Assignment due April 5th at 11:59 p.m.

Complete the following problems. These problems are related to Chapters 12 in the Book of Proof. Found at:
https://www.people.vcu.edu/ rhammack/BookOfProof/Main.pdf

Problem 1: For each of the following functions, determine whether the function is injective, surjective, bijective or none. Also, order the functions in a manner such that if a function $f(x)$ is to the left of $g(x)$ in the ordering, then $f(x)$ is $O(g(x))$. You do not need to prove any of your conclusions. Assume the domain and co-domain of each function is $\mathbb{R}$.

We say $f(x)$ is $\mathcal{O}(g(x))$ when f(x) is less than or equal to g(x) to within some constant multiple c

- $a(x) = 2x$

  $a(x)$ is $\mathcal{O}(2x)$

  Injective, Surjective and Bijective

- $b(x) = 2^x$

  $b(x)$ is $\mathcal{O}(2^x)$

  Injective, not Surjective and Bijective

- $c(x) = \log(x)$

  $c(x)$ is $\mathcal{O}()\log(x))$

  Injective, not Surjective and not Bijective

- $d(x) = x^x$

  not Injective, not Surjective and not Bijective

- $e(x) = \frac{1}{x}$

  Injective, Surjective and Bijective

- $f(x) = \frac{x^2}{5}$

  not Injective, not Surjective and not Bijective

- $g(x) = \frac{x^4-1}{7x+7}$

  Injective, Surjective and Bijective

- $h(x) = 918532$

  not Injective, Surjective and not Bijective

For problems 2 through 7, prove each of the statements.

Problem 2: The function $f : \mathbb{Z} \to \mathbb{N}$, $f(n) = n^2$ is not surjective and not injective.

Proof. Assume $a = b$, so $(f(a) = a^2) = (f(b) = b^2)$
we can say that a^2 is equal to b^2
now lets set $a = -1$ and $b = 1$
they both equal 1 yet are different values
therefore the function is not injective.
Now lets assume that $n = 0$
it follows that $f(0) = 0^2$ which is 0.
zero is not in $\mathbb{N}$
therefore, the function is not subjective.

$\square$

Problem 3: The function $f : \mathbb{N} \to \mathbb{Z}$, $f(n) = 3n - 17$ is injective.

Proof. Assume $f(a) = f(b)$ for some $a, b \in \mathbb{Z}$
It follows that $3a - 17 = 3b - 17$
we can simplify to $3a = 3b$ and then to $a = b$
Therefore the function is injective.

$\square$

Problem 4: The function $f : \mathbb{R} \to \mathbb{Z}$, $f(x) = \lfloor x \rfloor$ is surjective. *Note that the floor function, denoted $\lfloor x \rfloor$, just rounds down.*

Proof. $\square$

Problem 5: The function $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = -n + 4$, is a bijection. *Hint: first find its inverse.*

Proof. Lets start by finding the inverse of this f
so $y = -x + 4$, then switch x and y to get $x = -y + 4$
now rewrite with y on it's own side. $y = 4 - x$.
so $f^{-1}(x) = 4 - x$ and $f(x) = -n + 4$
now look at $f(f{-}1(x))$ which is $-(4 - x) + 4$. This is equal to x
They are inverses because this is equal to x.
Therefore f is bijective.

$\square$

Problem 6: $\frac{x^3+17}{4x+5}$ is $O(3x^2)$.

Proof. We say that $f(x)$ is $O(g(x))$ if there exists constants C and k such that $|f(x) \leq C(g(x))$ for all $x > k$
let $c = 1$ and $k = 2$, assume $x > k$.
Then $|\frac{x^3+17}{4x+5}| = \frac{x^3+17}{4x+5}$ because the function will be positive for any $x > 2$
likewise, $|\frac{x^3+17}{4x+5}| > x^3 + 17$ because removing the $4x + 5$ makes it a bigger function.
also, removing the $+17$ makes it a bigger function.
If we multiple the x^3 by 3, it is a larger function. Then we have $3x^3$
This can be written as $3|x^3|$. Therefore $f(x)$ is $O(g(x))$

□

Problem 7: $n \log(n)$ is not $O(100n)$. Assume log base 10.