Chapter 1

Motivations for $Z^{'}$ and $Z^{'} \to \tau\tau$ searches

In this chapter we outline the motivations for $Z^{'}$ and $Z^{'} \to \tau\tau$ searches. After a brief introduction of the Standard Model (SM) of particle physics and a little digression on the Higgs discovery, we focus on the open issues regarding its shortcomings. We then mention possible extentions that overcome these limits and we see how they imply the existence of new particles, $Z^{'}$-like. To conclude, we describe the main features of such new particles at the Large Hadron Collider (LHC).

1.1 The Standard Model of particle physics

The SM of particle physics [1] is the best theoretical model that we have today to explain phenomena of elementary particles. It describes the elementary particles that constitute known matter and three out of the four known fundamental interactions - the strong, the weak and the electromagnetic (the latter two unified to the electroweak) forces. It is supported by many experimetal measurements at high-level of precision that are found to be in good agreement with its previsions. At the same time, there are some shortcomings and phenomena not accounted by it (see section 1.2.1).

The known fundamental particles are leptons and quarks. They are fermions with spin-1/2 and are grouped into three "families", or "generations" (see table 1.1). Each fermion has an associated antifermion. It has the same mass as its corresponding fermion, but opposite electric charge, colour and third component of weak isospin. From the measured width of the Z^0 resonance, one can deduce that no further (fourth) massless neutrino exists.

Fermions	Family			El. Ch	Col	Weak Isospin		Spin
	1	2	3			left hd.	right hd.	
Leptons	ν_e	ν_μ	ν_τ	0	-	1/2	-	1/2
	e	μ	τ	-1	-	1/2	0	1/2
Quarks	u	c	t	+2/3	r,b,g	1/2	0	1/2
	d	s	b	-1/3	r,b,g	1/2	0	1/2

	Mass (GeV/c^2)			Mean life		
Lepton	$< 2 \times 10^{-6}$	$< 0.19 \times 10^{-3}$	$< 18.2 \times 10^{-3}$	(Mean life/Mass) $> 15.4\ s/eV$	$> 15.4\ s/eV$	$> 15.4\ s/eV$
	0.5110×10^{-3}	0.1057	1.777	$> 4.6 \times 10^{26}$ yr	$2.197 \times 10^{-6} s$	$291 \times 10^{-15} s$
Quarks	$1.5 to 3.3 \times 10^{-3}$	1.27	4.2	-	-	-
	$3.5 to 6.0 \times 10^{-3}$	104×10^{-3}	171	-	-	$0.5 \times 10^{-24} s$

Table 1.1: The fundamental fermions (spin 1/2) in the SM [11] and their properties.

The strong and the electroweak forces are mediated via the exchange of 12 vector bosons - eight massless gluons (g) for the strong force, one massless photon (γ) for the electromagnetic force, and three massive bosons for the weak force ($W^{\pm}$ and Z^0) (see table 1.2). Gluons carry colour and therefore interact with each other. Bosons of the weak interaction themselves carry weak charge and couple with each other as well. The range of the electromagnetic interaction is infinite since photons are massless. Because of the large mass of the exchange bosons of the weak interaction, its range is limited to 10^{-3} fm. Gluons have zero rest mass. Yet, the effective range of the strong interaction is limited by the mutual interaction of the gluons. The energy of the colour field increases as the distance grows. At distances ≥ 1 fm, it is large enough to produce real quark-antiquark pairs. “Free” particles always have to be colourless. The electromagnetic interaction and the weak interaction can be interpreted as two aspects of a single interaction: the electroweak interaction. The corresponding charges are related by the Weinberg angle (see section 1.1.3).

	Gravitational	Electromagnetic	Weak	Strong
Field boson	graviton	photon	$W^{\pm}$,Z^0	gluon
Spin-parity	2^+	1^-	$1^-,1^+$	1^-
Mass (GeV/c^2)	0	0	$M_W = 80.2$ $M_{Z^0} = 91.2$	0
Range (m)	∞	∞	10^{-18}	$\leq 10^{-15}$
Source	mass	electric charge	weak charge	colour charge
Coupling constant	$\frac{G_N M^2}{4\pi\hbar c}$ $= 5 \times 10^{-40}$	$\alpha = \frac{e^2}{4\pi\hbar c}$ $= \frac{1}{137}$	$\frac{G(Mc^2)^2}{(\hbar c)^3}$ $= 1.17 \times 10^{-5}$	$\alpha_s \leq 1$
Typical cross section (m^2)		10^{-33}	10^{-39}	10^{-30}
Typical lifetime (s)		10^{-20}	10^{-10}	10^{-23}

Table 1.2: Fundamental interactions and their properties [11].

Energy (E), momentum (p), angular momentum (L), charge (Q), colour (c), baryon number (B) and lepton number (L) are conserved in all three interactions. Parity (P), charge (C), and time (T) are conserved in the strong and in the electromagnetic interaction, but not in the weak interaction. For the charged current of the weak interaction, parity violation is maximal. The charged current only couples to left-handed fermions and right-handed antifermions. The neutral weak current is partly parity violating. It couples to left-handed and right-handed fermions and antifermions, but with different strengths. One case is known in which the combined CP parity is not conserved. Only the charged current of the weak interaction transforms one type of quark into another (quarks of a different flavour) and one kind of lepton into another. Thus, the quantum numbers determining the quark flavour (third component of isospin (I_3), strangeness (S), charm (C) etc.) are conserved in all other interactions. The magnitude of the isospin (I) is conserved in strong interactions.

1.1.1 From classical mechanics to quantum filed theory

The newtonian mechanics, which bases its foundations on the concept of forces, can be treated in a more elegant way by introducing two quantities: potential and kinetic energy. This approach is adopted in the Lagrangian mechanics.

In the Lagrangian mechanics there is a quantity, referred to as Lagrangian, given by the difference of kinetic and potential energy, $L = T - V$, which describes the dynamics of a given system. It is a function of the generalised coordinates q_i and their time derivatives (velocities) p_i (referred to as the "conjugate momentum", "canonical momentum" or "generalised momentum" of that specific coordinate)

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \tag{1.1}$$

If the evolution of the system is determined by time dependent constraints, the Lagrangian also depends on the time t

$$L(q, \dot{q}, t) = T(q, \dot{q}, t) - V(q, t) \tag{1.2}$$

The number of generalised coordinates equals the number of degrees of freedom of the system. Knowing the Lagrangian of a system means that we know the time evolution of that system through the Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1, 2, ..., d \tag{1.3}$$

In field theory the fields themselves, $\phi_r(x^\mu)$, $r = 1, 2, ..., N$, are the independent variables, and we exchange the Lagrangian L with the Lagrangian density $\mathcal{L}$ (although usually we refer to it as the Lagrangian), such that the Lagrangian density is the difference between the kinetic energy density and the potential energy density, $\mathcal{L} = \mathcal{T} - \mathcal{V}$.

The conjugate momenta (1.1) (now called the "conjugate fields")

$$\pi_r(x^\mu) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_r} \tag{1.4}$$

and the Euler-Lagrange equations (equation (1.3))

$$\frac{\partial \mathcal{L}}{\partial \phi_r} - \partial_\alpha \left(\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi_r)}\right) = 0, \quad \partial_\alpha = \frac{\partial}{\partial x^\alpha} \tag{1.5}$$

are the same as before, with the proper exchanges made as discussed above.

To go from classical fields to quantised fields, one interprets the generalised coordinates (fields and conjugate fields) as operators subjected to commutation relations

$$[\phi_r(x^\mu), \pi_s(x_\mu^{'})] = i\hbar \delta_{rs} \delta(x_\mu - x_\mu^{'}) \tag{1.6}$$

$$[\phi_r(x^\mu), \phi_s(x_\mu^{'})] = [\pi_r(x^\mu), \pi_s(x_\mu^{'})] = 0 \tag{1.7}$$

After that, it is a matter of finding the right Lagrangian.

One of the main reasons for introducing quantum fields is that the number of particles no longer needs to be constant. With this new formalism one can explore a new range of phenomena, like decaying particles and vacuum fluctuations.

1.1.2 Quantum electrodynamics

The first successful Quantum Field Theory (QFT) was that of electrodynamics - Quantum ElectroDynamics (QED) - developed from the 1920s to the 1940s. Feynman, Schwinger and Tomonaga recieved the 1965 Nobel Prize in Physics for this great achievement. In this section we start by reviewing this theory that describes fermions to help introduce the basic principles of QFT.

The free-fermion Lagrangian density $\mathcal{L}_0$ is given by

$$\mathcal{L}_0 = \bar{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x) \tag{1.8}$$

where ψ, γ are the Dirac spinor and matrix, and m is the rest mass of the spin 1/2 particles. The electromagnetic interaction is introduced by means of minimal substitution

$$\partial_{\mu} \to D_{\mu} = [\partial_{\mu} + iqA_{\mu}(x)] \tag{1.9}$$

where q is the charge of the fermion, and A_{μ} the 4-vector potential.

Since the potential A_{μ} does not have any physical meaning, while the fields do, the theory needs to be invariant under a gauge transformation of the potientials

$$A_{\mu}(x) \to A^{'}_{\mu}(x) = A_{\mu}(x) + \partial_{\mu} f(x) \tag{1.10}$$

where $f(x)$ is an arbitrary, real, and differentiable function.

The Lagrangian is invariant under this transformation only if the Dirac fields themselves $\psi(x), \bar{\psi}(x)$ undergo the transformations

$$\begin{aligned} \psi(x) &\to \psi^{'}(x) = \psi(x)e^{-iqf(x)} \\ \bar{\psi}(x) &\to \bar{\psi}^{'}(x) = \bar{\psi}(x)e^{iqf(x)} \end{aligned} \tag{1.11}$$

Any theory invariant under gauge transformations such as (1.10), is said to be a gauge theory, of which QED is the simplest example.

On the other hand, having the free-fermion Lagrangian density $\mathcal{L}_0$ we demand that it is invariant under local phase transformations (1.11), since the phase itself (i.e. $iqf(x)$) has no physical meaning. This makes our Lagrangian change

$$\mathcal{L}_0 \to \mathcal{L}^{'}_0 = \mathcal{L}_0 + q\bar{\psi}(x)\gamma^{\mu}\psi(x)\partial_{\mu} f(x) \tag{1.12}$$

which is not invariant. In order to do so we augment $\mathcal{L}_0$ by a term $\mathcal{L}_I$ in such a way that the new Lagrangian density $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$ is invariant under the gauge transformations (1.11) above. This is done by replacing the ordinary derivative $\partial_{\mu}\psi(x)$ by the covariant derivative $D_{\mu} = [\partial_{\mu} + iqA_{\mu}(x)]$, and thus $\mathcal{L}$ becomes

$$\begin{aligned} \mathcal{L} &= \bar{\psi}(x)\left(i\gamma^{\mu}D_{\mu} - m\right)\psi(x) \\ &= \mathcal{L}_0 - q\bar{\psi}(x)\gamma^{\mu}\psi(x)A_{\mu}(x) = \mathcal{L}_0 + \mathcal{L}_I \end{aligned} \tag{1.13}$$

The covariant derivative $D_{\mu}\phi(x)$ will, under the gauge transformations (1.11), undergo the transformations

$$D_{\mu}\psi(x) \to e^{-iqf(x)} D_{\mu}\psi(x) \tag{1.14}$$

leaving the Lagrangian density invariant.

$\mathcal{L}_0$ is the Lagrangian density of the free Dirac field, while $\mathcal{L}_I$ can be interpreted as the interaction Lagrangian density. In this way the form of the interaction between the spin 1/2 particle $\psi(x)$ and the gauge boson A_{μ} is completely specified. $\mathcal{L}_I$ couples the conserved electromagnetic current $-q\bar{\psi}(x)\gamma^{\mu}\psi$

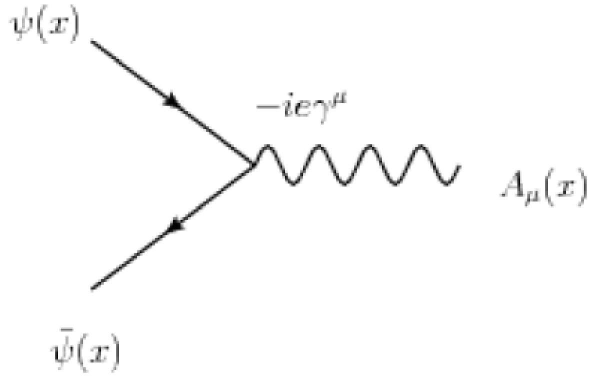


Figure 1.1: A Feynman diagram of the QED basic vertex.

to the electromagnetic field A_μ and gives rise to the vertex factor $-iq\gamma^\mu$ in the Feynman rules for QED (see figure 1.1). Note that this is a non-physical process to the first order as it does not conserve energy and momentum at the same time.

