Assignment due March 15th at 11:59 p.m.

Complete the following problems from Chapters 4, 5, and 6 of Book of Proof. Found at: https://www.people.vcu.edu/ rhammack/BookOfProof/Main.pdf

Prove each of the following statements:

Problem 4.10 Suppose a and b are integers. If $a|b$, then $a|(3b^3 - b^2 + 5b)$.
Problem 4.14 If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd.
Problem 4.26 Every odd integer is a difference of two squares. Example: $7 = 4^2 - 3^2$.
Problem 5.12 Suppose $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd.
Problem 5.28 If $n \in \mathbb{Z}$, then $4 \nmid (n^2 - 3)$.
Problem 6.8 Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even

4.10
Proposition Suppose a and b are integers. If $a|b$, then $a|(3b^3 - b^2 + 5b)$.

Proof. we say $a|b$ for some $z \in \mathbb{Z}$ for $b = az$
Therfore, we can write $3b^3 - b^2 + 5b$ as $3(az)^3 - (az)^2 + 5(az)$.
Then factor out a to get $a(3a^2b^3 - az^2 + 5z)$.
now a is multiplied to some constant and the product is $(3b^3 - b^2 + 5b)$
so the expression is true.

□

Proposition If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd

Proof. We say a number is odd if x=2k+1 for some integer k.
assume 2 cases, n can be either even or odd.
firstly, if n is odd.
then we can say that $5n^2$ can be written as $5(2k + 1)^2$
this can be rewritten to $5(2k + 1)(2k + 1)$ then to
$20k^2 + 10k + 10K + 2(2) + 1$, then $2(10k^2 + 10k + 2) + 1$
if we let $10k^2 + 10k + 2 = j$, then $5n^2$ is odd because it can be written as $2j + 1$
next, look at $3n$, if n is odd, it can be written as $3(2k + 1)$
multiply out to get $6k + 3$, rewrite this to $2(3k + 1) + 1$
let $3k + 1$ equal h, so $3n$ is odd because it can be written as $2h + 1$
Now that we know that $5n^2$ and $3n$ are both odd assuming that n is even
so adding these two terms together will result in a even number because
$(2j + 1) + (2h + 1)$ or $2j + 2h + 2$, which can be written as $2(j + h)$ which is a even number.
Now lets assume that n is even. An even number can be written as 2k for some interger k.
looking at $5n^2$, we can write it as $5(2k)^2$, or $2(10k^2)$
this number is even because if we write $10k^2$ as j, then it is $2j$
also, $3n$ is even because we can write it as $3(2k)$, or $6k$
which can then be written as $2(3k)$, if we make $3k$ h, then it would be $2h$
Now that they are both even if n is even, adding them together will also get a even number.
This is because, $2j + 2h$, can be written as $2(j + h)$

now that we know adding the first two terms will result in an even number wether n is even or odd
we can prove 7 is odd because it can be written as $2(3) + 1$,
we know the first 2 terms added together will be even, so lets write them as 2j,
so we have $2j + 2(3) + 1$, then a 2 can be factored out, $2(j + 3) + 1$.
Therfore, $5n^2 + 3n + 7$ is odd if $n \in Z$

$\square$

Proposition Every odd integer is a difference of two squares.

Proof. An integer is odd if it equals $2k + 1$ for some integer k.
now, lets look at 2 consecutive integers squared, written as k and k+1.
$(k + 1)^2 - k^2$, rewrite as $k^2 + 2k + 1 - k^2$. This is the same as $2k + 1$
thus, any prime can equal the difference of two squares.

$\square$

Proposition Suppose $a \in \mathbb{Z}$. if a^2 is not divisible by 4, then a is odd.

Proof. We say that $a|b$ for some $z \in \mathbb{Z}$ for $b = az$
lets assume that a is even by way of contradicton. $a = 2k$ by definition of an even number.
then a^2 would be equal to $4k^2$. a^2 is divisible by 4 because $a^2 = 4z$, with $z = k^2$.
So for a^2 to be divisble by 4, it must be even.

$\square$

Proposition If $n \in \mathbb{Z}$, then $4 \not| (n^2 - 3)$

Proof. By way of contradiction, assume $4|(n^2 - 3)$
we say that $a|b$ for some $z \in \mathbb{Z}$ for $b = az$
so $n^2 - 3 = 4z$.
Then there are two cases to check, n is even or odd.
first, assume n is even.
so $(2k)^2 - 3 = 4z$, and rewrite it as $4k^2 - 3 = 4z$ and then solve for z.
therefore $z = k^2 - 3/4$. this will not result in an integer
likewise, assume n is odd.
so $(2k + 1)^2 - 3 = 4z$, and rewrite it as $4k^2 + 4k - 2 = 4z$ and then solve for z.
Therefore $z = k^2 + 4k - \frac{1}{2}$. This case will also not result in an integer.

$\square$

Proposiiton Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even

Proof. By way of contradiction, lets assume that both a and b are odd.
then we can write it as: $(2k + 1)^2 + (2j + 1)^2 = c^2$
so $4(k^2 + k + j^2 + j) + 2 = c^2$
then we know c can be either even or odd.
assuming c is even, we get : $4h^2 = 4(k^2 + k + j^2 + j) + 2$
after dividing out the 4, there will be a fraction on one side.
now lets assume that c is odd.
so we get: $4(h^2 + h) + 1 = 4(k^2 + k + j^2 + j) + 2 = c^2$

after dividing out the four, there will be unequal fractions on both sides.
therfore it is not an integer for either case.

$\square$