1 Amplifiers

1.1 Active vs. Passive Circuits

We now move into the regime of so-called *active electronics*. Everything studied so far – resistors and capacitors, to voltage dividers and filters, to diode rectifiers has fallen under the realm of passive electronics. Active electronic devices, in addition to standard inputs and outputs, require additional power supplies in order to function. There are great benefits to this, as we shall see: active components can output more power than at their input, and can eliminate annoying effects such as voltage loading due to input and output impedance. The first active device we'll study is the amplifier.

1.2 The Ideal Voltage Amplifier

The idea of the basic amplifier is simple: take the input x_{in} and produce an output which is some number G times the output.

$$x_{out} = Gx_{in} \tag{1}$$

... where G is known as the gain.

There is the question of what input x_{in} is being amplified. In our course, normally x_{in} will be a voltage and so we will have a voltage amplifier. In this case the gain is often expressed in decibals as:
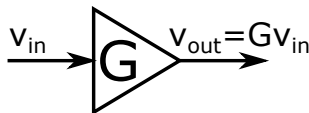
$$G_{dB} = 20 \log \frac{v_{out}}{v_{in}} \tag{2}$$

We will occasionally deal with power amplifiers whose input and output is a power. For these amplifiers, the gain is also in dB but with a 10 instead of a 20 multiplier[1]:

$$G_{dB} = 10 \log \frac{P_{out}}{P_{in}} \tag{3}$$

For now though, we will restrict outselves to voltage amplifiers. Diagrammed in figure 1. The output is descibed by equation 1.

Figure 1: The simplest of amplifiers: the output is the input multiplied by a constant gain G.

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[1]Recall that for a fixed load resistance, $P = v^2/R \propto v^2$. Thus $10 \log [P_{out}/P_{in}] = 10 \log \left[(v_{out}/v_{in})^2 \right] = 20 \log [v_{out}/v_{in}]$, thus yielding consistant definitions of gain

1.3 Reality Check: Amplifiers in Real Life

There are several limitations to the ideal amplifier that come into effect in practice. For example the amplifier will have to draw at least a small current from the source in order to operate. As we've learned, this is equivalent to a finite *input impedance* R_{in}. The amplifier will also have a limit to how much current it can output due to its internal resistances. This leads to a finite *output impedance* R_{out}. There are also limits to how fast the amplifier can respond to a changing input signal. This leads to a finite *gain bandwidth*, or in general, a frequency dependant gain $G(\omega)$. We will examine each of these effects presently.

1.3.1 Effects of Finite Input Impedance

We can model the non-ideal amplifier as an ideal amplifier inside a black box, precisely as we did with ideal vs real voltage sources. Inside the black-box, the input impedance is models as an internal resistance R_{in} to ground placed before the ideal amplifier. This forms a voltage divider with the input impedance from the rest of the circuit so that there is some voltage drop. The ideal amplifier thus sees a different voltage than v_{in}, which we denote v'_{in}. The output of the real amplifier is thus less than we would expect: $v_{\text{out}} = Gv'_{\text{in}} < Gv_{\text{in}}$

Figure 2: We model the finite input impedance of the amplifier as a internal resistor R_{in} to ground before an ideal amplifier all in a black box.
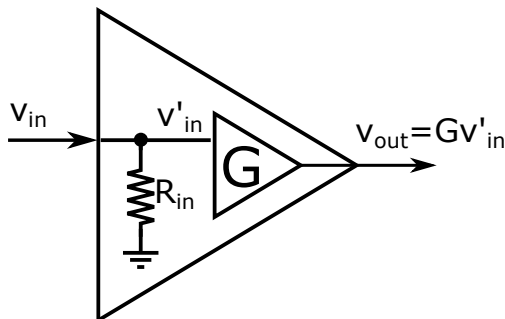

The true output, given the input impedance can be calculated given the output impedance of whatever follows. For example, with a $100\ \Omega$ input impedance amplifier with $G = 10$,

1.3.2 Effects of Finite Output Impedance

The operation of our ideal amplifier can be seen as a two part process: first it senses the voltage at its input (with the effect of finite input impedance causing some loading error), and second, it acts as a new voltage source, producing a voltage $v_{\text{out}}Gc_{\text{in}}$. As with any *real* source, it will be equivalent to an ideal source, in series with an output impedance R_{out}. This finite output impedance

leads to a maximum short-circuit current of $v_{\text{out}}/R_{\text{out}}$ and will lead to all of the loading issues discussed in chapter XXX. Figure 3 shows our improved model of how a real amplifier behaves.

