ES5205 – Project 1 Report

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Introduction and Motivation

There are many methods by which exoplanets can be detected, measured, and categorized. Finding these distant bodies helps to reach further into the growing universe and increase our knowledge of what else is out there. Also, with enough data, statistical trends can be identified regarding the formation and evolution of planets to increase our scientific understanding of those processes. This work is important not only for the purposes of exploration, which has been a side effect of the human condition since humanity began, but also to better understand our home planet of Earth.

This project was poised to grow the team's experience with data handling of identified exoplanets and to explore different detection techniques as well as their limits and applications. These methods include radial velocity, transits, and direct imaging, each of which have their own advantages and disadvantages.

Radial velocity involves measuring the Doppler shifts in the light of a central star due to gravitational influences of the exoplanets. It does the best with identifying large planets such as Hot Jupiters orbiting close to their star. Its key detection signal is a parameter known as the radial velocity semiamplitude. This method can tell observers the orbital period and planetary mass of the exoplanet. However, it is not capable of finding the exoplanet's size. The transits method, on the other hand, can measure the size and is often coupled with radial velocity to further find the density. This method measures the dimming of a star's light from planets passing between it and an observer. The key detection signal for transits is the maximum loss of light. Similar to radial velocity, it is optimal for larger, closely orbiting exoplanets.

The final detection method explored was direct imaging. It is still better for larger bodies but has the added advantage of being able to identify the exoplanets that are more separated from their star. This method involves taking an infrared light picture of the system and, after removing the overwhelming glare of the star with advanced post-processing, can reveal the light of the exoplanet. The key detection signal for this method is the exoplanet-star contrast.

Methods

Assumptions

For all detection methods, the following mass-radius relationships were used to visualize the detection limits in terms of both exoplanet mass and exoplanet radius. This relationship is specific to exoplanet type, with a different proportionality power value for rocky worlds, ice giants, and gas giants:

$$\text{Rocky planets: } R \sim M^{0.28} \Rightarrow M \sim R^{1/0.28}$$

$$\text{Neptunian worlds: } R \sim M^{0.59} \Rightarrow M \sim R^{1/0.59}$$

$$\text{Jovian worlds: } R \sim M^{0.04} \Rightarrow M \sim R^{1/0.04}$$

These relationship values are given in Chen & Kipping 2016 (from lecture slides). Planets were coarsely categorized into these 3 bins by mass, with planets of mass $M_P \leq 2M_\oplus$ classified as rocky terrestrial planets, mass of $2M_\oplus < M_P \leq 100M_\oplus$ as icy Neptunian worlds, and mass $M_P > 100M_\oplus$ classified as Jovian gas giants.

In the radial velocity formula, the following assumptions are made by fixing radial velocity, star mass, and observation angle to constant values:

$$K = 0.5\ m/s \text{ (State-of-the-art detection limit)}$$

$$M_\star = 0.5M_\odot$$

$$i = \frac{\pi}{2} \text{ (Edge-on orbit)}$$

These assumed values are consistent with those used in lecture.

Calculations

1. Transit Method

$$\left(\frac{R_P}{R_\oplus}\right)^2 = 3\sqrt{\frac{P}{T}} \quad (1)$$

Equation (1) is given from lecture notes. Rearranging (1) in terms of planetary radius R_P, we obtain:

$$R_P = R_\oplus \left\{3\left(\frac{P}{T}\right)^{1/2}\right\}^{1/2} \quad (2)$$

This allows for the plotting of the radius detection limit line over orbital period for the transit method.

From Kepler's Third Law, we establish the proportionality relationship between the semi-major axis and orbital period. Using this, the team is able to visualize the detection limit against the semi-major axis additionally.

$$a^3 \sim T^2 \Rightarrow a \sim T^{2/3} \quad (3)$$

The detection line is also computed in terms of planetary mass through the mass-radius relationship established prior in the Assumptions section. Applying the relationship to (2) yields:

$$M_P \sim R_P^{\ 1/x} = \left(R_\oplus \left\{3\left(\frac{P}{T}\right)^{1/2}\right\}^{1/2}\right)^{1/x} \quad (4)$$

Where x takes on the value 0.28, 0.59, or 0.04 for rocky, Neptunian, and Jovian planets respectively.

