

**Assignment due April 5th at 11:59 p.m.**

Complete the following problems. These problems are related to Chapters 12 in the Book of Proof. Found at:

<https://www.people.vcu.edu/~rhammack/BookOfProof/Main.pdf>

**Problem 1:** For each of the following functions, determine whether the function is injective, surjective, bijective or none. Also, order the functions in a manner such that if a function  $f(x)$  is to the left of  $g(x)$  in the ordering, then  $f(x)$  is  $O(g(x))$ . You do not need to prove any of your conclusions. Assume the domain and co-domain of each function is  $\mathbb{R}$ .

- $a(x) = 2x$   
Injective, Surjective and Bijective
- $b(x) = 2^x$   
Injective, not Surjective and Bijective
- $c(x) = \log(x)$   
Injective, not Surjective and not Bijective
- $d(x) = x^x$   
not Injective, not Surjective and not Bijective
- $e(x) = \frac{1}{x}$   
Injective, Surjective and Bijective
- $f(x) = \frac{x^2}{5}$   
not Injective, not Surjective and not Bijective
- $g(x) = \frac{x^4-1}{7x+7}$   
Injective, Surjective and Bijective
- $h(x) = 918532$   
not Injective, Surjective and not Bijective

*For problems 2 through 7, prove each of the statements.*

**Problem 2:** The function  $f : \mathbb{Z} \rightarrow \mathbb{N}$ ,  $f(n) = n^2$  is not surjective and not injective.

**Problem 3:** The function  $f : \mathbb{N} \rightarrow \mathbb{Z}$ ,  $f(n) = 3n - 17$  is injective.

**Problem 4:** The function  $f : \mathbb{R} \rightarrow \mathbb{Z}$ ,  $f(x) = \lfloor x \rfloor$  is surjective. *Note that the floor function, denoted  $\lfloor x \rfloor$ , just rounds down.*

**Problem 5:** The function  $f : \mathbb{Z} \rightarrow \mathbb{Z}$ ,  $f(n) = -n + 4$ , is a bijection. *Hint: first find its inverse.*

**Problem 6:**  $\frac{x^3+17}{4x+5}$  is  $O(3x^2)$ .

**Problem 7:**  $n \log(n)$  is not  $O(100n)$ . Assume log base 10.