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Matching Small Loop Antennas to rfPIC™ Devices

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INTRODUCTION

In close proximity to the human body, small loop antennas outperform small dipole and monopole antennas [1]. Their size, robustness and low manufacturing cost have made small loops the most popular antenna for use in miniature key fob transmitters. A small loop antenna typically consists of a circular, square or rectangular copper trace on a printed circuit board. In some cases, discrete wires are shaped into loops.

FIGURE 1: EQUIVALENT CIRCUIT MODEL OF A SMALL LOOP ANTENNA

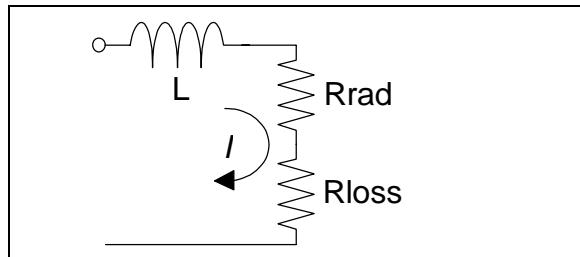


Figure 1 shows an equivalent circuit of a loop antenna consisting of two resistors and an inductor. The resistor R_{rad} , or radiation resistance, models the radio frequency energy actually radiated by the antenna. R_{rad} models the desired function of the antenna, which is to radiate RF power. Assuming a uniform current I flowing through the loop, the power consumed by R_{rad} (i.e., the radiated power) is shown in Equation 1.

EQUATION 1:

$$P_{radiated} = I^2 \cdot R_{rad}$$

The second resistor in the model, R_{loss} , models losses. R_{loss} models an undesired, but inevitable function of the antenna: to waste valuable RF energy by converting it to heat. If R_{loss} is larger than R_{rad} , the antenna is inefficient, since most of the available RF power will end up as heat. With current I flowing through the loop, the lost power (converted to heat) is given by Equation 2.

EQUATION 2:

$$P_{loss} = I^2 \cdot R_{loss}$$

Note that we assume that the current I is uniform around the small loop. This assumption is only valid if the loop circumference is smaller than one fifth of a wavelength.

For completeness, note that the total power delivered to the antenna is given by the sum of the radiated power and losses. From Equation 1 and Equation 2, we get Equation 3:

EQUATION 3:

$$P_{total} = P_{radiated} + P_{loss} = I^2 \cdot (R_{rad} + R_{loss})$$

In practice, the loop antenna designer has little control over R_{rad} and R_{loss} . R_{rad} is determined by the area of the loop antenna and R_{loss} is a function of conductor size and conductivity, as shown in Equation 4 and 5.

CALCULATING THE LOOP RADIATION RESISTANCE AND LOSS RESISTANCE

The radiation resistance R_{rad} of a small loop antenna is given by reference [2] as:

EQUATION 4:

$$R_{rad} = 320\pi^4 \left(\frac{A}{\lambda} \right)^2$$

where A is the area of the loop in square meter and λ is the wavelength in meters at the radiation frequency. It should be clear from Equation 4 that the radiation resistance of small loops will be in the milliohm range. The wavelength λ can be calculated as $\lambda = 3 \cdot 10^8/f$ where f is the radiating frequency in Hertz.

The loss resistance R_{loss} of a loop antenna is given by reference [2] as:

EQUATION 5:

$$R_{loss} = \frac{l}{2w} \cdot \sqrt{\frac{\pi f \mu}{\sigma}}$$

where l is the perimeter (circumference) of the loop in meters, w is the width of the PCB track in meters, f is the radiating frequency in Hertz, $\mu = 4\pi \cdot 10^{-7}$ and σ is the conductivity of the PCB track in Siemens per meter. Copper conductivity is typically $5.7 \cdot 10^7$ S/m.

Equation 5 is essentially the result of the 'skin effect' [2] at high frequency for nonmagnetic materials. In this case, the perimeter of the conductor, normally $2\pi r$ for a round wire, has been approximated by $2w$. In other words, its perimeter is 2 times the PCB trace width.