2. Radial Velocity Method

$$K = \frac{M_P}{M_\star}\sqrt{\frac{GM_\star}{a}} sin\ i \quad (5)$$

Equation (5) is given in lecture notes. Applying the assumption $i = \frac{\pi}{2}$ for an edge-on orbit case, the $sin\ i$ term in (5) becomes a factor of 1 and is no longer expressed explicitly. Rearranging in terms of planetary mass, the team obtained:

$$M_P = KM_\star\sqrt{\frac{a}{GM_\star}} \quad (6)$$

This result enables the team to plot the radial velocity limit in terms of planetary mass over semi-major axis. Applying the mass-radius relationships the team obtained the formula:

$$R_P \sim M_P^{\ x} = \left(KM_\star\sqrt{\frac{a}{GM_\star}}\right)^x \quad (7)$$

Where x is specific to planet type as established. The team used (7) to plot the radial velocity detection limit in terms of planetary radius. To visualize the detection limit against orbital period, the team recalled the semi-major axis-orbital period relationship from Kepler's Third Law. Rearranging (3) for period, the team was able to express the semi-major axis proportional to orbital period shown below in (8).

$$T \sim a^{3/2} \quad (8)$$

3. Direct Imaging

Direct imaging is the process of taking an image of the thermal (infrared) emissions of a planetary system. The light from the parent star is blocked out using a device called a coronagraph. Since planet's emit most of their radiation in the infrared spectrum, it makes it easier to resolve them from their parent star. The ratio of flux of the parent star and the planet gives us a value called 'contrast'. The contrast of a planet from its parent star can be calculated using the equation (9).

$$C_{lim} = \frac{f_{planet}}{f_{star}} = \left(\frac{R_{planet}}{R_{star}}\right)^2 \frac{e^{\frac{hc}{\lambda k T_{planet}}} - 1}{e^{\frac{hc}{\lambda k T_{star}}} - 1} \quad (9)$$

The contrast limit (C_{lim}) varies for each telescope. Currently, the world's best telescope, the James Webb Space Telescope (JWST), has a contrast limit of 10^{-6}[2]. We can rearrange equation (9) to solve for the radius of the planet. It is evident from equation (9) that we can detect planets with smaller radii if they have higher temperatures.

The James Webb telescope has an angular resolution of about 1" (1 arcsecond)[3]. So in order to find out the minimum semi-major axis required in order to see the exoplanet we can use the following equation,

$$a = \theta \times D \; (10),$$

where *a* is the semi-major axis and D is the distance of the exoplanet system from Earth.

