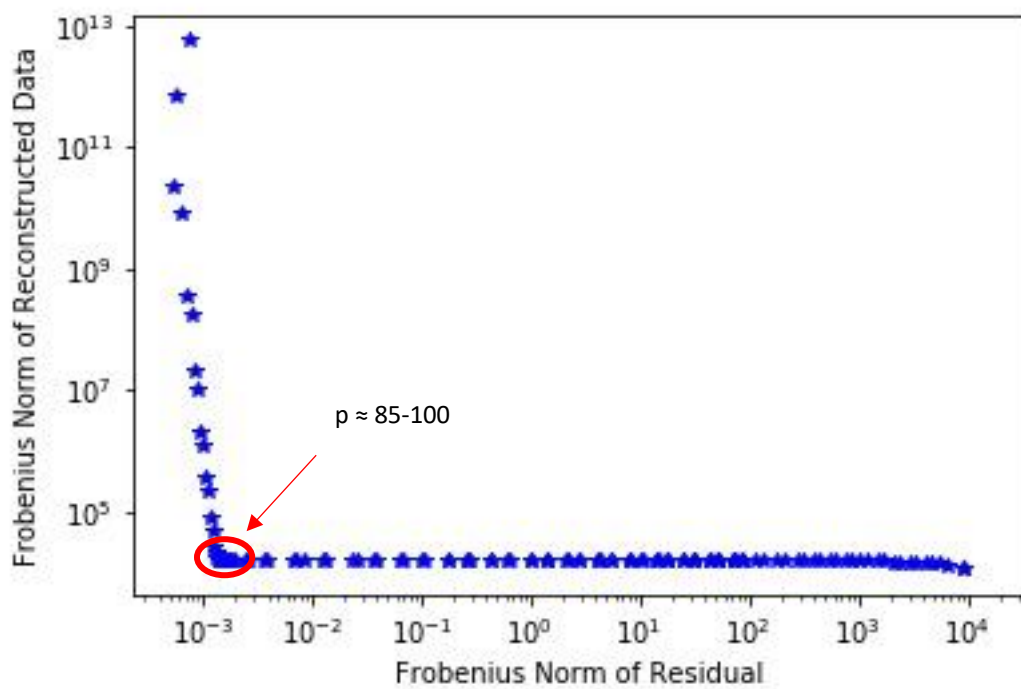


Figure 2: L-curve for TSVD technique

- Reconstructed image from Tikhonov regularization with  $\lambda \approx 5.8 \text{ E-6}$



Figure 3: Reconstructed image using Tikhonov regularization with  $\lambda \approx 5.8 \text{ E-6}$

Year: 1984

Inscriptions: IN GOD WE TRUST, LIBERTY

- Tikhonov L-curve

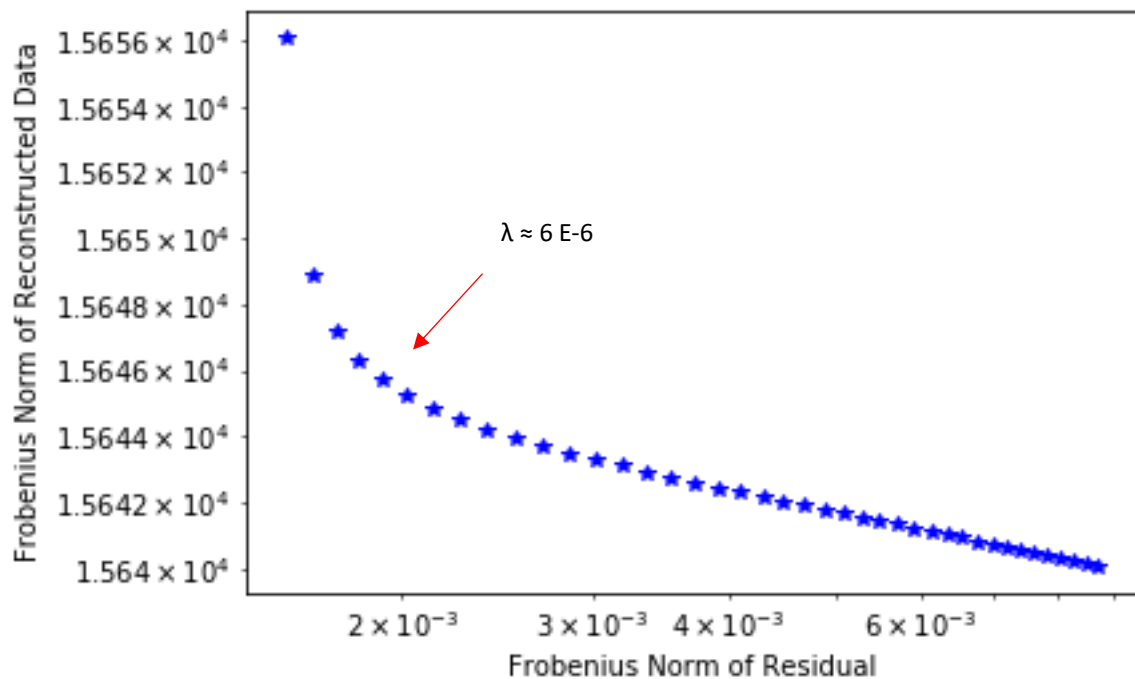


Figure 4: L-curve for Tikhonov regularization

- Singular values and filter numbers

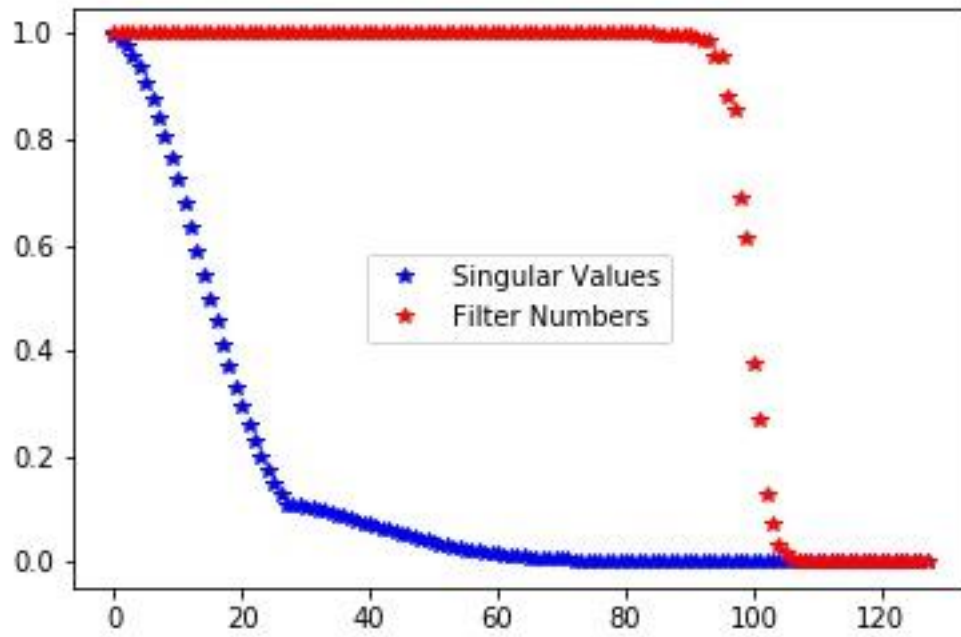


Figure 5: Plot of Singular Values and Filter numbers using  $\lambda \approx 5.8 \text{ E-}6$