CALCULATING THE INDUCTANCE OF THE LOOP

The third component in the model of Figure 1 is the loop inductance L . Inductance is primarily a magnetic effect, and general inductance formulas for even simple shapes are hard to derive. Several formulas for calculating the inductance of rectangular loops have been proposed. Most of these formulas are lengthy [2,3,4]. Grover's book [3], which is the primary reference work on inductance, provides one remarkably simple, but accurate formula for calculating the inductance of polygons. This formula includes, but is not limited to rectangular loops. The inductance formula given by Grover [3] is:

EQUATION 6:

$$L = \frac{\mu}{2\pi} \cdot l \cdot \ln\left(\frac{8 \cdot A}{l \cdot w}\right)$$

where $\mu = 4\pi \cdot 10^{-7}$, A is the area of the loop in square meters, l is the perimeter (circumference) of the loop in meters, and w is the width of the copper trace in meters.

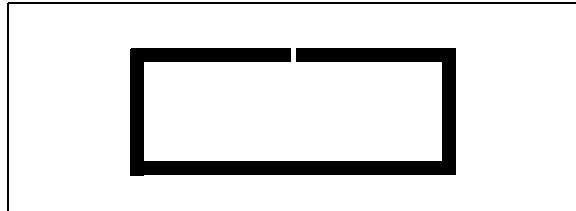
CALCULATION OF LOOP PARAMETERS

EXAMPLE 1:

Suppose a designer is constrained by PCB loop antenna dimensions of 34 mm x 12 mm. The copper track width is 1 mm.

The total loop resistance, that is the sum of radiation resistance and loss resistance, is calculated to be 0.249Ω .

**FIGURE 2: PCB COPPER LOOP
34 mm x 12 mm x 1 mm**



Using Equation 4, we calculate radiation resistance at 434 MHz as $R_{rad} = 0.0227 \Omega$.

Using Equation 5, we calculate loss resistance at 434 MHz as $R_{loss} = 0.227 \Omega$ (σ of copper $5.7 \cdot 10^7$).

Using Equation 6, we calculate loop inductance as $L = 52.914 \text{ nH}$.

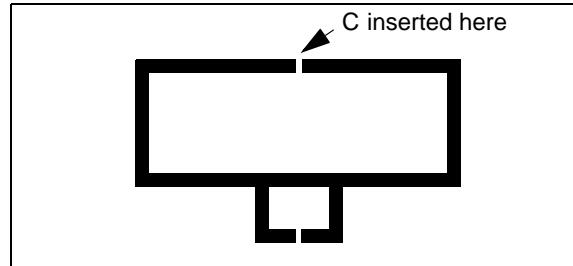
Summing the loss resistance and radiation resistance, total loop resistance is calculated to be $r = 0.249 \Omega$.

Matching the Loop to a 1 kΩ Source Impedance

A typical CMOS radio frequency integrated circuit, such as the rfPIC12C509AG, has a source impedance around $1 \text{ k}\Omega$. In the example above, the impedance of a typical loop has an inductance of 52.914 nH in series with a small resistance of 0.249Ω . To match such an antenna to the source, this low resistance and high inductive reactance must be transformed to $1 \text{ k}\Omega$.

The impedance transformation required is achieved by adding a second, smaller loop to our antenna, as well as a capacitor C , as shown in Figure 3.