All particle interactions in QED are constructed by combining several of these basic vertices. From that one can calculate the S-matrix.

The S-matrix contains all the information that one could possibly want from a collision process at an arbitrary order of perturbation theory. If a system is in the initial state $|i>$ at an initial time t_i, which in principle is - ∞ (in the microscopic world, a few seconds feel like an infinity) and ends up in the final state $|f>$ at some time $t_f = \infty$ after the scattering, then the S-matrix is the operator connecting the two states

$$S|i(t_i = -\infty) >= |f(t_f = \infty) > \tag{1.15}$$

What we usually get is the probability for that specific final state given our chosen initial state

$$| < f|(S|i>)|^2 = | < f|S|i > |^2 = S_{fi} \tag{1.16}$$

The S-matrix itself is the solution of the equations of motion for the fields given the initial and final states. It can be calculated at arbitrary order in perturbation theory

$$S = \sum_{n=0}^{+\infty} S^n \tag{1.17}$$

where

$$S^n = \frac{(-1)^n}{n!}\left(\frac{1}{i\hbar}\right)^n \int\int ... \int d^4x_1 d^4x_2 ... d^4x_n \mathcal{T}[\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)...\mathcal{H}_I(x_n)] \tag{1.18}$$

$\mathcal{H}$ is the Hamiltonian density $(\mathcal{H}(x^\mu) = \pi_r(x^\mu)\dot{\phi}_r(x) - \mathcal{L}(\phi_r, \partial_\alpha \phi_r, t))$ and the time-ordered $\mathcal{T}$ product only means that the operators are written in chronological order (the time runs from right to left).

From the S-matrix one can calculate the quantities that are experimentally observable, namely cross-sections, decay widths and lifetimes, all of which are of importance to the experimental particle physicist.

Till now we have dealt with free fermions and fermions that interact via the electromagnetic field. However, there is still another part to consider of the QED Lagrangian, the part describing the free electromagnetic field, the free Maxwell part, $\mathcal{L}_M$.

This part of the Lagrangian comes from Maxwells'equations, describing electric and magnetic fields. The covariant formulation uses the antisymmetric field tensor, $F^{\mu\nu}(x)$, that can be expressed in terms of A^{μ} as

$$F^{\mu\nu}(x) = \partial^{\nu} A^{\mu}(x) - \partial^{\mu} A^{\nu}(x) \tag{1.19}$$

This term is invariant under a gauge transformation of the potential $A_{\mu}(x)$ (as equation (1.10)) and can be added to the QED Lagrangian [1] making the total QED Lagrangian

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^{\mu} D_{\mu} - m)\psi(x) - \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \tag{1.20}$$

QED has been tested experimentally in many occasions [12]. For example the theoretical differential cross-section distribution for several processes (amongst others $e^{+}e^{-} \rightarrow \gamma\gamma$ and $e^{+}e^{-} \rightarrow e^{+}e^{-}$) has been compared to experimental results. No discrepancies have been found.

The principle of local phase invariance underpins the quantum field theories not only of the electromagnetic interaction (QED) but also those of electroweak (EW) and strong interactions (QCD) that are briefly described in the sections below.

1.1.3 Electroweak interactions

The following considerations require zero fermion and zero boson mass. The fact that both fermions and bosons have mass simply means that we need to consider a mechanism to let them acquire mass. This turns out to be the spontaneous symmetry breaking that brings the Higgs boson to life (see sections 1.1.5,1.1.6).

Experiments [13] have shown that the weak force acts only on left-handed particles, leaving the right-handed particles untouched. For massless particles the chirality (handedness) is the same as the helicity, which gives a more intuitive understanding of the concept.

Because of the fundamental difference between right- and left-handed fields, we write the free-lepton Lagrangian in an asymmetric way, with the left-handed fields grouped in a doublet and the right-handed fields in singlets:

$$\mathcal{L}_0 = i[\bar{\psi}^L(x)\ \partial\!\!\!/ \psi^L(x) + \bar{\psi}_l^R(x)\ \partial\!\!\!/ \psi_l^R(x) + \bar{\psi}_{\nu_l}^R(x)\ \partial\!\!\!/ \psi_{\nu_l}^R(x)], \quad \partial\!\!\!/ = \gamma^{\mu}\partial\mu \tag{1.21}$$

with

$$\psi^L(x) = \begin{pmatrix} \psi_{\nu_l}^L(x) \\ \psi_l^L(x) \end{pmatrix} \tag{1.22}$$

In analogy with what we did for the free-fermion Lagrangian density in QED, we now want to extend the principle of gauge invariance to $\mathcal{L}_0$ under local $SU(2)_L$ and $U(1)$ transformations

Introducing the Pauli matrices [2]

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1.23}$$

the local $SU(2)_L$ phase transformations are

$$\begin{aligned} \psi_l^L(x) &\rightarrow \psi_l^{L'}(x) = e^{\frac{1}{2} i g \tau_j \omega_j(x)} \psi_l^L(x) \\ \bar{\psi}_l^L(x) &\rightarrow \bar{\psi}_l^{L'}(x) = \bar{\psi}_l^L(x) e^{-\frac{1}{2} i g \tau_j \omega_j(x)} \end{aligned} \tag{1.24}$$

[1] In reality this is not the formulation used, as problems arise when one tries to do the canonical quantization. A formulation that works is $\mathcal{L} = -\frac{1}{2}(\partial_{\nu} A_{\mu}(x))(\partial^{\nu} A^{\mu}(x)) - \frac{1}{c} s_{\mu}(x) A^{\mu}(x)$. This Lagrangian also returns the Maxwell equations, but only if the potential $A^{\mu}(x)$ satisfies $\partial_{\mu} A^{\mu}(x) = 0$.

[2] They satisfy the commutation relations $[\tau_i, \tau_j] = 2i\varepsilon_{ijk}\tau_k$, with ε_{ijk} the usual antisymmetric tensor.

where $\omega_j(x)$, j=1,2,3, are three real differentiable functions of x, g is a real constant and the Pauli matrices τ_i are the generators of $SU(2)_L$. We define every right-handed lepton field to be invariant under any $SU(2)_L$ transformation.

In order to have the letpon Lagrangian density (1.21) invariant under these transformations, we first replace the ordinary derivatives $\partial^\mu \psi_l^L(x)$ with covariant derivatives

$$\partial^\mu \psi_l^L(x) \to D^\mu \psi_l^L(x) = [\partial^\mu + \frac{1}{2} ig\tau_j W_j^\mu(x)]\psi_l^L(x) \tag{1.25}$$

giving us the new expression for the Lagrangian density

$$\widetilde{\mathcal{L}}_0 = i[\bar{\psi}_l^L(x)\ \not{D}\psi_l^L(x) + \bar{\psi}_l^R(x)\ \not{\partial}\psi_l^R(x) + \bar{\psi}_{\nu_l}^R(x)\ \not{\partial}\psi_{\nu_l}^R(x)] \tag{1.26}$$

It is important to note that in doing this we have introduced three gauge fields $W_j^\mu(x)$, one for each $SU(2)_L$ generator. In QED we have introduced only one, $A_\mu(x)$. We will see that in QCD, which has an $SU(3)_C$ symmetry, we will introduce eight generators. In general for a group $SU(N)$, we need to introduce N^2-1 generators. The connection between symmetry group and number of force-carrying boson is intimate: the number of bosons equals the number of so-called generators of the group. From an experimental point of view this is very interesting: if we find a group $SU(N)$, $N>3$ that describes elementary particles, we would expect new particles to exist (see section 1.2).

Secondly, we need the covariant derivatives $D^\mu \psi_l^L(x)$ to transform in the same way as the fields themselves. This is achieved by requiring that the gauge fields transform according to

$$W_i^\mu(x) \to W_i^{\mu'}(x) = W_i^\mu(x) + \delta W_i^\mu(x) = W_i^\mu(x) - \partial^\mu\omega_i(x) - g\varepsilon_{ijk}\omega_j(x)W_k^\mu(x) \tag{1.27}$$

for small $\omega_j(x)$ [14]. Thus we have

$$D^\mu \psi_l^L(x) \to e^{\frac{1}{2}ig\tau_j\omega_j} D^\mu \psi_l^L(x) \tag{1.28}$$

Now that we have achieved $SU(2)_L$ invariance, the next step is to achive $U(1)$ invariance. The local phase transformations are:

$$\begin{aligned} \psi(x) &\to \psi'(x) = e^{ig'Yf(x)}\psi(x) \\ \bar{\psi}(x) &\to \bar{\psi}'(x) = \bar{\psi}(x)e^{-ig'Yf(x)} \end{aligned} \tag{1.29}$$

where Y is a conserved quantity called the weak hypercharge (explained below, see equation (1.40)). The Lagrangian density is invariant under the $U(1)$ transformations (1.29) if the ordinary derivatives are replaced by covariant derivatives

$$\partial^\mu \psi(x) \to D^\mu \psi(x) = [\partial^\mu + ig'YB^\mu(x)]\psi(x) \tag{1.30}$$

where $B^\mu(x)$ is a gauge field that transforms like this:

$$B^\mu(x) \to B^{\mu'}(x) = B^\mu(x) - \partial^\mu f(x) \tag{1.31}$$

which is exactly what we have seen in QED (equation (1.10)).

Making both replacements (1.30) and (1.28) simultaneously in (1.21) we get the Lagrangian density

$$\mathcal{L} = i[\bar{\psi}_l^L(x)\ \not{D}\psi_l^L(x) + \bar{\psi}_l^R(x)\ \not{D}\psi_l^R(x) + \bar{\psi}_{\nu_l}^R(x)\ \not{D}\psi_{\nu_l}^R(x)] \tag{1.32}$$

where now

$$\begin{aligned} D^{\mu}\psi_l^L(x) &= \left[\partial^{\mu} + \frac{1}{2}ig\tau_j W_j^{\mu}(x) - \frac{1}{2}ig^{'}B^{\mu}(x)\right]\psi_l^L(x) \\ D^{\mu}\psi_l^R(x) &= [\partial^{\mu} - ig^{'}B^{\mu}(x)]\psi_l^R(x) \\ D^{\mu}\psi_{\nu}^R(x) &= \partial^{\mu}\psi_{\nu}^R(x) \end{aligned} \tag{1.33}$$

Defining the fields $W_i^{\mu}(x)$ to be invariant under $SU(2)_L$ gauge transformations and $B^{\mu}(x)$ to be invariant under $U(1)$ gauge transformations the Lagrangian density $\mathcal{L}$ is invariant under both, and it is said to be $SU(2)_L \times U(1)$ gauge-invariant.

Before we advance any further, we should take a step back and look at global $U(1)$ and $SU(2)_L$ transformations.

