

# Sheet Stitching Problem Definition

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## Abstract

This write-up contains the formal problem definition of the Sheet Stitching Problem as a graph problem and a possible function to optimize for.

## 1 Introduction

The herculanuem papyri collection consists of a few hundred still unopened and charred scrolls that were buried by the vesuvius eruption in 79 AD. To read the papyri, they are scanned and a 3D reconstruction is created with computed tomography. Then the scroll is digitally unrolled and the ink made visible with AI. This write-up focuses on a specific step in the ThaumatoAnakalyptor unrolling pipeline. This specific step, called "Sheet Stitching" might be encountered by other, future automatic segmentation pipelines as well.

## 2 ThaumatoAnakalyptor Pipeline

The pipeline of automatic scroll CT papyrus surface extraction has multiple steps. First, extract 3D points of the surface from the raw scan  $V$ . Second, use PointCloud instance segmentation to extract small, instances of points belonging to the same local surface. Create a graph from the instance segmentations and solve it to extract the global sheet connectivity. Finally mesh the segmentation and digitally unroll the mesh.

You can find the complete automatic segmentation pipeline ThaumatoAnakalyptor at <https://github.com/schillij95/ThaumatoAnakalyptor>.

## 3 Instance Segmentation

Pointcloud Instance segmentation describes the step of extracting groups of point from the PointCloud which belong to the same sheet. The scroll volume  $V$  of size  $X, Y, Z$  is divided into overlapping subvolumes  $S$  of size 50, 50, 50. For

$s_{xyz} \in S, s_{xyz} \subseteq V$  the top-right-front corner is at position  $(x, y, z)$  in the scroll volume.

$$\forall s_{xyz} \in S, \quad \forall i \in \{x, y, z\}, \quad i \in \mathbb{Z}^+ \quad \wedge \\ i \bmod 25 \equiv 0 \quad \wedge \quad x \leq X, y \leq Y, z \leq Z$$

The instance segmentation is performed on each subvolumes. For all  $s_{xyz} \in S$  containing the Pointcloud  $PC_{xyz}$ , the instance segmentation model Mask3D creates instances set  $I_{xyz}$ .  $\forall i \in I_{xyz} : i \subseteq PC_{xyz}$  and  $\forall i, j \in I_{xyz}, i \neq j : i \cap j = \emptyset$ .

For an average scroll volume  $V$ , this process creates around 10 million instances.

## 4 Translation to Graph

Let's number the instance patches by integer  $p$ :

$$I_{xyz} = \{i_p \mid p = 0, 1, 2, \dots, n-1\}$$

All nodes can now be defined by:

$$N = \{(x, y, z, p) \mid s_{xyz} \in S, p \in I_{xyz}\}$$

Edges connect nodes from neighboring subvolumes:

$$E = \{((x_0, y_0, z_0, p_0), (x_1, y_1, z_1, p_1)) \mid \\ |x_0 - x_1|, |y_0 - y_1|, |z_0 - z_1| \leq 25\}$$

Let's define the directed, weighted, winding angle graph as  $G = (N, E, w, k)$ .

We define the winding angle difference function  $k : E \rightarrow \mathbb{R}$  as an estimation for the angular rotation that results of traversing the papyrus surface from the first to the second node of the edge with respect to a center axis (called umbilicus).

The estimation of the winding angle difference  $k$  is accurate up to a multiple  $o \in \mathbb{Z}$  of 360 to the actual winding angle difference  $k^*$ , since the position of the instance with respect to the umbilicus is known:

$$k^*(e) = k(e) + o \cdot 360$$

$k$  is antisymmetric:

$$k((n_0, n_1)) = -k((n_1, n_0))$$

The weight function  $w : E \rightarrow \mathbb{R}^+$  estimates if the assignment  $k(e)$  is true.

$w$  is symmetric:

$$w((n_0, n_1)) = w((n_1, n_0))$$

The resulting graph will have edges with conflicting information about the winding angle difference within the graph.

## 5 Sheet Stitching

We seek to find a winding angle assignment function  $f : N \rightarrow \mathbb{R}$ , which assigns each node a winding angle.

### 5.1 Desired Graph Node Assignment Characteristics

A good measure of the accuracy of the found solution  $\tilde{f}$  could be to optimize for:

$$f^* = \arg \max_{f \in F} \sum_{e \in E} w(e) \cdot c(e)$$

where

$$c((n_0, n_1)) = \begin{cases} 1, & \text{if } f(n_1) - f(n_0) = k((n_0, n_1)) \\ 0, & \text{otherwise} \end{cases}$$

Which translates to the least number of wrongly estimated edge winding angle differences.