

Assignment due April 19th at 11:59 p.m.

Complete the following problems. These problems are related to Chapters 11 in the Book of Proof. Found at:

<https://www.people.vcu.edu/~rhammack/BookOfProof/Main.pdf>

Problem 1: Consider the relation $R = \{(a, b), (a, c), (c, c), (b, b), (c, b), (b, c)\}$ on the set $A = \{a, b, c\}$. Is R reflexive, symmetric, and/or transitive? For each property, briefly explain your answer.

- Reflexive: The relation is not reflexive because a does not directly relate to a .
- Symmetric: The relation is not symmetric because not every pair has a exact reverse. For example, the pair (a, b) does not have a matching (b, a) pair.
- Transitive: The relation is transitive because a relates to c and c relates to b .

Problem 2: Let $S = \{a, b, c\}$. Let relation $R = \{(a, b), \dots\}$ be a relation on set S . For each of the following sets of properties, provide a complete list of ordered pairs in R .

1. Reflexive, Symmetric, Not Transitive

$$R = \{(a, a), (b, b), (c, c), (a, b), (b, a)\}$$

2. Not Reflexive, Symmetric, Transitive

$$R = \{(a, a), (b, b), (a, b), (b, c), (c, a)\}$$

3. Reflexive, Not Symmetric, Transitive

$$R = \{(a, a), (b, b), (c, c), (a, b), (b, c), (c, a)\}$$

4. Not Reflexive, Not Symmetric, Transitive

$$\{(a, b), (b, c), (c, a)\}$$

Problem 3: Prove that the divides relation on the set \mathbb{Z} is reflexive, transitive, and not symmetric. Note that the divides relation is denoted as $a|b$, which means $b = ka$ for some integer k .

Proof. For a relation to be reflexive, every element must relate to itself.

We know that $a|b$ means $b = ka$ for some $k \in \mathbb{Z}$.

Assume $k = 1$, so $b = (1)a$.

Therefore the divides relation is reflexive.

For a relation to be symmetric, every pair must have a matching pair in opposite order.

So lets say $a|b$ and $b = ka$ for some $k \in \mathbb{Z}$. For this to be true, $b|a$ must also be true.

$b|a$ mean $a = kb$ for some $k \in \mathbb{Z}$

So $b = ka$ and $a = kb$. This cannot be true because solving for a in the first equation would get $b/k = a$. this does not agree with the second function

therefore it is not symmetric.

□