Global $SU(2)_L$ transformations are the same as local ones (eqations (1.24)), except for the positional dependence of ω_j ($g\omega_j$ are now three real numbers, $\mathbf{g\omega} = (g\omega_1, g\omega_2, g\omega_3)$)

$$\begin{aligned} \psi_l^L(x) &\rightarrow \psi_l^{L^{'}}(x) = e^{\frac{1}{2}ig\omega_j\tau_j}\psi_l^L(x) \\ \bar{\psi}_l^L(x) &\rightarrow \bar{\psi}_l^{L^{'}}(x) = \bar{\psi}_l^L(x)e^{-\frac{1}{2}ig\omega_j\tau_j} \end{aligned} \tag{1.34}$$

When the right handed fields are defined as invariant under these global transformations, the whole free-lepton Lagrangian of equation (1.21) is clearly invariant as well. We can now use Noether's theorem, stating that the invariance of the Lagrangian implies some conserved quantities [3], to find the conserved currents. These are the weak isospin currents

$$J_i^{\mu}(x) = \frac{1}{2}\bar{\psi}_l^L(x)\gamma^{\mu}\tau_i\psi_l^L(x), \quad i = 1,2,3 \tag{1.35}$$

With the three conserved currents come three conserved quantities, the weak isospin charges I_i^W:

$$I_i^W = \int d^3x J_i^0(x) = \frac{1}{2}\int d^3x \psi_l^{L\dagger}(x)\tau_i\psi_l^L, \quad i = 1,2,3 \tag{1.36}$$

We need the expression for J_3^{μ}:

$$\begin{aligned} J_3^{\mu}(x) &= \frac{1}{2}\bar{\psi}_l^L(x)\gamma^{\mu}\tau_3\psi_l^L(x) \\ &= \frac{1}{2}\left(\bar{\psi}_{\nu}^L(x)\bar{\psi}_l^L(x)\right)\gamma^{\mu}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \psi_{\nu}^L(x) \\ \psi_l^L(x) \end{pmatrix} \\ &= \frac{1}{2}\bar{\psi}_{\nu}^L(x)\gamma^{\mu}\psi_{\nu}^L(x) - \frac{1}{2}\bar{\psi}_l^L(x)\gamma^{\mu}\psi_l^L(x) \end{aligned} \tag{1.37}$$

This is a neutral current - it couples either electrically charged leptons or neutral neutrinos, just like the electromagnetic current. This last part of J_3^{μ} is a part of the electromagnetic current, except for a constant factor. On this somewhat vague basis we define a new current, the weak hypercharge current J_Y^{μ}:

$$J_Y^{\mu} = \frac{1}{e}s^{\mu}(x) - J_3^{\mu}(x) = -\frac{1}{2}\bar{\psi}_l^L(x)\gamma^{\mu}\psi_l^L(x) - \bar{\psi}_l^R(x)\gamma^{\mu}\psi_l^R(x) \tag{1.38}$$

The conserved charge corresponding to the conserved current is called the weak hypercharge

$$Y = \int d^3x J_Y^0(x) \tag{1.39}$$

[3] The number of conserved quantities is equal to the number of generators of the symmetry

The hypercharge Y is closely related to the electric charge Q and the weak isocharge I_3^W (which are the conserved quantities of the electromagnetic current and the third weak hypercharge current respectively), as can be seen from equation (1.37)

$$Y = \frac{1}{e}Q - I_3^W \tag{1.40}$$

We are now equipped to return to the Lagrangian density of equation (1.32). If we write it explicitly, we find our original free-lepton Lagrangian, in addition to other terms:

$$\begin{aligned} \mathcal{L} = \mathcal{L}_0 + i\left[\bar{\psi}_l^L(x)\gamma_\mu\left(\partial^\mu + ig\tau_j W_j^\mu(x) - \frac{1}{2}ig' B^\mu(x)\right)\psi_l^L(x)\right] \\ + i\left[\bar{\psi}_l^R(x)\gamma_\mu(\partial^\mu - ig' B^\mu(x))\psi_l^R(x) + \bar{\psi}_\nu^R(x)\gamma_\mu\partial^\mu\psi_\nu^R(x)\right] \end{aligned} \tag{1.41}$$

We rewrite equation 1.41 in terms of the weak isospin currents (equation 1.36) and the weak hypercharge current (equation 1.38)

$$\mathcal{L} = \mathcal{L}_0 - gJ_i^\mu(x)W_{j\mu}(x) - g' J_Y^\mu(x)B_\mu(x) = \mathcal{L}_0 + \mathcal{L}_I \tag{1.42}$$

To proceed in any direction, we need to rewrite $\mathcal{L}_I$ in terms of the charged leptonic currents $J^\mu(x)$[4] and $J^{\mu\dagger}(x)$

$$\begin{aligned} J_\mu(x) &= \sum_l \bar{\psi}_l(x)\gamma_\mu(1-\gamma_5)\psi_{\nu_l}(x) \\ J_\mu^\dagger(x) &= \sum_l \bar{\psi}_{\nu_l}(x)\gamma_\mu(1-\gamma_5)\psi_l(x) \end{aligned} \tag{1.43}$$

and a new (non-hermitian) gauge-field

$$W_\mu(x) = \frac{1}{\sqrt{2}}[W_{1\mu}(x) - iW_{2\mu}(x)] \tag{1.44}$$

This gives us, after some straight forward mathematical manipulations (not shown), the first two terms of the interaction lagrangian of equation (1.42)

$$-g\sum_{i=1}^{2} J_i^\mu(x)W_{i\mu}(x) = \frac{-g}{2\sqrt{2}}[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W_\mu^\dagger(x)] \tag{1.45}$$

From here we can read off the basic vertices for the electroweak interactions (see figure 1.2). W^μ and $W^{\dagger\mu}$ are interpreted as the charged *physical* (but still massless) $W^\pm$ bosons. Now we return to equation (1.42) and rewrite $W_{3\mu}(x)$ and $B_\mu(x)$ as linear combinations of the two gauge fields $A_\mu(x)$ and $Z_\mu(x)$ in combination with the so-called weak mixing angle θ_W[5]

$$\begin{aligned} W_{3\mu}(x) &= cos\theta_W Z_\mu(x) + sin\theta_W A_\mu(x) \\ B_\mu(x) &= -sin\theta_W Z_\mu(x) + cos\theta_W A_\mu(x) \end{aligned} \tag{1.46}$$

From this in addition to $J_Y^\mu = \frac{1}{e}s^\mu(x) - J_3^\mu(x)$ we get for the rest of equation (1.42)

[4]Experiments assure that the weak charged current has this V-A structure [22].

[5]This gives the mixture of weak and electromagnetic interactions. Notice that $\theta_W = 0$ decouples them. This has been proven to be an incorrect description by experiment. The most current and exact value of $sin^2\theta_W$ is 0.23119(14) [11], corresponding to $\theta_W \sim 30^\circ$

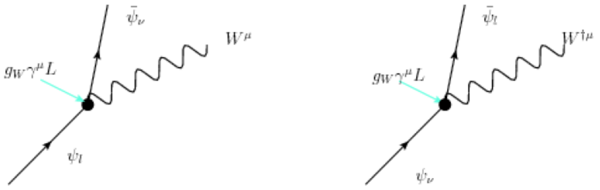


Figure 1.2: The Feynman diagrams corresponding to equation (1.45). g_W is equal to $\frac{g}{2\sqrt{2}}$.

$$\begin{aligned} &-J_3^\mu(x)W_{3\mu}(x) - g^{'}J_Y^\mu(x)B_\mu(x) = -\frac{g^{'}}{e}s^\mu(x)[-sin\theta_W Z_\mu(x) + cos\theta_W A_\mu(x) - \\ &J_3^\mu(x)\left(g[cos\theta_W Z_\mu(x) + sin\theta_W A_\mu(x)] - g^{'}[-sin\theta_W Z_\mu(x) + cos\theta_W A_\mu(x)]\right) \end{aligned} \tag{1.47}$$

Demanding that $A_\mu(x)$ is in fact the electromagnetic field and that it is coupled to the electric charges in the usual way (equation 1.13), i.e. $-s^\mu(x)A_\mu(x)$, we see that $J_3^\mu(x)A_\mu(x)$ must vanish and that we need to identify

$$g^{'}cos\theta_W = gsin\theta_W = e \tag{1.48}$$

This final step leads us to the expression for the interaction Lagrangian density

$$\begin{aligned} \mathcal{L}_I = -s^\mu(x)A_\mu(x) - \frac{g}{2\sqrt{2}}[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W^\dagger(x)] \\ -\frac{g}{cos\theta_W}[J_3^\mu(x) - \frac{1}{e}sin^2\theta_W s^\mu(x)]Z_\mu(x) \end{aligned} \tag{1.49}$$

We have now arrived at the $SU(2)_L \times U(1)_Y$ gauge-invariant interaction Lagrangian that Glashow proposed in 1961. The first term we know from QED - this is just the electromagnetic current coupling to the photon field. We interpret the $W(x)$ and $W^\dagger(x)$ gauge field's quanta as the physical (but still massless) $W^\pm$ vector bosons. The last term represents neutral currents. The quanta of the $Z_\mu(x)$ field is the (also still massless) physical Z^0 boson.

This is just a part of what is needed in the theory of electroweak interactions. Up to now we have seen how leptons interact with gauge fields. The situation is almost the same for quarks, just with up-type and down-type quarks playing the role of the neutrino and the lepton, i.e $\begin{pmatrix} u \\ d \end{pmatrix}_L$, $\begin{pmatrix} c \\ s \end{pmatrix}_L$, $\begin{pmatrix} t \\ b \end{pmatrix}_L$ and u_R, d_R etc.

One thing that is different for the quarks, in comparison to the leptons, is the flavour changing mechanism. The three quark generations transform into each other. The vertex factors involving quarks are equal to the ones involving leptons, except that it carries an extra factor representing the probability of the process in question. This probability is given by the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [11],

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.00049 \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 \\ 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 \end{pmatrix} \tag{1.50}$$

The probability of transition from flavour i to j is $|V_{ij}|^2$. We see that the largest probabilities are on the diagonal elements, meaning that the transition probability between quarks of the same generation is largest.

The complete Lagrangian also describes how gauge bosons interact when no leptons are present. We do not go into details about this. The interested reader can refer to ([23]).

1.1.4 Quantum Chromodynamics

The quarks also have, in addition to electroweak interactions, strong interaction, which this section addresses.

In the 1950s new particles were springing to life as never before. There were so many that people wondered if they could all be fundamental. In 1963 Murray Gell-Mann and George Zweig proposed that they were all made up of quarks, of which there were three flavours - up, down and strange. After the discovery of Ω^-, made of three strange quarks with parallel spins, a new quantum number was proposed - colour. At least three colours had to exist because of the three quarks in the Ω^- and the Pauli principle stating that fermions cannot be in the same quantum state.

Later on it was confirmed that there were in fact three colours looking at the differential cross-sections of the processes $e^+e^- \to hadrons$ and $e^+e^- \to \mu^+\mu^-$. When the collision energy E is well above the particle creating threshold, $E \gg mc^2$ (m being the mass of the particles created) the cross-section reduces to [24]

$$\sigma = \frac{\pi}{3}\left(\frac{\hbar Q c \alpha}{E}\right)^2 \tag{1.51}$$

where c is the speed of light in vacuum, α is electromagnetic coupling constant and Q is the electric charge of the created particles. The ratio of the rate of hadron production to the rate of muon pairs $R = \frac{\sigma(e^+e^- \to hadrons)}{e^+e^- \to \mu^+\mu^-}$ is thus given by $R = c\sum Q_i^2$, where c is the number of colours. Plotting this ratio as a function of the collision energy gives a staircase-shaped graph showing that the number c is equal to 3.

For a long time, quarks were considered a mere mathematical construct by some, because every search for them came up negative. Today we believe that they are indeed real particles, but forever imprisoned, or confined [6], in hadrons. The combination of quarks and gluons must be colourless ("white") which greatly reduces the number of possible configurations. Luckily, in addition to being confined they are also asymptotically free, meaning that the more energy is put into the system, the less bound the quarks appear and the smaller the effective coupling is. If this were not the case, we would not have had to use the perturbation theory, as we have done for QED and EW theories.

Because of this new quantum number, colour, the three versions of each quark are grouped in a triplet

$$\psi_q = \begin{pmatrix} \psi_{q_r} \\ \psi_{q_b} \\ \psi_{q_g} \end{pmatrix} = \begin{pmatrix} \psi_{q_1} \\ \psi_{q_2} \\ \psi_{q_3} \end{pmatrix} \tag{1.52}$$

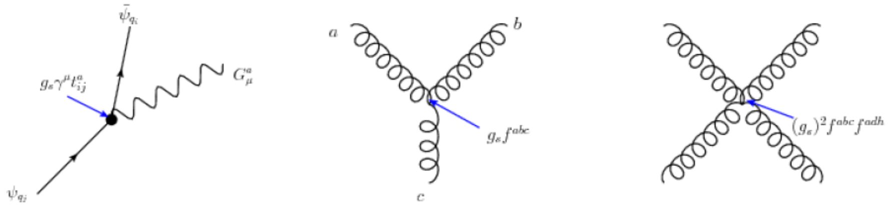
Now we simply follow the recipe of QED and claim that the free-quark Lagrangian is

$$\mathcal{L}^0_{QCD} = \bar{\psi}_q(\gamma^\mu i\partial_\mu - m_q)\psi_q \tag{1.53}$$

Because there are three versions of each quark, in contrast to QED, one postulated that the Lagrangian should be invariant under $SU(3)_C$ transformations:

$$U = e^{i\alpha^a(x)t^\alpha}, \quad t^\alpha = \frac{1}{2}\lambda^a \tag{1.54}$$

[6]Except for the heaviest quark, top, which decays before being able to hadronise



(a) (b)

Figure 1.3: Feynman diagrams of the basic vertices of QCD.

