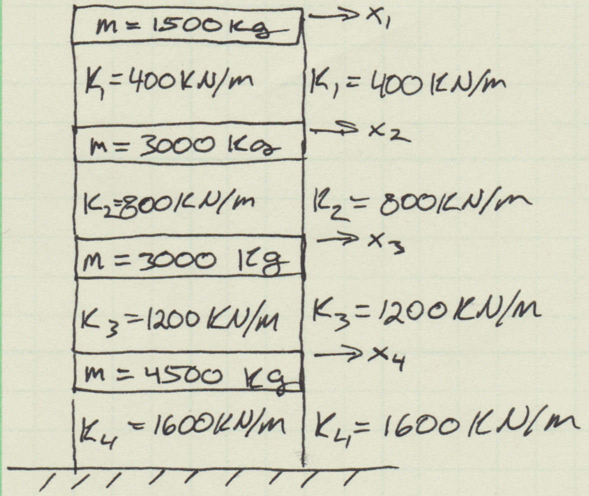
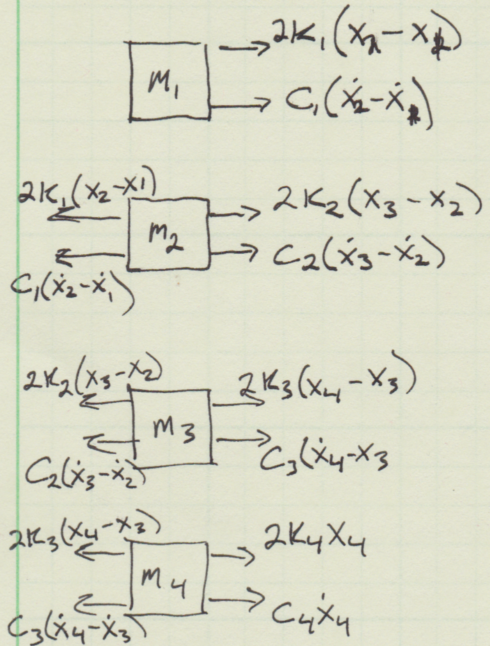
MEEM 4701 HW#1 | Andrew S Johnson | 12. September. 2018



Starting with copying the diagram from the handout to get a better feel for the components involved, then breaking each floor up into a free body diagram. (remembering that there are <u>two</u> spring forces for each floor.)



Equations of Motion

$$m_1\ddot{x}_1 = 2K_1x_2 - 2K_1x_1 + C_1\dot{x}_2 - C_1\dot{x}_1$$
$$m_1\ddot{x}_1 + 2K_1x_1 + C_1\dot{x}_1 - 2K_1x_2 - C_1\dot{x}_2 = 0$$

$$m_2\ddot{x}_2 = -2K_1x_2 + 2K_1x_1 - C_1\dot{x}_2 + C_1\dot{x}_1 + 2K_2x_3 - 2K_2x_2 + C_2\dot{x}_3 - C_2\dot{x}_2$$
$$m_2\ddot{x}_2 + C_1\dot{x}_2 + C_2\dot{x}_2 + 2K_1x_2 + 2K_2x_2 - C_1\dot{x}_1 + C_2\dot{x}_3 - 2K_1x_1 - 2K_2x_3 = 0$$

$$m_3\ddot{x}_3 = -2K_2x_3 + 2K_2x_2 - C_2\dot{x}_3 + C_2\dot{x}_2 + 2K_3x_4 - 2K_3x_3 + C_3\dot{x}_4 - C_3\dot{x}_3$$
$$m_3\ddot{x}_3 + C_2\dot{x}_3 + C_3\dot{x}_3 + 2K_2x_3 + 2K_3x_3 - C_2\dot{x}_2 - C_3\dot{x}_4 - 2K_2x_2 - 2K_3x_4 = 0$$

$$m_4\ddot{x}_4 = -2K_3x_4 + 2K_3x_3 - C_3\dot{x}_4 + C_3\dot{x}_3 - 2K_4x_4 - C_4\dot{x}_4$$
$$m_4\ddot{x}_4 + C_3\dot{x}_4 + C_4\dot{x}_4 + 2K_3x_4 + 2K_4x_4 - C_3\dot{x}_3 - 2K_3x_3 = 0$$

Mass Matrix

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix}$$

Stiffness Matrix

$$\begin{bmatrix} 2K_1 & -2K_1 & 0 & 0 \\ -2K_1 & 2K_1+2K_2 & -2K_2 & 0 \\ 0 & -2K_2 & 2K_2+2K_3 & -2K_3 \\ 0 & 0 & -2K_3 & 2K_3+2K_4 \end{bmatrix}$$

* Ignore damping for now . . .