Results

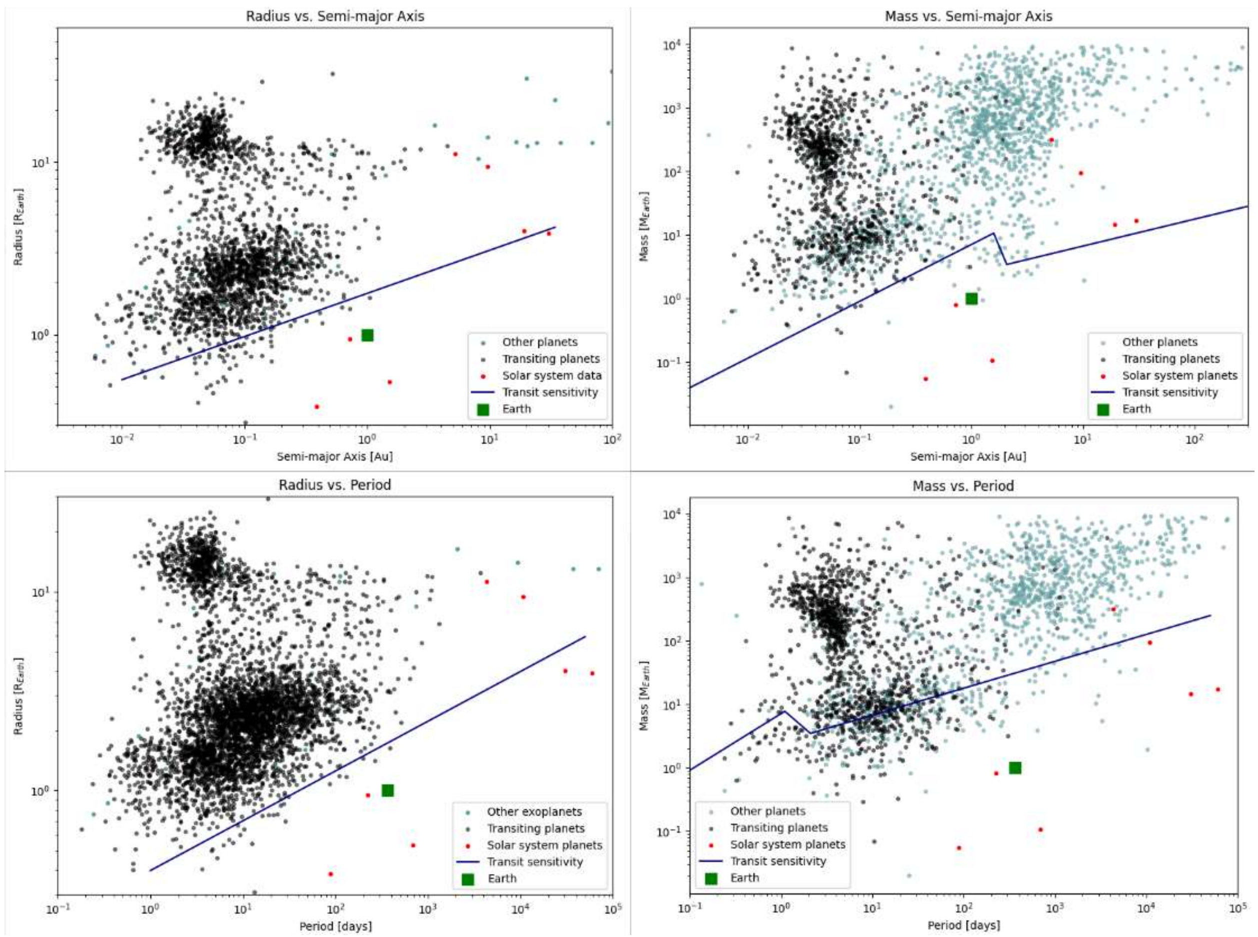

Figure 1. Transit Method Plots

Seen above in Figure 1, the team's results for the transit method are displayed. Each subplot shows the transiting planets in black, exoplanets detected by other methods than transit shown in light blue, solar system planets in red, and Earth in green. The sensitivity line in navy blue delineates the transit method sensitivity limit, with the area beneath the lines representing the parameter space that the transit method is insensitive to. The plots show the exoplanet data and transit method sensitivity in terms of mass and radius as functions of period and semi-major axis. Across these reference frames, the trend in sensitivity is shared with the transit method sensitivity positively correlated with both mass and radius, and negatively correlated with both period and semi-major axis. This result is intuitive as planets with larger periods and semi-major axes transit at lower frequency, and planets with lower masses and radii occlude a smaller portion of their star's flux when transiting, meaning there are fewer opportunities to observe such planets, and the magnitude of the transiting planet's effect is smaller and more challenging to detect.