U is a 3×3 matrix and λ^a are the eight Gell-Mann matrices, the generators of $SU(3)$. When trying to make $\mathcal{L}^0_{QCD}$ $SU(3)_C$ invariant, one finds it is necessary to exchange the ordinary derivative ∂_μ with the covariant derivative D_μ

$$\partial^\mu \psi_q(x) \rightarrow D^\mu \psi_q(x) = (\partial^\mu + ig_s t^a G^{\mu,a})\psi_q(x) \tag{1.55}$$

and that 8 new gauge fields $G^{\mu,a}$ are needed. These are the 8 gluons. When putting all together the Lagrangian looks like

$$\mathcal{L} = \bar{\psi}_q(x)(\gamma^\mu i\partial_\mu - m_q)\psi_q(x) - g_s j^a_\mu G^{\mu,a} \tag{1.56}$$

where the colour-octet current j^a_μ is introduced:

$$j^a_\mu = \bar{\psi}_q(x)\gamma^\mu t^a \psi_q, \quad \psi_q(x) = \begin{pmatrix} \psi_{q_1} \\ \psi_{q_2} \\ \psi_{q_3} \end{pmatrix} \tag{1.57}$$

This gives us a basic vertex for QCD, which is drawn in figure 1.3 (a). As for QED, the colour-octet current j^a_μ is a conserved current and the conserved quantity is the colour charge. The QCD interaction Lagrangian is

$$\mathcal{L}_G = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a}, \quad G^a_{\mu\nu} = F^a_{\mu\nu} - g_s f^{abc} G^b_\mu G^c_\nu \tag{1.58}$$

where $[t^a, t^b] = if^{abc}t^c$, $-ig_s t^a G^a_{\mu\nu} = [D_\mu, D_\nu]$ and $F^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu$

We now have the Lagrangian for the strong interaction [7]

$$\begin{aligned}\mathcal{L}_{QCD} = {} & \bar{\psi}_q(x)(\gamma^\mu i\partial_\mu - m_q)\psi_q(x) - g_s j^a_\mu G^{\mu,a} + F^a_{\mu\nu}F^{a,\mu\nu} \\ & - 2g_s F^a_{\mu\nu} f^{abc} G^{b,\mu} G^{c,\nu} + (g_s)^2 f^{abc} f^{adh} G^b_\mu G^c_\nu G^{d,\mu} G^{h,\nu}\end{aligned} \tag{1.59}$$

and the additional basic vertices as shown before in figures 1.3 (a) and 1.3 (b). The non-abelian nature of the $SU(3)_C$ group means the gluons have self-interactions, leading to the 3- and 4-gluon vertices in figure 1.3 (b). The presence of these self-intraction vertices explains asymptotic freedom, i.e. why the strong force between quarks diminishes at high energies/short distances.

[7] Keeping in mind that quarks have electromagnetic and weak interactions as well

1.1.5 The Higgs mechanism

There exists a model, the Glashow model, that simply introduces mass terms in the Lagrangian density. For leptons these are $-m_l \bar{\psi}_l(x)\psi_l(x)$. The result is a non-gauge invariant and a non-renormalizable theory. For some well chosen values of $sin\theta_W$ and m_z and at first order perturbation theory the model is able to fit the experiments reasonably well [25]. However we want a better explanation for the non-zero masses of our fermions and bosons. The answer comes in the form of spontaneous symmetry breaking.

In the last sections we reviewed the interaction Lagrangian density $\mathcal{L}$ consisting of a free lepton part and an interaction part, but without any mass term. To generate the obvious non-zero masses for the leptons, quarks and the $W^{\pm}$ and Z^0 bosons, we have to introduce the Higgs mechanism by letting the electroweak Lagrangian density contain a term $\mathcal{L}^H$. We need to break the $SU(2)_L \times U(1)_Y$ invariance spontaneously, leaving the $U(1)_Y$ invariance intact[8].

Spontaneous Symmetry Breaking for a U(1) theory

The following is an example of Spontaneous Symmetry Breaking (SSB) for a U(1) theory. Consider the Lagrangian density

$$\mathcal{L}(x) = [D^{\mu}\phi(x)][D_{\mu}\phi(x)] - \mu^2|\phi(x)|^2 - \lambda|\phi(x)|^4 \tag{1.60}$$

with

$$D_{\mu}\phi(x) = [\partial_{\mu} + igA_{\mu}(x)]\phi(x) \tag{1.61}$$

and

$$\phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) + i\phi_2(x)] \tag{1.62}$$

This Lagrangian is invariant under the U(1) gauge transformations

$$\begin{aligned} \phi(x) &\to \phi^{'}(x) = \phi(x)e^{-iqf(x)} \\ \phi^*(x) &\to \phi^{*'}(x) = \phi^*(x)e^{iqf(x)} \\ A_{\mu}(x) &\to A^{'}_{\mu}(x) = A_{\mu}(x) + \partial_{\mu}f(x) \end{aligned} \tag{1.63}$$

So far the situation is treated in a purely classical manner. Thus $\phi(x)$ should be thought of as a classical field, and μ should not be interpreted as the mass of a particle (equation (1.60)).

According to classical mechanics the Hamiltonian density is given by

$$\mathcal{H}(x) = [\partial^0\phi^*(x)][\partial_0\phi(x)] + [\nabla\phi^*(x)][\nabla\phi(x)] + \mathcal{V}(\phi) \tag{1.64}$$

To ensure that the field has a minimum value, we require $\lambda > 0$. For constant $\phi(x)$ the two first terms of equation (1.64) disappear. Thus the minimum value of $\mathcal{H}(x)$ (and the total energy of the field) corresponds to the value of $\phi(x)$ which minimises $\mathcal{V}(x)$. With $\lambda > 0$ and $\mu^2 > 0$ the potential energy density of the field, $\mathcal{V}(\phi) = \mu^2|\phi(x)|^2 + \lambda|\phi(x)|^4$, has exactly one minimum (see figure 1.4). For $\mu^2 < 0$ the situation is very different - the potiential now has non-unique lowest energy state that we were looking for. The minima are given by [9]

$$\phi(x) = \phi_0 = \left(\frac{-\mu^2}{2\lambda}\right)^{1/2} e^{i\theta}, \quad 0 \leq \theta < 2\pi \tag{1.65}$$

where θ is a direction in the $\phi_1 - \phi_2$ plane. Seeing as the Lagrangian is invariant under the $U(1)$ phase

[8] Because the photons should still be massless and the other three bosons should not.

[9] Just set the derivative of $\mathcal{V}(\phi)$ equal to zero.

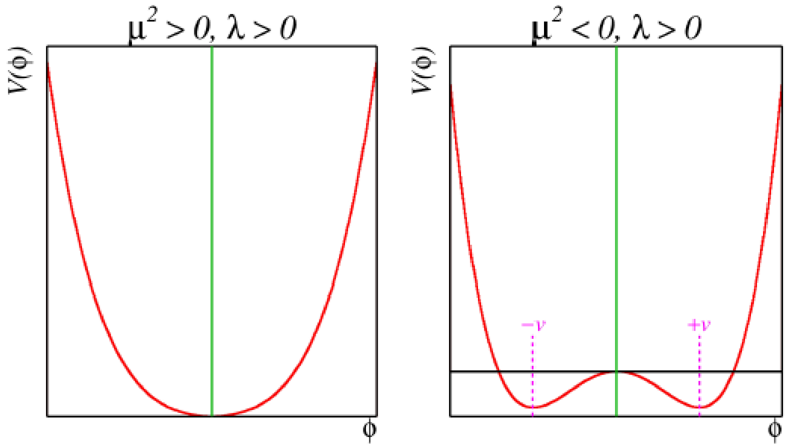


Figure 1.4: One minimum (left), one local maximum and indefinately many minima (right) for $\mathcal{V}(\phi)$ against ϕ.

transformations, the chosen value of θ is not significant, thus we are free to choose it at our convenience. $\theta = 0$ is the obvious choice such that

$$\phi_0 = \left(\frac{-\mu^2}{2\lambda}\right)^{1/2} = \frac{1}{\sqrt{2}}v, \quad v^2 = \frac{-\mu^2}{2\lambda} \tag{1.66}$$

We now rewrite $\phi(x)$ as a function of two new real fields $\sigma(x)$ and $\eta(x)$, which can be interpreted as deviations from the ground state ϕ_0:

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \sigma(x) + i\eta(x)) \tag{1.67}$$

The Lagrangian density of equation (1.60) then becomes

$$\begin{aligned}\mathcal{L}(x) = &\frac{1}{2}[\partial^\mu \sigma(x)][\partial_\mu \sigma(x)] - \frac{1}{2}(2\lambda v^2)\sigma^2(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) \\ &+ \frac{1}{2}(qv)^2 A_\mu(x)A^\mu(x) + \frac{1}{2}[\partial^\mu \eta(x)][\partial_\mu \eta(x)] + qvA^\mu(x)\partial_\mu \eta(x) \\ &+ cubic \quad and \quad quartic \quad terms \quad + \quad const\end{aligned} \tag{1.68}$$

Now we need to eliminate the scalar field $\eta(x)$. This can be done because for a complex field $\phi(x)$ there always exists a gauge choice (called the unitary gauge) that transforms it into a real field of the form

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \sigma(x)] \tag{1.69}$$

When this is introduced into the equation (1.60) it can easily be shown that the result is

$$\mathcal{L}(x) = \mathcal{L}_0(x) + \mathcal{L}_I(x) \tag{1.70}$$

where $\mathcal{L}_0(x)$ is made up of all quadratic terms

$$\mathcal{L}_0(x) = \frac{1}{2}[\partial^\mu \sigma(x)][\partial_\mu \sigma(x)] - \frac{1}{2}(2\lambda v^2)\sigma^2(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}(qv)^2 A_\mu(x)A^\mu(x) \tag{1.71}$$

and $\mathcal{L}_I(x)$ contains the rest, i.e. the higher-order interaction terms (the constant term has been dropped as it has no significance)

$$\mathcal{L}_I(x) = -\lambda v\sigma^3(x) - \frac{1}{4}\lambda\sigma^4(x) + \frac{1}{2}q^2 A_\mu(x)A^\mu(x)[2v\sigma(x) + \sigma^2(x)] \tag{1.72}$$

Now comes the main point: $\mathcal{L}_0(x)$ (equation (1.72)) can be interpreted as the free Lagrangian density of a real Klein-Gordon field ($\sigma(x)$) and a real massive vector field $A_\mu(x)$. These will, after quantization, give rise to bosons of mass $\sqrt{2\lambda v^2}$ and $|qv|$ respectively. This is the famous *Higgs mechanism*, and the ripples in the $\sigma(x)$ field is the *Higgs boson*.

The standard electroweak theory

The last section gave an example of the Higgs mechanism in which the photon actually gains a mass. We know that this is certainly not the case in our world. The SM incorporates this mechanism, but uses it on the full Lagrangian density, and the broken symmetry is $SU(2)_L \times U(1)_Y$ to give masses to the Z^0 and W bosons as well as to the fermions, while keeping the $U(1)_Y$ symmetry exact and therefore the photon massless.

The electroweak Lagrangian (excluding the quarks for simplicity) is made of two parts, $\mathcal{L} = \mathcal{L}^L + \mathcal{L}^B$, where the leptonic Lagrangian $\mathcal{L}^L$ is given by equation 1.32 and the bosonic lagrangian $\mathcal{L}^B$ describes how the gauge bosons interact when leptons are not present [23]. We now introduce a scalar field with a non-zero vacuum expectation value, a Higgs field that is not invariant under the $SU(2)_L \times U(1)_Y$ transformations. The $\phi(x)$ field is a doublet, as we want to break the $SU(2)_L$ symmetry

$$\phi(x) = \begin{pmatrix} \phi_a(x) \\ \phi_b(x) \end{pmatrix} \tag{1.73}$$

and where $\phi_a(x)$ and $\phi_b(x)$ are scalar fields.

