Electrically Small Antenna Simulation and Capacitive Matching Network Design

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I. Introduction

Electrically small antennas play a critical role in low frequency systems, where size constraints necessitate the design of antennas many times smaller than their intended operational wavelength. This report illustrates the design of a 10MHz, electrically small, 30x20cm loop antenna on 60mil Rogers3003 substrate, matched to a 50Ω load. Due to it's extremely small size ($D_{max} \sim \lambda/100$), designing this antenna presents a unique set of challenges. Namely, extremely low radiation resistance and highly inductive input impedance. This report solves these issues through the design of an RF matching network, informed by analytical calculation and ADS co-simulation. Through these methods, the antenna's Q-Factor, efficiency, impedance and radiation pattern are also characterized.

In short, this work provides a complete workflow, from initial theoretical estimates, to EM simulation, to impedance-matching network synthesis, that illustrates the practical considerations and performance trade-offs involved in designing electrically small loop antennas.

II. Theory

An electrically small antenna (ESA) is defined in terms of its largest physical dimension, D_{max}, which relates to the circumscribing sphere radius a and wavelength λ. The antenna is considered electrically small when:

$$D_{max} = 2a < \frac{\lambda}{4} \tag{1}$$

From Chu's Limit [1], a lossless, electrically small antenna's quality factor Q is lower bounded by the expression:

$$Q \; > \; \frac{1}{(ka)^3} + \frac{1}{(ka)} \tag{2}$$

Where k is angular wavenumber and a is the circumscribing sphere radius. Further, Q may be decomposed into Q_{loss} and Q_{rad} quality factors, describing bandwidth from ohmic loss and radiation respectively:

$$Q_{loss} = \frac{Q}{\eta} \qquad Q_{rad} = \frac{Q}{1-\eta} \tag{3}$$

Where η is radiation efficiency. Assuming uniform current around the loop, the equivalent circuit model for an electrically small loop antenna, as described in Microchip AN831 [2], is shown in Figure 1:

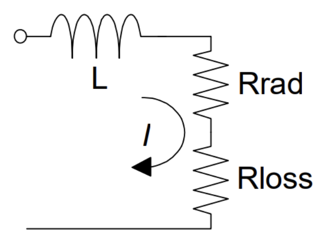

Fig. 1. Equivalent circuit model for an electrically small loop antenna, from Microchip [2]

Where R_{rad} and R_{loss} are the antennas radiation and loss resistance respectively, and L models the antennas high inductance.

From [3], R_{rad} and R_{loss} may be calculated as:

$$R_{rad} = 320\pi^4 (\frac{A^2}{\lambda^4}) \tag{4}$$

$$R_{loss} = \frac{l}{2w}\sqrt{\frac{\pi f \mu}{\sigma}} \tag{5}$$

Where A is the area of the loop, l is the perimeter of the loop, w is the width of the PCB track, f is the radiating frequency, σ is the conductivity of the PCB trace, and μ is free space magnetic permeability.

From 6, L may be calculated as:

$$L = \frac{\mu}{2\pi} ln(\frac{8A}{lw}) \tag{6}$$

Where A is the area of the loop, l is the perimeter of the loop, w is the width of the PCB track, and μ is free space magnetic permeability.

Typically, electrically small loop antennas have very small R_{rad} and R_{loss}, and very high L. This requires a design with capacitive compensation to account for the loops high inductance, and a suitable matching network/structure to match to a typical feed-line (50Ω).

III. Design Specifications

The loop antenna was designed to operate at 10 MHz on a Rogers RO3003 substrate. This section summarizes the key physical and electrical parameters used in simulation (see Table I).