Figure 3: A better model of a real amplifier has an input resistance to ground in parallel with the inpput and an output resistance in series with its output.
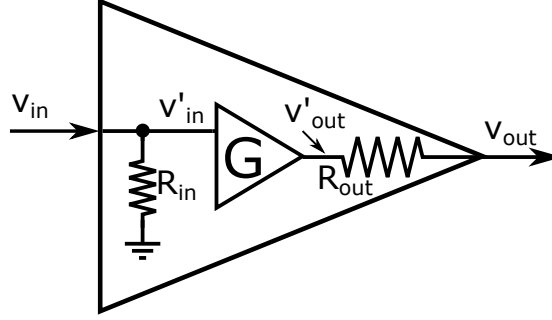
Design Example: Consider a real amplifier with $R_{\text{in}} = 10$ kΩ, $R_{\text{F}} = 100$ Ω, and G = 25. We use it to amplify a 10 mV signal from a photodetector having $R_{\text{out,PD}} = 1250$ Ω and send it to a (lousy) voltmeter with $R_{\text{in,LVM}} = 500$ Ω (see figure 4). What is the voltage measured (a) assuming an ideal $G = 25$ amplifier, and (b) with our real amplifier?

Answer: For part (a) the answer is simply $v_{\text{meas}} = 25 \times 10$ mV $= 250$ mV. For our real amplifier, we must consider the input and output stage. The input now forms a voltage divider with R_{in} (figure 4b so that the inner ideal amplifier sees:

$$v'_{\text{in}} = \frac{R_{\text{in}}}{R_{\text{out,PD}} + R_{\text{in}}} = \frac{10 \text{ k}\Omega}{11.25 \text{ k}\Omega} \times 10 \text{ mV} \approx 8.9 \text{ mV}$$

In our model, this is then amplified by the ideal internal amplifier producing

$$v'_{\text{out}} = 8.9 \text{ mV} \times 25 \approx 222 \text{ mV}$$

Finally, this is then sent to our lousy voltmeter, through its internal output resistance, forming another voltage divider (figure 4c) to yeild the final voltage:

$$v_{\text{out}} = \frac{R_{\text{in,LVM}}}{R_{\text{out}} + R_{\text{in,LVM}}} v'_{\text{out}} = \frac{500 \text{ }\Omega}{600 \text{ }\Omega} \times 222 \text{ mV} \approx 185 \text{ mV}$$

We thus see that the effect of finite input and output impedance is to lessen the effective gain of the real amplifier. Even if we used an ideal voltmeter with infinite input impedance, we'd still see 222 mV $<$ 250 mV. Equivalently, we could try to build an amplifier with $R_{\text{out}} \to 0$ Ω and again we'd see 222 mV, regardless of the impedance of the voltmeter. To get the full 250 mV however, we'd need to also have $R_{\text{in}} \to \infty$, so that $v'_{\text{in}} = v_{\text{in}} = 25$ mV.

Figure 4: Example Problem.
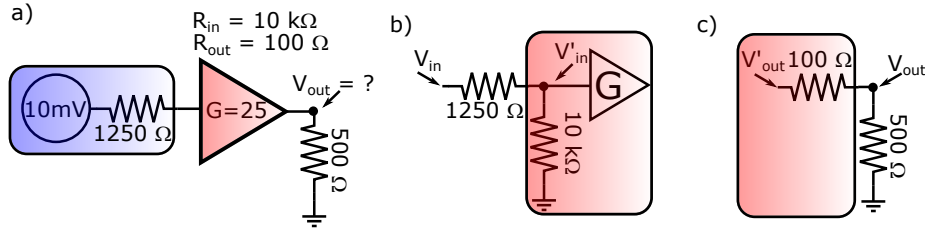
From this we can infer the ideal characteristrics of a real amplifier: we want the input impedance as high as possible while having output impedance as low as possible. This can be accomplished using solid state transistors, discussed later in the course.

1.4   Effects of Noise: Noise Factor and SNR

The ideal amplifier will amplify the voltage present at it's input, regardless of whether we consider that voltage to be useful signal or to be noise. This can lead to fundamental issues in isolating very weak signals since eventually, we will just be scaling up Johnson, Shot, or technical noise.