We want to modify the Lagrangian to contain an additional part, a Higgs part, $\mathcal{L}^H$, in analogy with equation (1.60)

$$\begin{aligned} \mathcal{L}^H &= [D^\mu\phi(x)]^\dagger [D_\mu\phi(x)] - \mu^2\phi^\dagger(x)\phi(x) - \lambda[\phi^\dagger(x)\phi(x)]^2 \\ D^\mu\phi(x) &= [\partial^\mu + \frac{1}{2}ig\tau_j W_j^\mu(x) + ig' YB^\mu(x)]\phi(x) \end{aligned} \tag{1.74}$$

This new addition to the family must be $SU(2)_L \times U(1)_Y$ invariant (see for example [[14], p.289] for more information).

As before one finds the Higgs field of the vacuum state (see the discussion regarding equation (1.64))

$$\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad v = \sqrt{\frac{-\mu^2}{\lambda}} \tag{1.75}$$

which is not invariant under $SU(2)_L \times U(1)_Y$ transformations, but must be invariant under $U(1)_Y$ transformations alone since the photon is masseless. Then, in analogy with (1.67), we rewrite the Higgs field in terms of deviations from the vacuum

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1(x) + i\eta_2(x) \\ v + \sigma(x) + i\eta_3(x) \end{pmatrix} \tag{1.76}$$

Rewriting the Lagrangian, one finds that the Higgs part is a function of the four real fields $\sigma(x)$, $\eta_i(x)$, i=1,2,3.

One can specify $\phi(x)$ in the so-called unitary gauge (meaning a specific transformation, in this case first a $SU(2)$ then a $U(1)$ transformation) so that it becomes

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \sigma(x) \end{pmatrix} \tag{1.77}$$

implying that the three η_i fields are unphysical. The fourth, $\sigma(x)$, will on quantization give rise to a massive and electrically neutral scalar particle, the Higgs boson.

The whole Lagrangian must now be transformed into the unitary gauge, but before doing this we add the fermion masses by simply introducing another gauge-invariant term, $\mathcal{L}^{FH}$, to the Lagrangian

$$\mathcal{L} = \mathcal{L}^L + \mathcal{L}^B + \mathcal{L}^H + \mathcal{L}^{FH} \tag{1.78}$$

where the Lagrangian for fermions consists of a lepton part and a quark part. The lepton part is given by

$$\begin{aligned} \mathcal{L}^{LH} = -g_l[\bar{\psi}_l^L(x)\psi_l^R(x)\phi(x) + \phi^\dagger(x)\bar{\psi}_l^R(x)\psi_l^L(x)] - \\ g_v[\bar{\psi}_l^L(x)\psi_v^R(x)\widetilde{\phi}(x) + \widetilde{\phi}^\dagger(x)\bar{\psi}_v^R(x)\psi_l^L(x)] \end{aligned} \tag{1.79}$$

with g_l and g_v as dimensionless coupling constants and

$$\widetilde{\phi}(x) = -i[\phi^\dagger(x)\tau_2]^T \tag{1.80}$$

The situation is equivalent for quarks (up-type and down-type playing the role of the lepton and the neutrino) with the slight complication due to quark mixing.

The full Lagrangian now consists of four parts: the leptonic (L) and bosonic (B) parts, the Higgs part (H) and the interaction between the fermions and the Higgs field (FH).

After the transformation is complete (not shown, the interested reader is referred to chapter 14 of reference [14]), the Lagrangian contains mass terms for fermions and bosons, giving predictions for the masses of the Z^0 and W bosons

$$m_W = \sqrt{\frac{\alpha\pi}{G\sqrt{2}}}\frac{1}{sin\theta_W}, \quad m_Z = \sqrt{\frac{\alpha\pi}{G\sqrt{2}}}\frac{2}{sin2\theta_W} \tag{1.81}$$

where $\alpha \sim 1/137.036$ is the well-known fine structure constant [11], $G \sim 1.166 \times 10^{-5}$ is the fermi constant [11] and θ_w is the weak mixing angle ($sin^2\theta(M_Z^0) \sim 0.231$) [11]. Thus, the prediction for the masses to the first order is $m_W = 76.9\ GeV/c^2$, $m_Z = 87.9\ GeV/c^2$. When using higher order in perturbation theory one gets $m_W = 79.8 \pm 0.8\ GeV/c^2$ and $m_Z = 90.8 \pm 0.6\ GeV/c^2$ [14]. The experimental masses are 80.398(25) GeV/c^2 and 91.1876(21) GeV/c^2 respectively, making the predictions very good indeed.

The fermion masses are functions of the free parameters g from equation (1.79), $m_l = \frac{v_g}{\sqrt{2}}$ and the exact values are not predicted by the SM. Sadly this is also true for the Higgs scalar, whose mass is a function of μ (equation (1.75)), $m_H = \sqrt{-2\mu^2}$, and it is a free parameter of the theory.

1.1.6 Observation of a Higgs-like excess at the LHC

With the 4.8 fb^{-1} of integrated luminosity at $\sqrt{s} = 7\ TeV$ collected in 2011, both ATLAS [15] and CMS [16] reported excesses which were compatible with SM Higgs boson production and decay in the mass

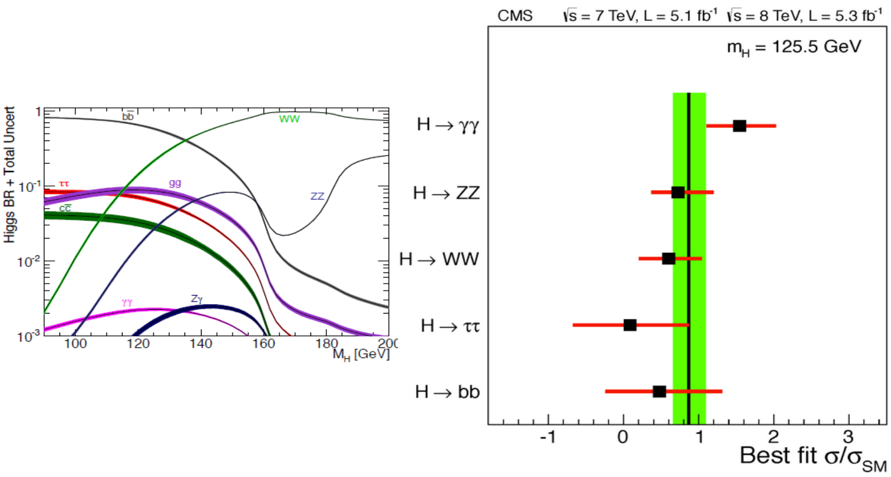


Figure 1.5: (Left) The branching ratios with which the SM Higgs decays with estimated theoretically uncertainties shown by the bands [20], [21]. (Right) Values of σ/σ_{SM} for the combination (solid vertical line) and for individual decay modes (points). The vertical band shows the overall σ/σ_{SM} value 0.87±0.23. The symbol σ/σ_{SM} denotes the production cross section times the relevant branching fractions, relative to the SM expectation. The horizontal bars indicate the ± 1 standard deviation uncertainties in the σ/σ_{SM} values for individual modes; they include both statistical and systematic uncertainties [18].

range $\sim$ 124-126 GeV/c^2, with significances of 2.9 and 3.1 standard deviations (σ) respectively. By the summer of 2012, the LHC had delivered about 5.3 fb^{-1} of integrated luminosity at $\sqrt{s} = 8\ TeV$. Combining results from searches for the Higgs boson with both the 2011 and 2012 datasets, on July 4, 2012, the ATLAS [17] and CMS [18] experiments independently announced the discovery of a new particle consistent with a SM Higgs boson with $m_H \sim 125\ GeV/c^2$, with significances of 5.9 and 5.8 σ respectively.

Through the Yukawa couplings and the EW interactions, the SM Higgs has many ways in which it can decay, especially within the preferred Higgs mass range $115 - 160\ GeV/c^2$. Figure 1.5 (left) shows a plot of the Higgs branching fractions as a function of the Higgs mass. Both the ATLAS and CMS experiments search for the Higgs boson in $\gamma\gamma$, ZZ^*, WW^*, $\tau\tau$ and $b\bar{b}$ decays. The significance of the excesses reported in the July 2012 observation is dominated by the $H \to \gamma\gamma$, $H \to ZZ^* \to 4l$ searches. The $H \to \gamma\gamma$, $H \to ZZ^* \to 4l$ channels fully reconstruct the decay products of the Higgs boson and have mass resolutions better than one percent. Figure 1.6 shows the reconstructed Higgs mass distributions for the $H \to \gamma\gamma$ (left) and $H \to ZZ^* \to 4l$ (right) searches.

Figure 1.7 shows the confidence-level (CL) values for the SM Higgs boson hypothesis as a function of the Higgs boson mass in the range 110-145 GeV/c^2 (left) and the observed local p-value for 7 TeV and 8 TeV data as a function of the SM Higgs boson mass (right).

The CMS $H \to \gamma\gamma$, $H \to ZZ^* \to 4l$, having precise mass resolution, are combined to measure the mass of the excess, giving a best-fit mass of $m_H = 125.3 \pm 0.4(stat.) \pm 0.5(syst.) GeV/c^2$.

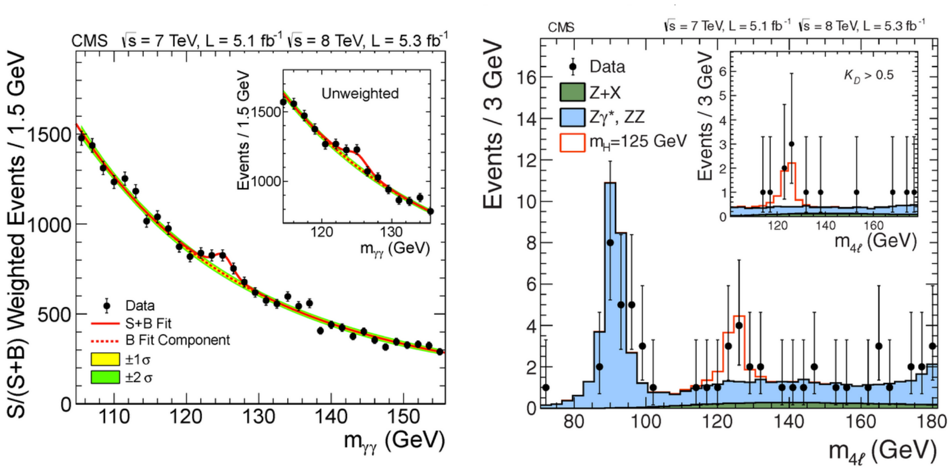


Figure 1.6: (Left) The diphoton invariant mass distribution with each event weighted by the S/(S+B). The lines represent the fitted background and signal, and the coloured bands represent the ±1 and ±2 standard deviation uncertainties in the background estimate. The inset shows the central part of the unweighted invariant mass distribution. (Right) Distribution of the four-lepton invariant mass for the $ZZ \to 4l$ analysis. The points represent the data, the filled histograms represent the background, and the open histogram shows the signal expectation for a Higgs boson of mass $m_H = 125\ GeV/c^2$, added to the background expectation. The inset shows the m_{4l} distribution.

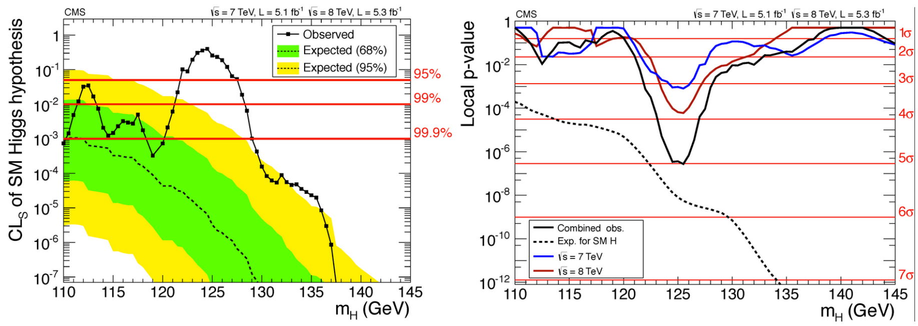


Figure 1.7: (Left) The CLs values for the SM Higgs boson hypothesis as a function of the Higgs boson mass in the range 110-145 GeV/c^2. The background-only expectations are represented by their median (dashed line) and by the 68% and 95% CL bands. (Right) The observed local p-value for 7 TeV and 8 TeV data as a function of the SM Higgs boson mass. The dashed line shows the expected local p-values for a SM Higgs boson with mass m_H.