TABLE I
DESIGN PARAMETERS

Parameter	Value
Substrate Material	Rogers RO3003 (60 mil)
Relative Permittivity ε_r	3.00
Loss Tangent $\tan\delta$	0.0013
Substrate Thickness	60 mil (1.524 mm)
Operating Frequency	10 MHz
Target Impedance	50 Ω
Radiating Loop Size	30 × 20 cm
Simulation Tools	ADS Schematic + ADS Momentum EM

IV. Antenna Geometry and Initial Calculations

A. Series Capacitive Tuning

The antenna is designed to operate at 10 MHz on a 60 mil RO3003 substrate. As a result, it is very inductive, deeply subwavelength, and will necessarily be very inefficient. To compensate for this inefficiency, a trackwidth of 10mm was selected. R_{rad}, R_{loss} and L where calculated using equations 4, 5 and 6. From these values, a matching capacitor value was calculated to compensate for inductive reactance (Table II):

TABLE II
ANALYTICAL RADIATING LOOP PARAMETERS

Parameter	Value
Radiation Resistance R_{rad}	1.39×10^{-4} Ω
Loss Resistance R_{loss}	4.16×10^{-2} Ω
Loop Inductance L	0.774 μH
Reactance at 10 MHz, X_L	48.7 Ω
Resonating Capacitance	≈ 330 pF

B. Impedance Matching to 50Ω at Resonance

The matching capacitor eliminates the reactive component of the antennas total impedance, but it's total resistance, $R_{loss} + R_{rad}$, is still very small. Matching this to a 50Ω feed-line required the addition of a matching network. While an L-type network would be sufficient for matching, it adds extra cost and complexity. For this reason, a physical PCB level matching structure was chosen. Microchip AN831 ([2]) describes such a network:

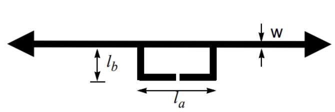

Fig. 2. Matching network structure, from [2]

In Figure 2, l_a and l_b are the dimensions of a smaller loop, attached to the larger radiating loop, modeled here as infinite length. If $l_b = 2w$, where w is the PCB track-width, l_a may be written as:

$$l_a = \frac{\sqrt{r \cdot R_p}}{\mu \cdot f \cdot ln(9)} \tag{7}$$

Where $r = R_{loss} + R_{rad}$, R_p is the load impedance, f is the radiating frequency, and μ is free space magnetic permeability. The matching structure design centers around a secondary loop, by which the primary radiating loop is driven. This "transformer" action increases input impedance and eliminates the need for extra components. Using equation 7, the dimensions of the matching loop where calculated (Table III):

TABLE III
ANALYTICAL MATCHING LOOP PARAMETERS

Dimension	Size
Width l_a	52 mm
Height l_b	20 mm

These analytical results will serve as a starting point for empirical EM co-simulation in ADS.

V. ADS Modeling Workflow

A. Substrate Setup

Figure 3 shows the substrate setup in ADS. The copper conductor layer was modeled with physical dimension, above the dielectric layer to improve simulation accuracy.

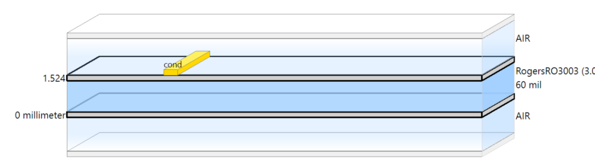

Fig. 3. Substrate layup for EM simulation. From bottom to top: Air, Rogers3003 (60 mils, 1.524 mm), Copper (35 μm, 1 oz), Air.

B. Schematic Model

Figure 4 shows the schematic setup for the circuit level simulation of the antenna and matching capacitor:

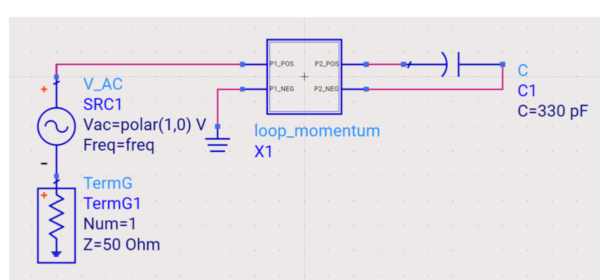

Fig. 4. Initial circuit schematic setup

C. EM Model

The antenna was initially modeled in ADS Momentum using the parameters calculated in Section IV. Two differential ports where placed at the capacitor and feed gaps (top and bottom of Figure 5, respectively).

