

Assignment due March 15th at 11:59 p.m.

Complete the following problems from Chapters 4, 5, and 6 of Book of Proof. Found at:
<https://www.people.vcu.edu/~rhammack/BookOfProof/Main.pdf>

Prove each of the following statements:

Problem 4.10 Suppose a and b are integers. If $a|b$, then $a|(3b^3 - b^2 + 5b)$.

Problem 4.14 If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd.

Problem 4.26 Every odd integer is a difference of two squares. Example: $7 = 4^2 - 3^2$.

Problem 5.12 Suppose $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd.

Problem 5.28 If $n \in \mathbb{Z}$, then $4 \nmid (n^2 - 3)$.

Problem 6.8 Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even

4.10

Proposition Suppose a and b are integers. If $a|b$, then $a|(3b^3 - b^2 + 5b)$.

Proof.

□

Proposition If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd

Proof. We say a number is odd if $x=2k+1$ for some integer k .

assume 2 cases, n can be either even or odd.

firstly, if n is odd.

then we can say that $5n^2$ can be written as $5(2k+1)^2$

this can be rewritten to $5(2k+1)(2k+1)$ then to

$20k^2 + 10k + 10k + 2(2k+1) + 1$, then $2(10k^2 + 10k + 2) + 1$

if we let $10k^2 + 10k + 2 = j$, then $5n^2$ is odd because it can be written as $2j + 1$

next, look at $3n$, if n is odd, it can be written as $3(2k+1)$

multiply out to get $6k + 3$, rewrite this to $2(3k+1) + 1$

let $3k+1$ equal h , so $3n$ is odd because it can be written as $2h + 1$

Now that we know that $5n^2$ and $3n$ are both odd assuming that n is even

so adding these two terms together will result in a even number because

$(2j+1) + (2h+1)$ or $2j+2h+2$, which can be written as $2(j+h)$ which is a even number.

Now lets assume that n is even. An even number can be written as $2k$ for some interger k .

looking at $5n^2$, we can write it as $5(2k)^2$, or $2(10k^2)$

this number is even because if we write $10k^2$ as j , then it is $2j$

also, $3n$ is even because we can write it as $3(2k)$, or $6k$

which can then be written as $2(3k)$, if we make $3k$ h , then it would be $2h$

Now that they are both even if n is even, adding them together will also get a even number.

This is because, $2j+2h$, can be written as $2(j+h)$

now that we know adding the first two terms will result in an even number wether n is even or odd

we can prove 7 is odd because it can be written as $2(3) + 1$,

we know the first 2 terms added together will be even, so lets write them as $2j$,

so we have $2j + 2(3) + 1$, then a 2 can be factored out, $2(j+3) + 1$.

Therefore, $5n^2 + 3n + 7$ is odd if $n \in \mathbb{Z}$

□

Proposition Every odd integer is a difference of two squares.

Proof. An integer is odd if it equals $2k + 1$ for some integer k .

now, let's look at 2 consecutive integers squared, written as k and $k+1$.

$(k+1)^2 - k^2$, rewrite as $k^2 + 2k + 1 - k^2$. This is the same as $2k + 1$

thus, any prime can equal the difference of two squares.

□

Proposition Suppose $a \in \mathbb{Z}$. if a^2 is not divisible by 4, then a is odd.

Proof. We say that $a|b$ for some $z \in \mathbb{Z}$ for $b = az$

let's assume that a is even by way of contradiction. $a = 2k$ by definition of an even number.

then a^2 would be equal to $4k^2$. a^2 is divisible by 4 because $a^2 = 4z$, with $z = k^2$.

So for a^2 to be divisible by 4, it must be even.

□

Proposition If $n \in \mathbb{Z}$, then $4 \nmid (n^2 - 3)$

Proof. By way of contradiction, assume $4|(n^2 - 3)$

we say that $a|b$ for some $z \in \mathbb{Z}$ for $b = az$

so $n^2 - 3 = 4z$.

Then there are two cases to check, n is even or odd.

first, assume n is even.

so $(2k)^2 - 3 = 4z$, and rewrite it as $4k^2 - 3 = 4z$ and then solve for z .

therefore $z = k^2 - 3/4$. this will not result in an integer

likewise, assume n is odd.

□