1.2 Limits and extensions of the Standard Model

1.2.1 Shortcomings of the Standard Model

Although the SM is a very successful theory, it is believed not to be an ultimate theory of nature but rather a low-energy remnant of some more fundamental theory. The reasons are twofold: both theoretic and experimental. From a theoretical point of view there is a number of problems which either remain unsolved or the solutions incorporated in the SM do not have any fundamental justifications:

The hierarchy problem. It arises from the fact that the weak scale ($M_{weak} \sim 100\ GeV/c^2$) and the Planck scale ($M_P \sim 10^{19}\ GeV/c^2$) differ by 17 orders of magnitude. The Higgs mass is quadratically divergent when one loop of self-interactions of the Higgs boson is considered. For these divergences to be cancelled an additional mass counterterm, δm_h^2, needs to be introduced. At the lowest order in perturbation theory, the Higgs mass is $m_H^2 = m_0^2 + \delta m_H^2 \sim m_0^2 - g^2\lambda^2$ where m_0 is the "ground" Higgs mass, g is a dimensionless coupling constant, and λ is the energy scale. Taking into account Higgs mass (see section 1.1.6), and assuming that $g \sim 1$ and λ is around the Planck scale, then m_0 must be adjusted so that $m_0^2 - g^2\lambda^2 \sim m_H^2$. This requires a precise adjustment of the SM parameters and it is referred to as the fine-tuning problem.

The unification problem. The three gauge groups within the SM, $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$, are associated with individual running coupling constants. It was believed that they converge at some scale, but precision measurements have shown that this is not the case within the SM [29].

Unanswered questions. Why $SU(3) \times SU(2) \times U(1)$? Why $Q_e = -Q_p$? Why do exactly three families of fermions exist? Is it a coincidence that within every family the fermions which carry more charge (strong, electromagnetic, weak) have larger masses? Are baryon number and lepton number strictly conserved (not supported by any known underlying symmetry)? What is the origin of P violation? What is the origin of CP violation? What is the origin of the mixture of lepton families, described by the Pontecorvo-Maki-Nakagava-Sakata matrix? What is the origin of the mixture of quark families, described by the Cabibbo-Kobayashi-Maskawa matrix? Why are there just four interactions? What determines the magnitudes of the coupling constants of the different interactions? Is it possible to unify the strong and electroweak interactions, as one has unified the electromagnetic and weak interactions? Will it be possible to include gravity in a complete unification?

From an experimental point of view there are a number of phenomena which are not explained within the SM framework:

Neutrino masses. The neutrinos are massless within the SM. However experiments with solar and atmospheric neturinos have shown that the three generations of neutrinos mix with each other [26]. This would not be possible if their mass were exactly zero. The most current limit on the neutrino masses is $m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau} < 0.28\ eV/c^2$ (95 %CL) [27].

Dark matter and dark energy. Cosmological observations, such as the cosmic microwave background and the structure and movements of galaxies, show that the energy content of the universe consists of roughly 5% baryonic matter, 25% dark matter and 70% dark energy [28]. The SM describes only the baryonic matter.

Gravity. The fourth type of interaction, gravity, is not incorporated in the SM. At the electroweak scale, gravity is so weak to be negligible. The scale at which effects of quantum gravity are expected to become important is of the order of $10^{19}\ GeV$ and is referred to as the Planck scale, M_p.

Baryogenesis. The baryon-anti-baryon asymmetry observed in the universe is not explained by the

SM. It does not offer a satisfactory explanation for the matter-dominated Universe we observe. CP-violation incorporated in the CKM matrix is indeed not enough.

1.2.2 Models beyond the Standard Model

In attempts to resolve the problems outlined in section 1.2.1, several Beyond the SM (BSM) theories have been developed. General concepts involve:

Unification of Gauge Interactions. In Grand Unified Theories (GUTs) [2] one imagines that the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry we have today in the SM originates from a larger symmetry group relating quarks and leptons. This addresses the unification problem: the symmetry is unbroken at some higher energy where all interactions are described by a local gauge theory with one running coupling. The scale which this happens is thought to be 10^{16} GeV. In most of these models the electroweak symmetry is broken by introducing the Higgs field.

Extra-dimensions. The idea is that the four-dimensional world we live in, is embedded in a higher dimensional space. Since extra dimensions have not been detected so far, they have to be either compactified as in the N.Arkani-Hamed, S.Dimopoulos, and G.Dvali (ADD) [3] and Universal Extra Dimension (UED) models or have a strong curvature, which makes it hard to escape into them as in the Randall-Sundrum (RS) model [4]. In these models, gravity can propagate in the extra dimensions and SM particles therefore experience only a small fraction of the total gravitational force. In this way, the hierarchy problem is solved because the fundamental scale of gravity (and therefore the ultimate limit up to which the SM is valid) lies around the TeV region. In scenarios where SM particles are allowed to propagate in extra dimensions (such as in the UED model), for every SM particle there is a series of particles, the so-called Kaluza-Klein (KK) excitations. The lightest of these KK modes is stable and it is therefore a good candidate for Dark Matter [28].

Modification of existing particles and their interactions. An example is String Theory, which assumes particles to be string-like and interactions to be extended, rather than point-like. This theory naturally incorporates gravity. Another example is provided by composite models in which quarks and leptons are built of more fundamental constituents.

Supersymmetry. Supersymmetry (SUSY) [5] is a theory which postulates the existence of a symmetry between fermions and bosons. It predicts that for every SM particle, there exist a supersymmetric partner with the same mass but with a spin differing by 1/2. Since such particles have not yet been discovered, the symmetry must be broken at some scale. SUSY offers a solution to the hierarchy problem - the correction loops that should have made Higgs very heavy are now cancelled by the SUSY partners. Models that conserve R-parity [10] have a stable Lightest Susy Particle (LSP), providing a candidate for Dark Matter. The most popular model is the Minimal Supersymmetric SM (MSSM) [6].

Dynamical symmetry breaking. The dynamic approach to electroweak symmetry breaking has been developed in order to eliminate elementary Higgs boson. The idea was motivated by the premise that every fundamental energy scale should have a dynamical origin and thus the weak scale should reflect the characteristic energy of a new strong interaction called technicolour. Technicolour (TC) and Extended Technicolour (ETC) models are asymptotically free gauge theories of fermions with no elementary scalars. The electroweak symmetry is dynamically broken through the new, technicolour interaction [30].

[10] All SM particles are given a R-parity of 1, while the superpartners have R-parity -1. When the R-parity is conserved the lightest supersymmetric particle cannot decay.

The goal of experimental physics is to verify the correctness of the models or alternatively to put constraints on the Beyond the SM (BSM) theories. Many hundreds of models, however, can be constructed in some way that allows them to pass the electroweak precision tests. One can vary the number, the size and the shape of the extra dimension, alter the rank of an extra gauge group, impose sypersymmetry or add explicitly extra particles. A generic feature of most SM extensions is the existence of new heavy particles. The general names given to them are $W^{'}$ for electrically charged gauge bosons and $Z^{'}$ for neutral ones.

1.2.3 New extra neutral gauge bosons

$Z^{'}$ particles can be classified into two broad categories, depending on whether they arise in GUT scenarios or not [7]. The few examples given in this section are meant to illustrate the various possibilities rather then to provide an exhaustive, fully exclusive list of models. For a detailed review of the models the reader is referred to [7], [31], [32].

GUT models. In Grand Unified Theories strong and electroweak interactions are merged into a single interaction, described by a higher symmetry group. The choice of symmetry group, however, varies. The simplest one,

$$SO(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y = SM \tag{1.82}$$

predicts rapid proton decay [34]. Experiments however have established that the lifetime of the proton is at least $10^{31} - 10^{33}$ seconds [35] thereby invalidating SU(5)-based theories [36]. It appears evident that a rank higher than 5 is needed. The two most popular scenarios with $SU(N), N > 5$, are the E_6 and the Left Right symmetric Model (LRM) models ([7], [37]). In the E_6 scenario, the symmetry is broken down into the following pattern:

$$E_6 \to SO(10) \times U(1)_\psi \to SU(5) \times U(1)_\chi \times U(1)_\psi \to SM \times U(1)_{\theta_{E_6}} \tag{1.83}$$

where $U(1)_{\theta_{E_6}}$ is believed to be broken at the TeV scale [33]. Only one linear combination remains light

$$Z^{'} = Z^{'}_\chi cos_{\theta_{E_6}} + Z^{'}_\psi sin_{\theta_{E_6}} \tag{1.84}$$

where θ_{E_6} is a free parameter in the range $-90^\circ \leq \theta_{E_6} \leq 90^\circ$. Four popular models are $Z^{'}_\chi$ ($\theta_{E_6} = 0$), $Z^{'}_\psi$ ($\theta_{E_6} = 90^\circ$), $Z^{'}_\eta$ ($\theta_{E_6} = sin^{-1}(\sqrt{3/8}) \sim 37.76^\circ$) and $Z^{'}_I$ ($\theta_{E_6} = sin^{-1}(\sqrt{5/8})$).

The LRM is based on the group SO(10). It breaks down via the following chain:

$$SU(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \tag{1.85}$$

In this model the electric charge of a particle is given by

$$Q = I_{3L} + I_{3R} + \frac{1}{2}(B-L) \tag{1.86}$$

where I_{3L} and I_{3R} are the weak isospin components of the fields and $B-L$ is the baryon number minus the lepton number. Comparing to equation (1.40) which connects electric charge to weak hypercharge and isospin in the SM, we see that in this theory $Y = I_{3R} + \frac{1}{2}(B-L)$. This model gives rise to two new gauge bosons, one charged and one neutral, that both couple to right handed quarks and leptons. In addition, a Dark Matter candidate is created - the right-handed neutrino [38].

Stueckelberg models. In the Stueckelberg extension [39] of the SM particle, masses are generated without the Higgs mechanism. This is done by adding a new kinetic term for a massive $U(1)$ gauge field to the Lagrangian, i.e. it is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. After symmetry breaking a second Z boson emerges, namely the $Z^{'}$. A prototype Stueckelberg Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}(mC^{\mu} + \partial^{\mu}\sigma)(mC_{\mu} + \partial_{\mu}\sigma) \tag{1.87}$$

where C_{μ} is a massive gauge boson, $F_{\mu\nu}$ is the field strength tensor, σ is a pseudoscalar field, known as axion[11]. Requiring

$$\delta C_{\mu} = \partial_{\mu}\varepsilon, \quad \delta\sigma = -m\varepsilon \tag{1.88}$$

makes the lagrangian gauge-invariant. By adding a gauge-fixing term,

$$-\frac{1}{2\xi}(\partial_{\mu}C^{\mu} + \xi\sigma)^2 \tag{1.89}$$

the total Lagrangian becomes

$$\mathcal{L}_{Stu} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{m^2}{2}C_{\mu}C^{\mu} - \frac{1}{2\xi}(\partial_{\mu}A^{\mu})^2 - \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - \xi\frac{m^2}{2}\sigma^2 \tag{1.90}$$

The Stueckelberg mechanism is only compatible with Abelian gauge symmetries. Consequently, in the minimal Stueckelberg extension of the SM (StSM), a new $U(1)_X$ group must be added in order for the Abelian factors (Y and X) to be able to couple to the real pseudoscalar, σ.

It can be shown [41] that an analogous

$$\mathcal{L}_{Stu} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + g_X C_{\mu}J^{\mu}_X - \frac{1}{2}(\partial_{\mu}\sigma + M_1C_{\mu} + M_2B_{\mu})^2 \tag{1.91}$$

in which M_1 and M_2 ($M_1/M_2 = \varepsilon$) are the mass parameters of the axion, can be added to the well-known SM Lagrangian without compromising renormalisability. A similar Stueckelberg-type extension can be applied to the MSSM. Furthermore, Stueckelberg interactions can appear in extra-dimensional theories (e.g., Kaluza-Klein model, string theories).