FIGURE 3: ADDING A SMALL SECOND LOOP AND CAPACITOR



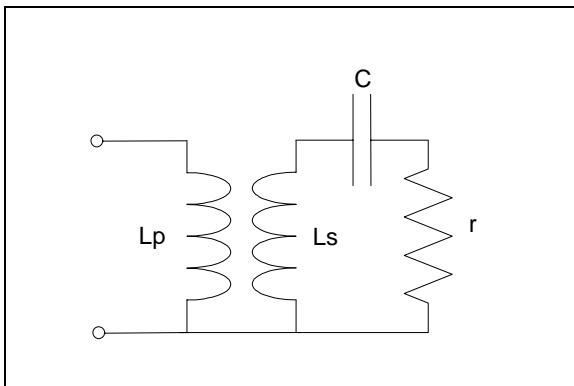
The magnetic coupling between the large loop and small loop results in transformer action. The large loop, or loop antenna, makes up the secondary winding of our transformer. The small loop becomes the primary winding of our transformer.

Figure 4 is a revised circuit model of the loop antenna, showing the transformer action. The loop antenna's total resistance r , consisting of $R_{rad} + R_{loss}$, forms the resistive load in the secondary circuit. Also note a capacitance C in the secondary circuit. Capacitor C is primarily used to cancel the loop inductance L_s . The capacitance may be approximated as follows:

EQUATION 7:

$$C \approx \frac{L_s}{\omega^2}$$

FIGURE 4: REVISED LOOP ANTENNA EQUIVALENT CIRCUIT



MAGNETIC COUPLING

Magnetic coupling between the primary and secondary windings is at the root of the impedance transformation. We now write the basic voltage and current equations for the above magnetically coupled circuit:

EQUATION 8:

$$\begin{aligned} V_p &= j\omega L_p I_p + j\omega M I_s \\ 0 &= j\omega M I_p + j\omega L_s I_s - j \frac{1}{\omega C} I_s + r I_s \end{aligned}$$

where V_p is the primary voltage, I_p the primary current, I_s the secondary current and ω the angular frequency, equal to $2\pi f$. M is the mutual inductance, which is a function of the degree of magnetic coupling between the two loops.

By using Equation 8, the real part (resistive portion) of the load impedance as seen from the primary side of the transformer can be derived:

EQUATION 9:

$$Re(Z_p) = (\omega M)^2 \cdot \frac{r}{\left[r^2 + \left(\omega L_s - \frac{1}{\omega C} \right)^2 \right]}$$

Near resonance the term

$$\left(\omega L_s - \frac{1}{\omega C} \right)^2$$

becomes small, so that it is possible to estimate the resistive load as seen from the primary side by:

EQUATION 10:

$$R_p \approx (\omega M)^2 \cdot \frac{1}{r}$$

Equation 10 shows how the transformer action translates the low loop resistance r of the small loop antenna:

- The resistance is inverted.
- The inverted resistance is then multiplied by the square of the mutual reactance, $(\omega M)^2$.

Equation 10 can be rewritten as:

EQUATION 11:

$$M \approx \frac{\sqrt{r \cdot R_p}}{\omega}$$

Equation 11 shows the mutual inductance needed to transform a loop impedance of r ohm into a needed source impedance of R_p ohm.

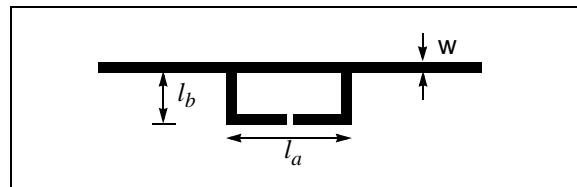
Next, we find a way to calculate mutual inductance M as a function of loop dimensions.

Obtaining a Given Mutual Inductance

A formula for the calculation of mutual inductance between two off-center, coplanar rectangles is daunting. However, two reasonable assumptions simplify the calculation significantly. The assumptions are:

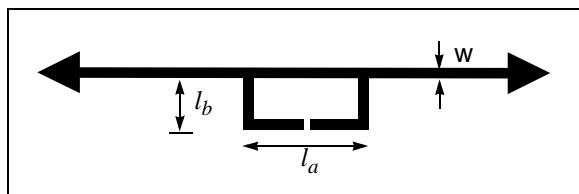
1. Assume that only one side of the loop antenna couples magnetically to the small loop. The other three sides are much further away, so we may neglect their effect. This assumption simplifies the mutual inductance calculation problem to that of mutual inductance between a straight wire (or PCB track), and the small loop, as drawn in Figure 5.