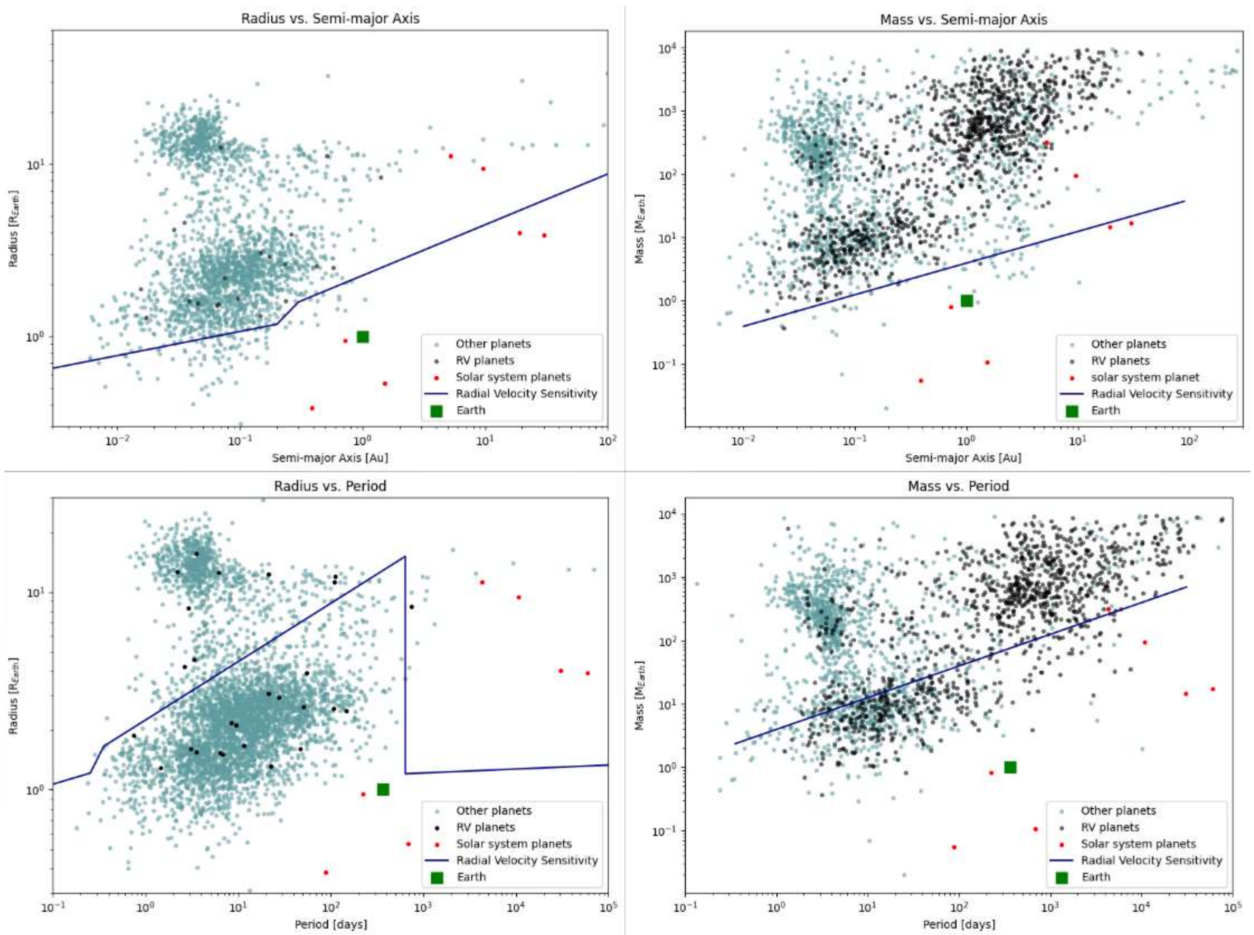

Figure 2. Radial Velocity Method Plots

Above, Figure 2 catalogues the team's results for the radial velocity method with common format to the transit method plots in Figure 1. Overall, a similar trend in sensitivity limits to the transit method is observed, with the radial velocity method insensitive to smaller mass planets and by proxy smaller radius, as well as to planets with larger semi-major axis, and by association larger period. This is also intuitive as a smaller semi-major axis results in a more frequent change in the star's radial velocity and lesser distance attenuation in gravitational influence between the star and planet, and a larger planetary mass directly influences a stronger gravitational force on the star to induce a greater magnitude radial velocity.