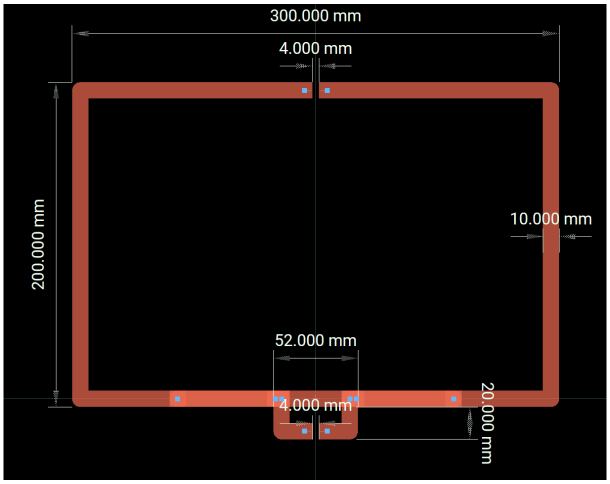

Fig. 5. Initial loop geometry

D. Tuning

Simulation with the initial setup described above yielded the following input impedance curve:

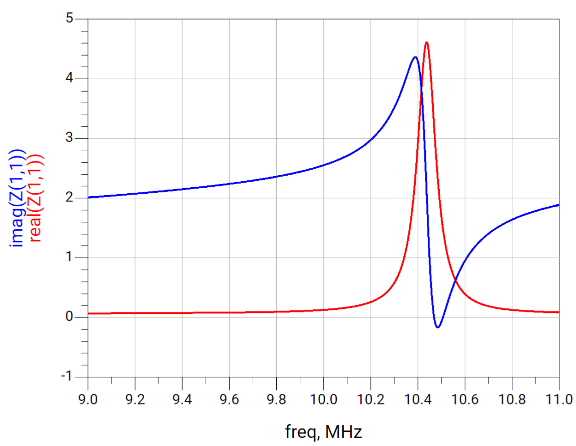

Fig. 6. Imaginary and real part of initial input impedance Z_{11} plotted against frequency

The first issue addressed was the incorrect center frequency. Since equation 6 is not exact, the matching capacitor and loop resonated roughly 0.4MHz off center. To address this, a value sweep was run on the matching capacitor, from 200 pF to 400 pF, yielding the family of curves in Figure 7:

From these curves and with additional fine tuning, a 360.4 pF capacitor was selected for the design. To address the low input impedance, the loop was tuned manually by increasing and decreasing the length and width dimensions of the smaller transformer loop. This manual tuning produced dimensions

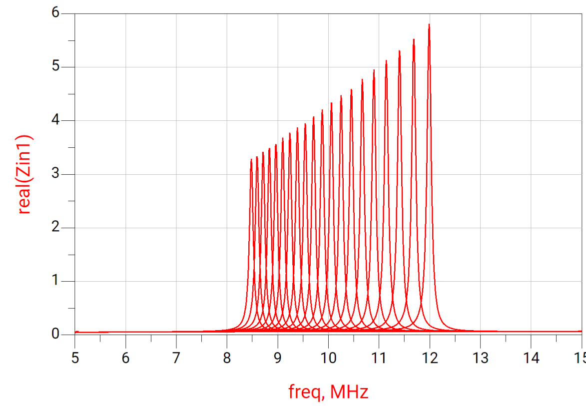

Fig. 7. Capacitor value sweep from 200 pF to 400 pF (right to left)

$l_a = 180$mm and $l_b = 25$mm (Figure 8) and a simulated antenna with ˜50 Ω impedance.