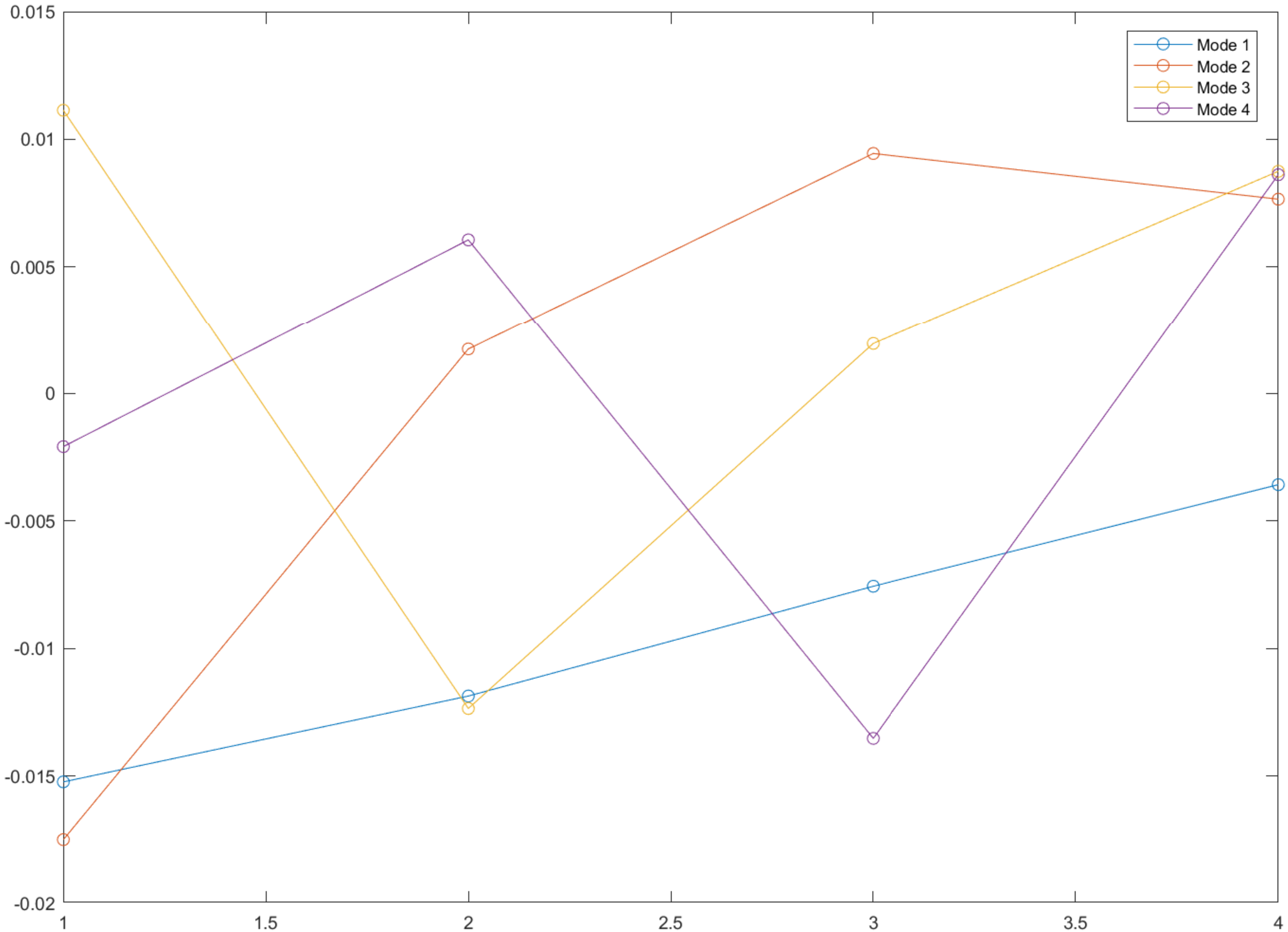
Equations of motion derived from free body diagrams, put into mass and stiffness matrices. (damping ignored for now) Matrices and values were entered into a matlab script (attached) and the "eig()" function was used to compute the eigenvalues and eigen vectors, thereby uncovering the natural frequencies and mode shapes.

Natural Frequencies (Hz)

$$\begin{bmatrix} 1.73 & 0 & 0 & 0 \\ 0 & 3.85 & 0 & 0 \\ 0 & 0 & 5.34 & 0 \\ 0 & 0 & 0 & 7.26 \end{bmatrix}$$

Mode Shapes

$$\begin{bmatrix} -4.25 & -10.04 & 5.68 & -1.0 \\ -3.31 & 1.0 & -6.31 & 2.90 \\ -2.11 & 5.42 & 1.0 & -6.48 \\ -1.0 & 4.39 & 4.46 & 4.13 \end{bmatrix}$$



0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
1
1.5
2
2.5
3
3.5
4
Mode 1
Mode 2
Mode 3
Mode 4

```
% Modal Analysis
% Homework 1
% 12.September.2018
% Andrew S. Johnson

% Housekeeping
clear
clc
clf

% Calculated Element Dimensions and Properties
m1 = 1500;      % Kilograms
k1 = 400000 * 2;  % N/M

m2 = 3000;      % Kilograms
k2 = 800000 * 2;  % N/M

m3 = 3000;      % Kilograms
k3 = 1200000 * 2;  % N/M

m4 = 4500;      % Kilograms
k4 = 1600000 * 2;  % N/M

% Mass Matrix
M = [m1 0 0 0;
     0 m2 0 0;
     0 0 m3 0;
     0 0 0 m4];

 % Stiffness Matrix
 K = [k1 -k1 0 0;
      -k1 k1+k2 -k2 0;
      0 -k2 k2+k3 -k3;
      0 0 -k3 k3+k4];

 % Damping Matrix
 for i=1:4
     for k=1:4
         C(k,i) = 0.01*M(k,i)+0.0005*K(k,i);
     end
 end

 % Eigenland
 [V,D] = eig(K,M);

 % Natural Frequencies
 for i=1:4
     F(i) = sqrt(D(i,i))/(2*pi);
 end
 disp('Natural Frequencies (Hz):');
 disp(F);
```

```
% Mode Shapes
for i=1:4
    for k=1:4
        S(k,i) = V(k,i)/min(abs(V(:,i)));
    end
end
disp('Mode Shapes:');
disp(S);

% Plot
X = linspace(1, 4, 4);
for i=1:4
  plot(X, V(:,i), 'o-');
  hold on
end
```