Sequential Standard Model. This model includes a neutral gauge boson $Z^{'}_{SSM}$ with the same couplings to quarks and leptons as the SM Z^0 boson, and it decays only to the three known families of fermions. Such a model is not gauge-invariant [37] unless it has different couplings to exotic fermions, or if it occurs as an excited state of the ordinary Z^0 in models with extra dimensions at the weak scale [32]. However, it is a useful reference case when comparing constraints from various sources, and is traditionally used in the experimental analysis. The $Z^{'}$ decay width is equal to the width of the SM Z^0 boson scaled by a factor of $M_{Z'}/M_{Z^0}$:

$$\Gamma_{Z'} = \Gamma_Z \times M_{Z'}/M_{Z^0} \tag{1.92}$$

Models with non-universal couplings. A number of models are characterised by $Z^{'}$ bosons with non-universal couplings favoring the third generation, i.e., they lead to larger branching ratios to top quarks and τ leptons. Some of these models are: *topcolour*, *non-universal extended technicolour*, and string-theory derived $Z^{'}$ models (Kaluza-Klein excitations in extra-dimensional theories in which the fermion families are spatially separated) [32].

The couplings of a $Z^{'}$ boson to the first-generation of fermions are given by

$$Z^{'}_{\mu}(g^L_u\bar{u}_L\gamma^{\mu}u_L + g^L_d\bar{d}_L\gamma^{\mu}d_L + g^R_u\bar{u}_R\gamma^{\mu}u_R + g^R_d\bar{d}_R\gamma^{\mu}d_R + g^L_{\nu}\bar{\nu}_L\gamma^{\mu}\nu_L + g^L_e\bar{e}_L\gamma^{\mu}e_L + g^R_e\bar{e}_R\gamma^{\mu}e_R) \tag{1.93}$$

[11] A hypothetical elementary particle appearing in the *Peccei – Quinn* theory. Under very specific mass conditions, axions are cold dark matter candidates [40]

where u, d, ν and e are the quark and lepton fields in the mass eigenstate basis, and the coefficients $g_u^L, g_d^L, g_u^R, g_d^R, g_\nu^L, g_e^L, g_e^R$ are real dimensionless parameters. If the Z' couplings to quarks and leptons are generation-independent, then these seven parameters describe the couplings of the Z' boson to all SM fermions. In models with non-universal couplings that represent the most general case, the Z' couplings to fermions are generation-dependent, in which case equation (1.93) may be written with generation indices $i, j = 1,2,3$ labelling the quark and lepton fields, and with the seven coefficients promoted to 3×3 Hermitian matrices (e.g., $g^L_{e_{ij}}\bar{e}^i_L\gamma^\mu e^j_L$, where e^2_L is the left-handed muon, etc.) [42].

1.3 New extra neutral gauge bosons at the Large Hadron Collider

1.3.1 Production cross-section

The primary discovery mode for a Z' at a hadron collider is the resonant Drell-Yan production of opposite-sign pairs of leptons. For this to happen, the beam energy has to be greater than $M_{Z'}$. For the LHC, the reaction to look for is:

$$pp \to l^+l^-X \tag{1.94}$$

or, in the parton language,

$$q\bar{q} \to Z' \to l^+l^- \tag{1.95}$$

with the invariant mass of the lepton pairs peaked at $M_{Z'}$. If the Z' couples to all the fermions of the SM, then it can decay into pairs of any of them. In general, the $q\bar{q}$ pair in (1.95) is not at rest in the centre of mass reference of the colliding protons, thus resulting in a continuous spectrum for the rapidity, y, of the Z' w.r.t. the beam axis. Therefore, it makes sense to consider the differential cross-section per unit rapidity for the production of a Z'. The rapidity of the intermediate boson is equal to that of the final fermions (because energy and momentum are conserved), so that it can be defined as:

$$y = \frac{1}{2} ln \frac{\sum E_i + \sum p_i^L}{\sum E_i - \sum p_i^L} \tag{1.96}$$

If the four-momenta of the interacting quarks in protons A and B are a fraction x_1 and x_2 of the respective protons, then the resonant condition for them to produce a Z' on the $mass-shell$ is

$$x_1 x_2 s = Q^2 = M^2_{Z'} \tag{1.97}$$

where $\sqrt{s}/2$ is the beam energy. Conditions (1.96) and (1.97) imply that

$$x_{1,2} = \frac{M_{Z'}}{\sqrt{s}} e^{\pm y} \tag{1.98}$$

The Born cross-section for the reaction (1.94) is the folding of the parton cross-sections for the underlying process (1.95) (where any intermediate vector boson should be considered) with the parton density functions of the colliding protons A and B:

$$\sigma_T^{ll} = \sum_q \int_0^1 dx_1 \int_0^1 dx_2 \sigma(sx_1x_2; q\bar{q} \to l^+l^-)(f_q^A(x_1) f_{\bar{q}}^B(x_2) + f_{\bar{q}}^A(x_1) f_q^B(x_2)) \tag{1.99}$$

where f_q and $f_{\bar{q}}$ are the structure functions of the quark and antiquark in the proton, evaluated at $Q^2 \sim M^2_{Z'}$. Now, if we switch from (x_1, x_2) to (Q^2, y) using (1.98), and we approximate $\sigma(sx_1x_2; q\bar{q} \to l^+l^-)$ with a a Breit-Wigner:

$$\sigma(Q^2; q\bar{q} \to l^+l^-) \sim \frac{12\pi}{N_C^2} \frac{Q^2}{M^2_{Z'}} \frac{\Gamma^2_{Z'} Br^q Br^l}{(Q^2 - M^2_{Z'})^2 + \Gamma^2_{Z'} M^2_{Z'}} \tag{1.100}$$

it is then possible to simplify the integral in Q^2 using the narrow width approximation ([43] and [44]). Finally, equation (1.99) becomes:

$$\sigma_T^{ll} = \frac{4\pi^2}{3s}\frac{\Gamma_{Z'}^2}{M_{Z'}^2} Br^q Br^l f(\tau) \tag{1.101}$$

where $\tau = \frac{\sqrt{s}}{M_{Z'}}$ and $f(\tau)$ is the parton luminosity. A good approximation for $f(\tau)$ up to $\tau < 10$ has been found to be $f(\tau) = Ce^{-A/\tau}$ [31], with $C \sim 600$ and $A \sim 32$ for pp collisions. With this choice we have:

$$\sigma_T^{ll} = \frac{N_{Z'}}{L} \sim \frac{1}{s} c_{Z'} C e^{-A\frac{M_{Z'}}{\sqrt{s}}} \tag{1.102}$$

where L is now the integrated luminosity collected by the experiment. In equation (1.102), all the dependence on the mass and couplings of the $Z^{'}$ to fermions is included in $c_{Z'}$. Typical values for $c_{Z'}$ for the canonical models range from 0.6×10^{-3} to 2.3×10^{-3}. Equation (1.102) is useful to estimate the sensitivity of an experiment to $M_{Z'}$: suppose that $N_{Z'}$ di-lepton events from a $Z^{'}$ decay are necessary to have a significant signal. Then, for a given L and $\sqrt{s}$, $M_{Z'}$ must not exceed $M_{Z'}^{lim} \sim \sqrt{s}\{0.386 + \frac{1}{32} ln(\frac{L}{N_{Z'}s}\, 1000\, c_{Z'})\}$ [31]. For the LHC, choosing $L = 100 fb^{-1}$, $N_{Z'} = 10$ and $\sqrt{s} = 14\ TeV$, we find that $M_{Z'}^{lim} \sim 4.5\ TeV/c^2$ for SSM model and $3 < M_{Z'}^{lim}(TeV/c^2) < 5$ for all E_6 models. If the $Z^{'}$ resonance will be found in the l^+l^- channel at the LHC, then its mass and width will be measured by a fit to the invariant mass distribution of the lepton pairs, giving model independent information about the new neutral boson. If the width of the new $Z^{'}$ were much smaller than the resolution on the di-lepton invariant mass, then only $M_{Z'}$ can be reasonably measured. If a new resonance is found, the next step would be to establish its spin-1 nature [32]. The more direct way would be to check the $(1 + cos\theta_{CM})$ angular distribution w.r.t. the beam axis for the final leptons, which is typical of a spin-1 decay and insensible to the left-right direction of the initial quark.

1.3.2 Forward-backward and polarization asymmetry of $Z^{'}$

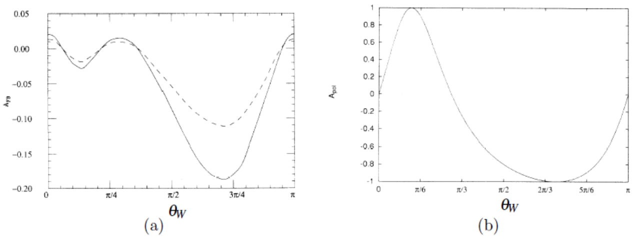
Another important aspect of a new particle is the study of couplings to the fermions. This is an important tool in establishing the model in which the new boson belongs. In this perspective, a useful observable is the forward-backward asymmetry

$$\hat{A}_{FB} = \frac{\hat{\sigma}^{qf}(cos\theta > 0) - \hat{\sigma}^{qf}(cos\theta < 0)}{\hat{\sigma}^{qf}(cos\theta > 0) + \hat{\sigma}^{qf}(cos\theta < 0)} \tag{1.103}$$

At the $Z^{'}$ peak, neglecting the small contributions from γ and Z exchange, this asymmetry can be expressed as [45]:

$$\hat{A}_{FB} = \frac{3}{4} \underbrace{\frac{(g_L^q)^2 - (g_R^q)^2}{(g_L^q)^2 + (g_R^q)^2}}_{A_q} \underbrace{\frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}}_{A_f} \tag{1.104}$$

where g_L and g_R are the neutral current chiral couplings, related to the vector and axial-vector couplings through the relations $g_L = g_V + g_A$ and $g_R = g_V - g_A$. The A_q and A_f are hence called the initial-state and the final-state fermion polarization (chiral) asymmetries. At hadronic colliders the forward-backward asymmetry, A_{FB}, depends on the initial state quarks momenta. The $\hat{A}^{FB}$ therefore needs to be convoluted with the parton density functions as explained in Sect. 1.3.1. The asymmetry A_{FB} is, thus, a complicated function of the couplings of the $Z^{'}$ to both leptons and quarks and of the structure functions of the proton. Furthermore for the A_{FB} to be a measurable quantity at symmetric proton-proton collisions, one needs to



(a) (b)

Figure 1.8: The forward-backward asymmetry (a) and the polarization asymmetry (b) as a function of the mixing angle θ_W defined in section 1.1.3. In plot (b) A_{pol} stands for A_f. The dashed and the solid lines in (a) correspond to a lepton rapidity cut $|\eta| < 2.5$ and $|\eta| < 5.0$ respectively. The plots were made assuming the Z' to be produced at 40 TeV (centre-of-mass energy) pp collider [49].

assume that the quarks (since being harder than $\bar{q}$) are travelling in the same direction as the Z' [46]. The measurable asymmetry is therefore somewhat washed out. Additional information is available for decay channels in which the helicities of the final state fermions are experimentally accessible. At the Z' peak, the polarization asymmetry A_f and the longitudinal polarization, P_f , of fermion f are connected through the relation:

$$A_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2} = -P_f \tag{1.105}$$

Measurement of A_f , which depends only on couplings to the final state fermions, does not involve any knowledge of the proton structure function. In theories where more than one extra gauge boson is predicted, a single measurement of its cross section is not enough to determine its coupling strength to fermions. In the E_6 scenario, for example, the Z' couplings depend on a mixing angle θ_{E_6} (see equation 1.84), which is a free parameter of the theory. This parameter can be determined by measuring the forward-backward or the polarization asymmetry (in the $Z' \to \tau\tau$ events) as illustrated in figure 1.8. Note that A_f is a much steeper function of θ_{E_6} for most of the mixing angle range and therefore a measurement of A_f will result in a better determination of θ_{E_6} than an equally precise measurement of A_{FB} [47] would.