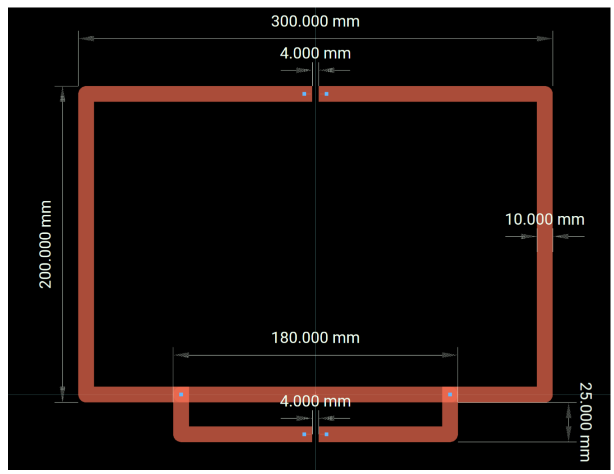

Fig. 8. Final antenna layout geometry

VI. Simulation Results

With the final antenna parameters described above, the following simulation results where achieved through simulation in ADS Momentum Co-simulation.

A. Input Impedance vs Frequency

The impedance curves in Figure 6 suggest a good impedance match, but extremely narrow bandwidth and high Q. Small shifts in frequency result in a poor load match, and high signal reflection.

B. S-Paramters

Figure 10 confirms the narrow bandwidth noted in Figure 9. Using raw simulation data from Figure 10, the half-power bandwidth was estimated at 1.32KHz and total quality factor was calculated to be $Q_{total} = 7.576 \times 10^3$. Most of this bandwidth comes from ohmic losses. Radiation quality factor, Q_{rad}, is calculated in the following sections.

