

Assignment due April 5th at 11:59 p.m.

Complete the following problems. These problems are related to Chapters 12 in the Book of Proof. Found at:

<https://www.people.vcu.edu/~rhammack/BookOfProof/Main.pdf>

Problem 1: For each of the following functions, determine whether the function is injective, surjective, bijective or none. Also, order the functions in a manner such that if a function $f(x)$ is to the left of $g(x)$ in the ordering, then $f(x)$ is $O(g(x))$. You do not need to prove any of your conclusions. Assume the domain and co-domain of each function is \mathbb{R} .

We say $f(x)$ is $O(g(x))$ when $f(x)$ is less than or equal to $g(x)$ to within some constant multiple c

- $a(x) = 2x$
 $a(x)$ is $O(2x)$
 Injective, Surjective and Bijective
- $b(x) = 2^x$
 $b(x)$ is $O(2^x)$
 Injective, not Surjective and Bijective
- $c(x) = \log(x)$
 $c(x)$ is $O(\log(x))$
 Injective, not Surjective and not Bijective
- $d(x) = x^x$
 not Injective, not Surjective and not Bijective
- $e(x) = \frac{1}{x}$
 Injective, Surjective and Bijective
- $f(x) = \frac{x^2}{5}$
 not Injective, not Surjective and not Bijective
- $g(x) = \frac{x^4-1}{7x+7}$
 Injective, Surjective and Bijective
- $h(x) = 918532$
 not Injective, Surjective and not Bijective

For problems 2 through 7, prove each of the statements.

Problem 2: The function $f : \mathbb{Z} \rightarrow \mathbb{N}$, $f(n) = n^2$ is not surjective and not injective.

Proof. Assume $a = b$, so $(f(a) = a^2) = (f(b) = b^2)$

we can say that a^2 is equal to b^2

now lets set $a = -1$ and $b = 1$

they both equal 1 yet are different values

therefore the function is not injective.

Now lets assume that $n = 0$

it follows that $f(0) = 0^2$ which is 0.

zero is not in \mathbb{N}

therefore, the function is not surjective.

□

Problem 3: The function $f : \mathbb{N} \rightarrow \mathbb{Z}$, $f(n) = 3n - 17$ is injective.

Proof. Assume $f(a) = f(b)$ for some $a, b \in \mathbb{Z}$

It follows that $3a - 17 = 3b - 17$

we can simplify to $3a = 3b$ and then to $a = b$

Therefore the function is injective.

□

Problem 4: The function $f : \mathbb{R} \rightarrow \mathbb{Z}$, $f(x) = \lfloor x \rfloor$ is surjective. *Note that the floor function, denoted $\lfloor x \rfloor$, just rounds down.*

Proof.

□

Problem 5: The function $f : \mathbb{Z} \rightarrow \mathbb{Z}$, $f(n) = -n + 4$, is a bijection. *Hint: first find its inverse.*

Proof. Lets start by finding the inverse of this f

so $y = -x + 4$, then switch x and y to get $x = -y + 4$

now rewrite with y on it's own side. $y = 4 - x$.

so $f^{-1}(x) = 4 - x$ and $f(x) = -x + 4$

now look at $f(f^{-1}(x))$ which is $-(4 - x) + 4$. This is equal to x

They are inverses because this is equal to x .

Therefore f is bijective.

□

Problem 6: $\frac{x^3+17}{4x+5}$ is $O(3x^2)$.

Problem 7: $n \log(n)$ is not $O(100n)$. Assume \log base 10.