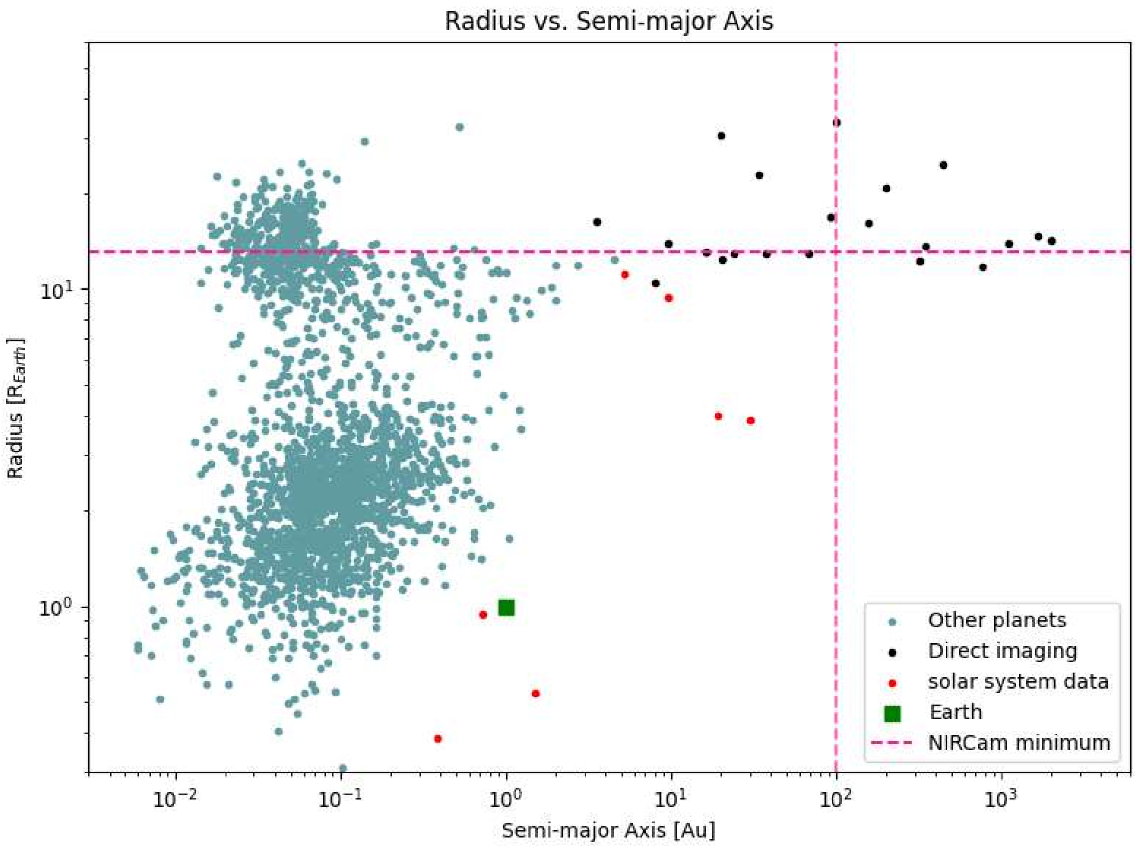

Figure 3. Direct Imaging Method Plot

Figure 3 displays the team's results for the current direct imaging limitations. The pink, vertical dashed-line displays the minimum semi-major axis limit, calculated using equation (10), for an exoplanet system which is 100 parsecs away from the Earth. The pink, horizontal dashed line displays the minimum limit on exoplanet radii. This has been calculated using equation (9), we have used values from JWST documentation as well as assuming that the exoplanet is revolving around a sun-like star. The black points in the top right of figure 3 display the exoplanets that have been detected using direct imaging methods.

The group chose to use a Radius vs. Semi-major Axis plot since it is the best plot to represent the limits of current technology in the detection of exo-planets using direct imaging. This is because the 2 limits can be easily represented as functions of planetary radius (equation 9) and semi-major axis (equation 10).