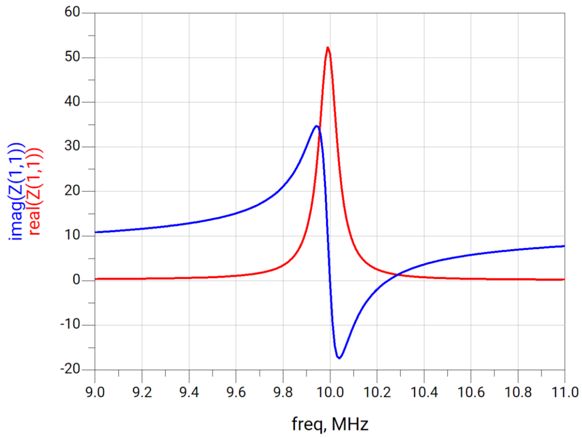

Fig. 9. Final input impedance curve, $Z_{11} = 50.823 - 0.560j$ at 10MHz

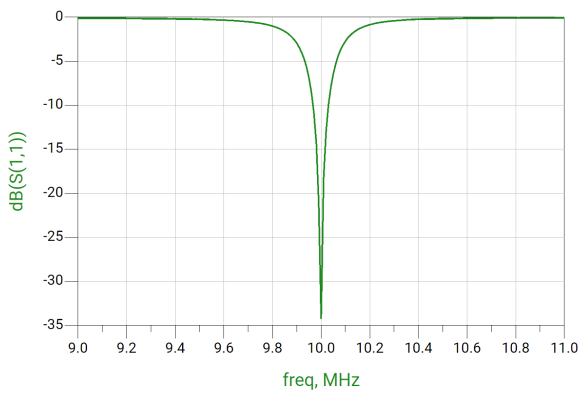

Fig. 10. Reflection S parameter, S_{11} at 10MHz

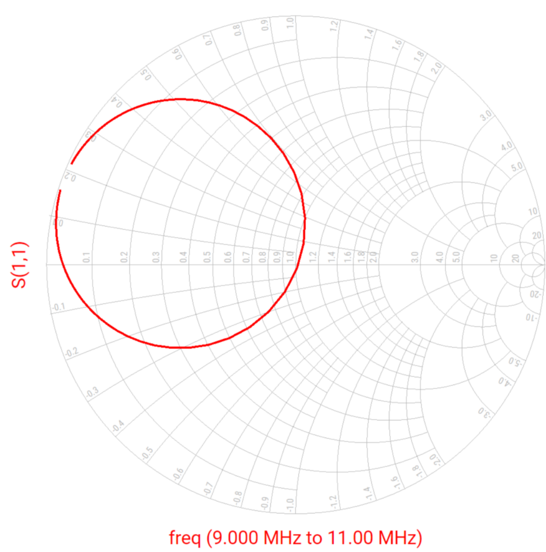

Fig. 11. Reflection S parameter, S_{11} plotted on a smith chart over 9-11MHz

C. Current Density

Figure 12 shows current density on the antenna geometry. The high current density on the smaller driving loop indicates high reactivity, and strong circulating fields. These fields don't radiate, but they couple to the larger loop. The larger loop shows near uniform current, affirming the assumption made in Section II.

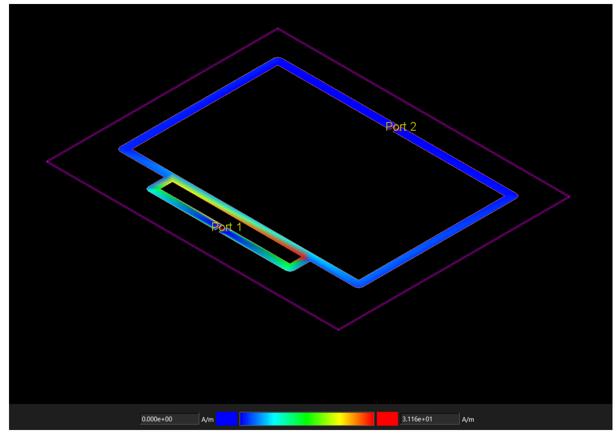

Fig. 12. Current density on final geometry at 10MHz

D. Far Field Patterns

Figures 13 - 16 show the 3D gain pattern and $\phi = 0$ far field cuts of the antenna radiation pattern. As expected for a small loop antenna operating in the deep subwavelength regime, the radiation pattern exhibits a classic magnetic dipole radiation pattern. The radiation intensity is maximum in the plane of the loop and approaches zero along the loop axis, producing a donut shaped distribution.

In the principal cuts, the pattern maintains the familiar sinusoidal shape characteristic of small loop antennas. The elevation cut demonstrates a broad main lobe with nulls at $\theta = \pm 90$. The azimuthal cut is isotropic, consistent with a magnetic dipole. Overall, simulated results agree with theoretical expectations.

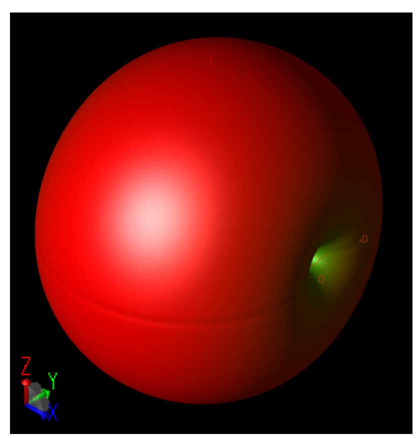

Fig. 13. 3D antenna gain pattern, at 10MHz

E. Directivity, Gain, and Efficiency

Quality factor was decomposed into loss and radiation terms according to equation 2. As shown in Table IV, directivity,