FIGURE 5: USING ONLY ONE SIDE OF THE LARGE LOOP



2. It is assumed that the straight wire of Figure 5 stretches to infinity, as drawn in Figure 6. This is reasonable because the part of the wire close to the loop is the dominating contributor of magnetic flux in the small loop.

FIGURE 6: INFINITE WIRE AND SMALL LOOP



The two assumptions greatly simplify the calculation of mutual inductance. The mutual inductance of the loop-and-wire of Figure 6 is a popular college physics [5] problem with a compact result:

EQUATION 12:

$$M = \frac{\mu}{2\pi} \cdot l_a \cdot \ln\left(1 + \frac{4 \cdot l_b}{w}\right)$$

Where M is mutual inductance in Henry, l_a is the rectangle dimension parallel to the wire, l_b is the rectangle dimension perpendicular to the wire, and w is the width of the PCB track. All dimensions are in meter, and $\mu = 4\pi \cdot 10^{-7}$.

By setting the rectangle dimension l_b to 2 times the PCB track width, in other words, setting $l_b = 2w$, we find that Equation 12 simplifies to:

EQUATION 13:

$$M = \frac{\mu}{2\pi} \cdot l_a \cdot \ln\left(1 + \frac{8w}{w}\right)$$

$$M = \frac{\mu}{2\pi} \cdot l_a \cdot \ln(1 + 8)$$

By combining Equations 11 and 13, an expression for loop dimension l_a is found as follows:

EQUATION 14:

$$l_a = \frac{\sqrt{r \cdot R_p}}{\mu \cdot f \cdot \ln(9)}$$

Equation 14 is the final result and provides a simple method to match to a small loop antenna, which is summarized below.

1. Calculate the loop series resistance r using Equations 4 and 5.
2. Determine the required impedance R_p .
3. Calculate l_a of a small matching loop using Equation 14.

EXAMPLE 2:

Continuing our loop antenna of Example 1:

1. From Example 1, we calculated the loop series resistance as $r = 0.249 \Omega$
2. The needed antenna impedance is $1 \text{ k}\Omega$.
3. From Equation 14, we calculate $l_a = 13.2 \text{ mm}$, using $f = 433.92 \text{ MHz}$.

CONCLUSION

A simple method to match a small loop antenna has been found. By adding a small primary loop and by controlling the mutual inductance of the resulting transformer, the low loop resistance is transformed to the value desired for maximum power transfer.

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APPENDIX A: COMPLEX IMPEDANCE

Equation 9 shows only the real part of the primary side impedance. The entire complex impedance as seen on the primary side is:

EQUATION 15:

$$Z_p = \omega^2 M^2 \cdot \frac{r}{\left[r^2 + \left(\omega L_s - \frac{1}{\omega C} \right)^2 \right]} + j \left[\omega L_p - \frac{\omega^2 M^2 \cdot \left(\omega L_s - \frac{1}{\omega C} \right)}{\left[r^2 + \left(\omega L_s - \frac{1}{\omega C} \right)^2 \right]} \right]$$

M is mutual inductance, *r* is loop resistance, *L_s* is secondary (large) loop inductance and *L_p* is primary loop inductance. *L_p* and *L_s* are both calculated using Equation 6.

An exact value for the capacitance *C* at resonance can be found by setting the imaginary part of Equation 15 to zero.

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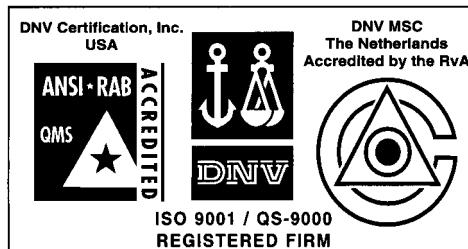
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