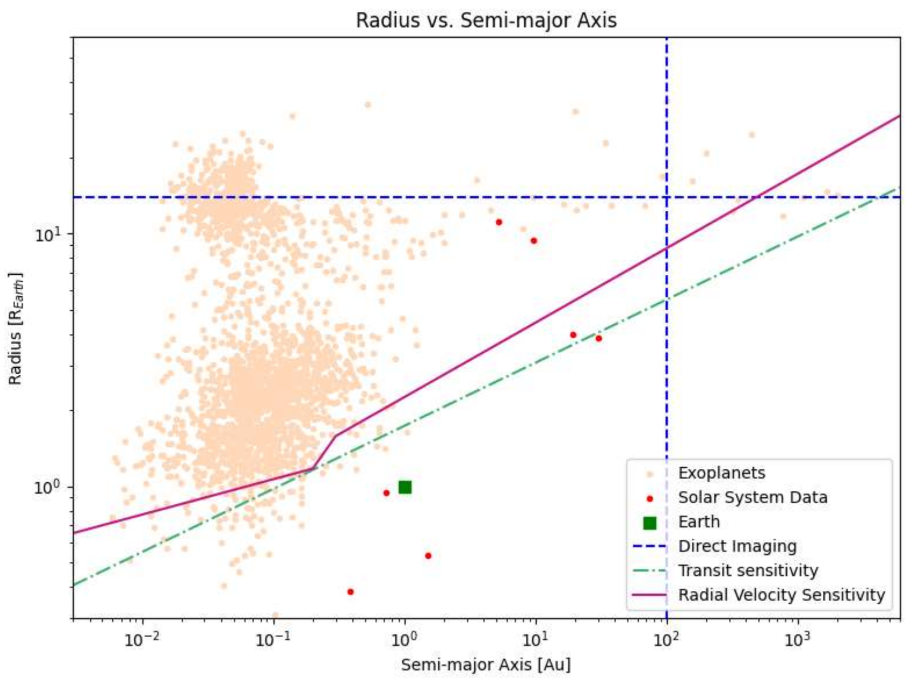

Figure 4. Exoplanet Detection Sensitivity Limits Plot

Regarding part II of the project, calculations for the detection signal of the three methods were performed and can be found in the attachments section below. For the Earth-Sun system the radial velocity semiamplitude works out to be approximately 0.09. The calculation of this parameter involves the orbital period, mass of the planet, and mass of the star. For the Jupiter-Sun system the maximum loss of light measured in transits is approximately 1%, assuming that the planet passes directly between the star and observer. For the Earth-Sun contrast calculations, values of 6000 K and 500 nm for the temperature and thermal radiation wavelength of the Sun were assumed. The contrast comes out to an order of magnitude of 10^{-5}, which is supported by the brightness contrast found in [1].

Conclusions

As seen in Figure 4, Earth is not large enough at its distance from the Sun to be detected by any of the three methods examined in this report. The only solar system planets which are detectable are Saturn and Jupiter, by both radial velocity and transits, and Uranus by transits alone. This is largely due to the fact that these three methods are better fit for detecting larger exoplanets like gas giants. In order to better detect habitable, Earth-like planets advancements in detection technology are required. Specifically to note is the limited capability of direct imaging with only five of the detected exoplanets within its bounds. Finally, transits make up approximately 74% of exoplanet detections (with radial velocity coming in second at 20%), making them the most valuable method by far. Furthermore, direct imaging methods need to be improved to be more effective since current technology only allows us to detect large gas-giants with a large semi-major axis.

Individual Contributions

Technical Work

Part 1 data extraction from NEA	Suhas
Part 1 plotting (exoplanets + solar system)	Suhas
Part 1 plotting (sensitivity limits)	Jackson (RV + Transits) and Suhas (DI)
Part 2 calculations	Andy

Project Presentation

Motivation	Andy
Methods	Jackson
Results	Jackson and Suhas
Conclusions	Andy

Project Report

Introduction and Motivation	Andy
Methods	Jackson and Suhas
Results	Jackson and Suhas
Conclusions	Andy

Attachments

II, Calculating Detection Signal of Different Methods for one of the following cases:
1, A temperate Earth-like planet around a Sun-like star.