1.3.3 The Z' width and decay channels

In the absence of any exotic decay channels, the total decay width of the Z' is given by the sum of partial decay widths to neutrino, leptons and quarks of all three generations. Decays such as $Z' \to W^+W^-$ are expected to be rare [48] and therefore neglected in the current discussion. The partial width of the Z' decaying into a pair of fermions is given (at the lowest order assuming the fermions to be massless) by [45]:

$$\Gamma_{Z'} = \frac{1}{12\pi} M_{Z'} g_{Z'}^2 N_c^q [(g_V^f)^2 + (g_A^f)^2] \tag{1.106}$$

Z' width is typically of the order of 3% of the mass of the resonance. If exotic decay modes are kinematically allowed, the Z' width becomes larger and, more significantly, the branching ratios to conventional fermions smaller. Considering only decays into known particles, observation of extra gauge bosons in a proton-proton collider may be possible in the following channels:

e^+e^- **or** $\mu^+\mu^-$ **channel.** $Z' \to e^+e^-$ and $Z' \to \mu^+\mu^-$ decays have the highest sensitivity, due to the lowest backgrounds. A high mass resolution in these two channels enables measurement of the Z' width.

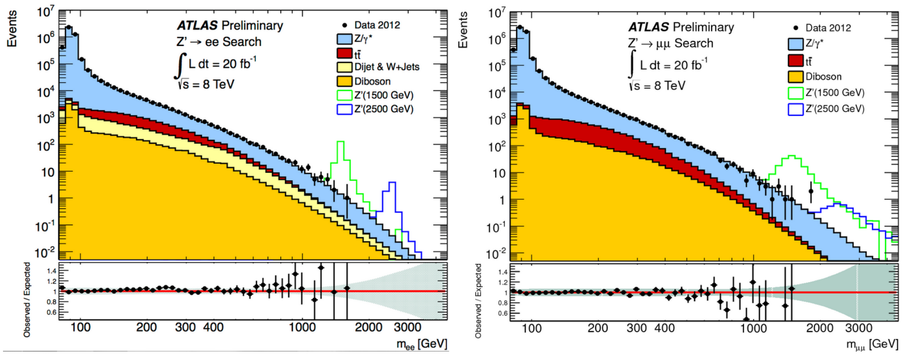


Figure 1.9: (Left) Dielectron invariant mass (m_{ee}) distribution with statistical uncertainties after final selection, compared to the stacked sum of all expected backgrounds, with two selected $Z^{'}$ signals overlaid. The SSM bin width is constant. Bottom inset: The black points show the ratio of observed to expected events with statistical uncertainty, while the shaded band indicates the mass-dependent systematic uncertainty on the sum of the backgrounds. (Right) Ditto, for $m_{\mu\mu}$.

Model	$Z^{'}_{SSM}$	$Z^{'}_{\psi}$	$Z^{'}_{\chi}$
Limit (TeV/c^2)	2.86	2.38	2.54

Table 1.3: Upper limits, in TeV/c^2, on various $Z^{'}$ models at 95% CL for ATLAS experiment [50].

$\tau^+\tau^-$ **channel.** The $\tau^+\tau^-$ channel has a considerably larger background than e^+e^- or $\mu^+\mu^-$ final states do. It also yields an invariant mass resolution not good as in the other leptonic channels, due to the neutrinos into the final state. Nevertheless, it enables the extraction of further information by measuring the polarization asymmetry A_f. Moreover, this channel enables observation of a $Z^{'}$ with enhanced couplings to a third generation of fermions.

$t\bar{t}$ **or** $b\bar{b}$ **channel.** Studies of the heavy flavour decay modes $Z^{'} \to t\bar{t}$ or $Z^{'} \to b\bar{b}$ have shown low sensitivity due to overwhelming QCD background.

$q\bar{q}$ **channel.** QCD backgrounds are far too large to allow observation of a $Z^{'}$ decaying into light quark pairs.

1.3.4 Experimental bounds on $Z^{'}$ from direct searches at LHC

Theories which postulate the existence of extra gauge bosons do not predict the masses of these particles. Several collider experiments have, however, put experimental constraints on their masses. In this subsection we show the most updated limits on the masses of new $Z^{'}$-like particles derived from direct production at the LHC. We show results for $Z^{'} \to e^+e^-$ and $Z^{'} \to \mu^+\mu^-$ decays that have the highest sensitivity, both from ATLAS [50] and CMS [51] experiments.

Figure 1.9 shows the invariant mass spectrum of ee (left) and $\mu\mu$ (right) events for $Z^{'}$ searches at ATLAS, while figure 1.10 shows the upper limits as a function of resonance mass M on the production ratio R_σ of cross section times branching fraction into lepton pairs for the ATLAS experiment. Results are summarised in table 1.3.

Figure 1.11 shows the invariant mass spectrum of ee (left) and $\mu\mu$ (right) events for $Z^{'}$ searches

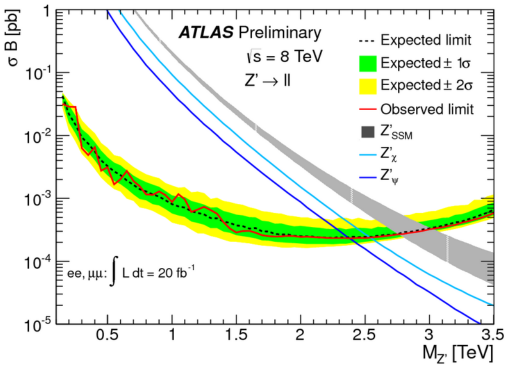


Figure 1.10: Median expected (dashed line) and observed (solid red line) 95% CL limits on σB and expected σB for $Z^{'}$ production and the two E_6-motivated $Z^{'}$ models with lowest and highest σB for the combination of the dielectron and dimuon channels. These limits are conservative for the E_6-motivated $Z^{'}$ models due to their narrower intrinsic width. The inner and outer bands show the range in which the limit is expected to lie in 68% and 95% of pseudo-experiments, respectively. The thickness of the $Z^{'}$ SSM theory curve represents all theoretical uncertainties and holds for the other theory curves.

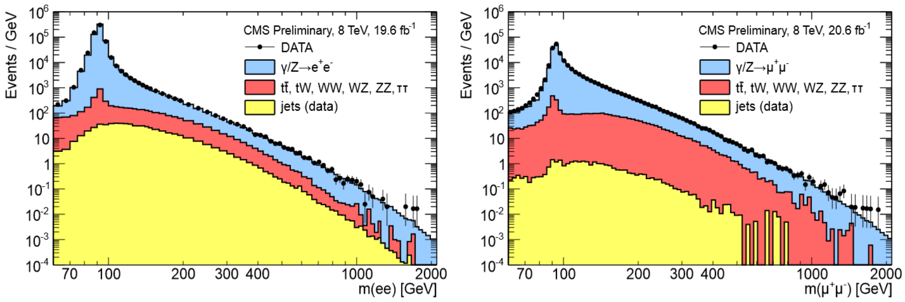


Figure 1.11: (Left) The invariant mass spectrum of ee events. The points with error bars represent the data. The histograms represent the expectations from standard model processes: Z/γ^{*}, $t\bar{t}$ and other sources of prompt leptons (tW, diboson production, $Z \to \tau\tau$), and multi-jet backgrounds. Multi-jet backgrounds contain at least one jet that has been misreconstructed as a lepton. The Monte-Carlo (MC) simulated backgrounds are normalised to the data in the region of $60 < m(ll) < 120\ GeV/c^2$, with the electron channel using events collected with a pre-scaled lower threshold trigger for this purpose. (Right) Ditto, for $m_{\mu\mu}$.

at CMS, and figure 1.12 shows the upper limits as a function of resonance mass M on the production ratio R_σ of cross section times branching fraction into lepton pairs for the CMS experiment. Results are summarised in table 1.4.

1.4 Summary

In this chapter we have introduced the SM of particle physics and we have discussed its shortcomings. It turns out that many of the extensions proposed to overcome its limits imply the existence of new particles.

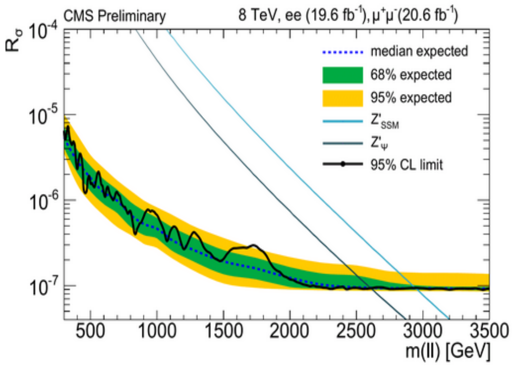


Figure 1.12: Upper limits as a function of resonance mass M on the production ratio $R\sigma$ of cross section times branching fraction into lepton pairs for Z'_{SSM} and Z'_{χ} boson production to the same quantity for Z bosons for the combined dilepton final state. Shaded green and yellow bands correspond to the 68% and 95% quantiles of the expected limits.

Model	Z'_{SSM}	Z'_{ψ}
Limit (TeV/c^2)	2.96	2.6

Table 1.4: Upper limits, in TeV/c^2, on various Z' models at 95% CL for CMS experiment [51].

New charged particles are generically referred to as W', while new neutral ones as Z'. $Z' \rightarrow e^+e^-$ and $Z' \rightarrow \mu^+\mu^-$ decays have the highest sensitivity, due to the lowest backgrounds. However, $Z' \rightarrow \tau\tau$ decay enables observation of a Z' with enhanced couplings to third generation of fermions and it also enables extracting important information by measuring the polarization asymmetry A_f. For these reasons $Z' \rightarrow \tau\tau$ channel becomes crucial in understanding the properties of any new hypothetical neutral particle, wheter it is discovered first or not in another final state. This motivates the $Z' \rightarrow \tau\tau$ search that is the main subject of this thesis and will be detailed in the next sections.

Chapter 2

The Large Hadron Collider and the Compact Muon Solenoid

In this chapter, after a short introduction to the Large Hadron Collider (LHC) [9] accelerator, we describe the Compact Muon Solenoid (CMS) [8] experiment. Emphasis will be put on the main subdetectors used for particle reconstruction and identification, and a brief description of the trigger and data acquisition system will be given. In the final section we discuss the baselines of CMS computing and software.

2.1 The Large Hadron Collider

The LHC ([9]) machine is the most powerful hadron collider ever built by man for proton-proton (pp) and heavy ion (HI) collisions. It was built at CERN in Geneva, Switzerland, at an underground depth of 100 m, using the tunnel 27 Km in circumference that was formerly used by LEP (Large Electron Positron) collider (see Fig.2.1).

The LHC accelerator can produce collisions between proton beams of up to $\sqrt{s} = 14\ TeV$. This energy in the centre of mass frame is reached with subsequent accelerations. First, a proton current (about 250 mA) is generated from the source and reaches an energy of 50 MeV through a linear accelerator. In succession, these protons are accelerated by the PSB (Proton Synchroton Booster) up to 1.4 GeV, by the PS (Proton Synchrotron) up to 26 GeV and by the SPS (Super Proton Synchrotron) up to 450 GeV. Protons are then injected into the LHC ring where 2 beams rotate in opposite directions inside vacuum pipe lines and are further accelerated by radio-frequency cavities up to 7 TeV.

The circular trajectory is kept by about 8000 superconducting dipole magnets with magnetic field of ~8.4 T that are located along the circumference of the ring and cooled with superfluid helium down to 1.9 K.

The instantaneous luminosity $\mathcal{L}$ at the collision points is given by [55]

$$\mathcal{L} = \frac{\gamma f k_B N_p^2}{4\pi\varepsilon_n\beta^*}F \tag{2.1}$$

where $\gamma = E_{beam}/m_p$, f is the bunch frequency, k_B is the number of bunches per beam, N_p is the number of protons per bunch, F is a reduction factor due to non-π intersecting angle, ε_n is the normalized transverse beam emittance, and β^* is the beta function at the IP. By construction, the rate of events $\frac{dN}{dt}$ (Hz) expected from a process with cross section σ (cm^2) is given by

$$\frac{dN}{dt} = \mathcal{L}\sigma \tag{2.2}$$