For example, if you have a 10 nV signal atop 25 nV of Johnson noise, you can send your signal to a 120 dB amplifier to get 100 mV of signal, but it will still be lost in the Johnson noise, which is now at 250 mV. We amplified the signal level, but have not improved the signal level as compared to the signal. This ratio of signal vs noise has, in a spark of linguistic creativity, been dubbed the "signal to noise ratio" or SNR. since the voltage of the noise is usually stochastic [2], usually the signal to noise ratio is usually specified in terms of power, rather than voltage[3] (ie. v_{RMS}^2/R)

$$\text{SNR} \equiv \frac{P_\text{sig}}{P_\text{noise}} = 10 \log_{10} \left[\frac{P_\text{sig}}{P_\text{noise}} \right] \text{dB} \tag{4}$$

So an amplifier on its own will not help you discern signals dominated by noise and the SNR must be improved though separate means. If your signal exits within a given range of frequencies but your noise exists at all frequencies, a filter can be applied to the input of the amplifier to greatly improve the SNR. If the signal and noise coexist at the same range of frequencies, you have to be more clever. For example, a common source of noise is thermal motion of electrons through a load resistance. To improve the SNR, circuits are often

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[2]A high-brow term, effectively meaning "random"

[3]...wait, what R should we use here? Well power is energy per unit time and that energy has to be delivered to something. That something is the load resistance, so that a given voltage noise will deliver more power to a 100 Ω than a 100 kΩ resistor

cooled to -40 °C or even as low as several Kelvin, when extremely faint signals are being detected.

To make matters worse, the amplifier often adds its own noise to the system - that is the SNR of the output is typically even higher than that of the input! This is quantified by the so-called noise figure:

$$\text{NF} \equiv \frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}} = 10 \log_{10} \left[\frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}} \right] \tag{5}$$

Good amplifiers will have NF 2 dB or less.

1.4.1 Effects of Finite Frequency Response

Since every circuit element has some finite capacitance and inductance, we can replace the internal input and output resistances with an input impedance Z_{in} and output impedance Z_{out}. The net effect of this is to produce a frequency dependant gain $G(\omega)$. This can be modelled as internal frequency filter that is applied to the input or output. This bug can be rebranded as a feature as an amplifier may actually filter out high frequency noise present in the signal, as discussed in the previous section.

1.5 Differential Amplifiers

Often in physics, it is a small deviation from a fixed value that is of interest rather than the absolute value itself. For example, a beam of light may pick up a small oscillation in intensity as compared to a reference beam when passed through a sample. In this case, a *differential amplifier* may be used. As diagrammed in figure XXX, a differential amplier has two inputs and a single output corresponding to the difference between the signals. This is operation can be seen as first inverting one input and then adding it to the other. The input which is inverted V_- is fitting called the "inverting input", while the normal input is verbosely titled the "non-inverting input", V_+. The multiplication factor is known as the "differential gain" G_{diff} and is defined through the relation

$$v_{\text{out}} = G_{\text{diff}} \left(v_+ - v_- \right). \tag{6}$$

Example: A differential amplifier with $G_{\text{diff}} = 40$ dB has, at its inputs: $v_+ = 3.7$ mV and $v_- = 4.2$ mV. What is the output voltage?

Solution: The gain of 40 dB implies that $20 \log_{10} \frac{v_{\text{out}}}{v_+ - v_-} = 40$, so that $v_{\text{out}} = 100 \left(v_+ - v_- \right)$. Since, $\left(v_+ - v_- \right) = (3.7 - 4.2)$ mV $= -0.5$ mV, we will find $v_{\text{out}} = -500$ mV.

1.5.1 Common Mode Gain

From equation 6, we expect that if $v_+ = v_-$, we will have $v_{\text{out}} = 0$ regardless of the value of v_+ and v_-. In practice however, this is not the case. This can

be due, for example, to differences in the input impedances of the individual inputs so that internally, $v'_- \neq v'_+$ even though $v_- = v_+$ (see the previous section 1.3.1). To quantify this, we define the *common-mode voltage* as the average of the inverting vs. non-inverting input voltages:

$$v_{\text{com}} \equiv \frac{v_+ - v_-}{2} \tag{7}$$

The amount of signal that gets through defines the *common-mode gain*

$$v_{\text{out}} = G_{\text{com}} \frac{v_+ + v_-}{2} \tag{8}$$

An ideal differntial amplifier would have $G_{\text{com}} = 0$ or $-\infty$ dB. The true output of the *real* differential amplifier is thus:

$$v_{\text{out}} = G_{\text{diff}} (v_+ - v_-) + G_{\text{com}} \frac{v_+ + v_-}{2}. \tag{9}$$

1.5.2 Common Mode Rejection Ratio

The "goodness" of a differential amplifier is how much it behaves like one! Mathematically we can quantify this by saying that for a good differential amp, $G_{\text{com}} \ll G_{\text{diff}}$. THis is represented by the so-called "common-mode rejection ratio" or CMRR. The CMRR is usually stated in decibels and is given by:

$$\text{CMRR} \equiv 20 \log_{10} \left[\frac{G_{\text{diff}}}{G_{\text{com}}} \right] \tag{10}$$

Decent differential amplifiers have CMRR[s] of at least 70 dB but some applications require 120 dB or higher.