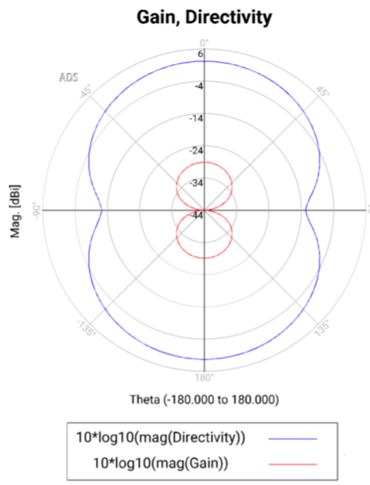

Fig. 14. Antenna Gain and Directivity plotted for $\phi = 0$ cut

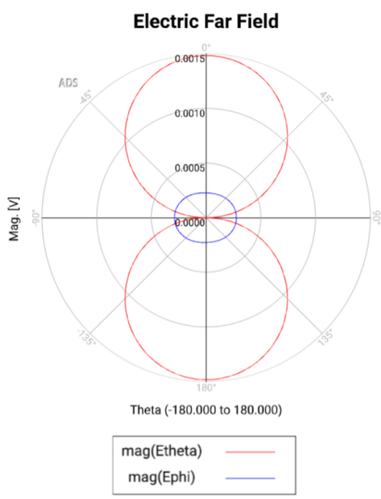

Fig. 15. Electric far field plotted for $\phi = 0$ cut

gain, and efficiency fall inline with theoretical expectations. Gain and radiation efficiency are very low, and quality factor is very high, indicating an extremely narrow band antenna.

TABLE IV
ANTENNA RADIATION RESULTS

Dimension	**Size**
Directivity	2.176 dBi
Gain	-29.105 dBi
Radiation Efficiency	0.0745%
Q_{total}	7.576×10^3
Q_{rad}	1.018×10^7
Q_{loss}	7.581×10^3

VII. Validation and Discussion

From equation 3, Chu's Limit was calculated as $Q_{chu} = 1.868 \times 10^4$. While total quality factor Q_{total} is significantly below this lower bound, Chu's limit does not take into account

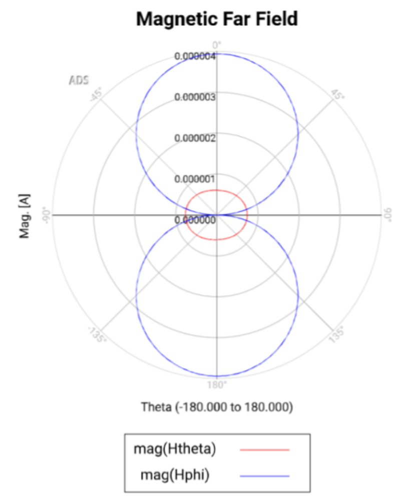

Fig. 16. Magnetic far field plotted for $\phi = 0$ cut

ohmic loss. Comparing to Q_{rad} shows that this antenna's quality factor is indeed 3 orders of magnitude greater than Chu's limit. This is an unavoidable consequence for an antenna this size.

Further, this antenna's extremely low radiation efficiency makes it impractical in most applications. This antenna may be useful in space-limited environments with very high SNR. Improvements to radiation efficiency may be realized through wider PCB track widths or thicker copper layers.

VIII. Conclusions

In sum, the antenna behaves as expected: a characteristic magnetic dipole radiation pattern, with very low efficiency and very narrowband operation. Matching structures on the antenna and circuit level capacitive loading provide effective compensation for the antennas highly inductive nature and strict size requirements.

References

[1] L. J. Chu, "Physical limitations of omni-directional antennas," Journal of Applied Physics, vol. 19, no. 12, pp. 1163–1175, 1948.

[2] J. van Niekerk, "Matching Small Loop Antennas to rfPICTM Devices," Microchip, 2002.

[3] K. Fujimoto, A. Henderson, K. Hirasawa, and J. R. James, Small Antennas. Research Studies Press, 1987, ISBN 0-86380-048-3; John Wiley & Sons, 1987, ISBN 0-471-91413-4.

[4] F. W. Grover, "Inductance Calculations Working Formulas and Tables," Dover Publications Inc. 1946