Radial Velocity

RV Semiamplitude

$$\boxed{K_1 = \frac{28.4329\ ms^{-1}}{\sqrt{1-e^2}} \frac{m_2 \sin(i)}{M_{jup}} \left(\frac{m_1 + m_2}{M_\odot}\right)^{-1/2} \left(\frac{a}{1\,AU}\right)^{-1/2}}$$

Earth – Sun System:

$m_1 = M_\odot$

$m_2 = M_\oplus$

$i = 90°$ (defined as perpendicular to the line of sight)

$e = 0.0167$

neglect

$$K_1 = \frac{28.4329\ ms^{-1}}{\sqrt{1-e^2}} \frac{M_\oplus \sin(i)}{M_{jup}} \underbrace{\left(\frac{M_\odot + \cancel{M_\oplus}}{M_\odot}\right)^{-1/2}}_{=1} \underbrace{\left(\frac{1\,AU}{1\,AU}\right)^{-1/2}}_{=1}$$

$$= \frac{28.4329}{\sqrt{1-(0.0167)^2}} \left(\frac{1}{317.83}\right) \sin(90°) = 0.0895$$

NSSDC Jupiter Fact Sheet

$$\boxed{K_1 \sim 0.09}$$

2, A Jupiter-like planet around a Sun-like star.

Transits

Transit Depth – Maximum Loss of Light

$$\boxed{\delta_{tra} = k^2 \left[1 - \frac{I_p(t_{tra})}{I_*}\right] \approx \left(\frac{R_p}{R_*}\right)^2}$$

* assuming 90° orbital inclination (passes directly between sun and observer)

$$\delta = \left(\frac{R_\oplus}{R_\odot}\right)^2 = \left(\frac{66854\ km}{695700\ km}\right)^2 = 0.009 \sim 1.0\%$$

$$\boxed{\delta \sim 1.0\ \%}$$

There is an approximate 1% dip in light flux when Jupiter directly transits the sun. The transit method can only reveal the ratio of star to planet radii. An additional detection technique, such as radial velocity, is required to determine the mass and radius of the exoplanet.

Direct Imaging

1, A temperate Earth-like planet around a Sun-like star.

Star - Planet Contrast

$$\boxed{f = \left(\frac{R_p}{R_*}\right)^2 \frac{e^{\left(\frac{h\nu}{k_b T_*}\right)} - 1}{e^{\left(\frac{h\nu}{k_b T}\right)} - 1}}$$

$h = 6.62607015(10^{-34})$ J/Hz (Planck Constant)

$k = 1.380649(10^{-23})$ J/K (Boltzmann Constant)

Allen's Astrophysical Quantities by Arthur Cox

$T_\odot = 6000$ K

$\lambda = 500$ nm

$T_\oplus = 290$ K

$\lambda = 20$ μm

$\nu = \frac{c}{\lambda}$ w/ $c = 3(10^8)$

$$f = \left(\frac{R_\oplus}{R_\odot}\right)^2 \frac{e^{\left(\frac{h\frac{c}{\lambda_\oplus}}{k_b T_*}\right)} - 1}{e^{\left(\frac{h\frac{c}{\lambda_\oplus}}{k_b T_\oplus}\right)} - 1} = 9.2(10^{-4}) \sim 10^{-5}$$

$$\boxed{f \sim 10^{-5}}$$

Sun to Earth brightness contrast $= 63{,}000 \sim 10^5$

The earths brightness is a factor of 10^{-5} less than the suns.

References

[1] Cox, A. N., Allen's astrophysical quantities. 2000.

[2] HCI NIRCam Limiting Contrast (Updated 1 August 2024), https://jwst-docs.stsci.edu/methods-and-roadmaps/jwst-high-contrast-imaging/jwst-high-contrast-imaging-supporting-technical-information/hci-contrast-considerations/hci-nircam-limiting-contrast#gsc.tab=0

[3] NIRCam Coronagraphic Imaging (Updated 19 July 2023), https://jwst-docs.stsci.edu/jwst-near-infrared-camera/nircam-observing-modes/nircam-coronagraphic-imaging#gsc.tab=0

Solar System Planetary data:

[4] Planetary Fact Sheet, NASA (Updated 22 March 2024), https://nssdc.gsfc.nasa.gov/planetary/factsheet/

AI Statement:

We have used AI (ChatGPT) to get a better understanding of how exoplanet detection methods work, especially direct imaging.