Assignment due April 19th at 11:59 p.m.

Complete the following problems. These problems are related to Chapters 11 in the Book of Proof. Found at:
https://www.people.vcu.edu/ rhammack/BookOfProof/Main.pdf

Problem 1: Consider the relation $R = \{(a, b), (a, c), (c, c), (b, b), (c, b), (b, c)\}$ on the set $A = \{a, b, c\}$. Is R reflexive, symmetric, and/or transitive? For each property, briefly explain your answer.

- Reflexive: The relation is not reflexive because a does not directly relate to a.
- Symmetric: The relation is not symmetric because not every pair has a exact reverse. For example, the pair (a,b) does not have a matching (b,a) pair.
- Transitive: The relation is transitve because a relates to c and c relates to b.

Problem 2: Let $S = \{a, b, c\}$. Let relation $R = \{(a, b), ...\}$ be a relation on set S. For each of the following sets of properties, provide a complete list of ordered pairs in R.

1. Reflexive, Symmetric, Not Transitive
   $R = \{(a, a), (b, b), (c, c), (a, b), (b, a)\}$
2. Not Reflexive, Symmetric, Transitive
   $R = \{(a, a), (b, b), (a, b), (b, c), (c, a)\}$
3. Reflexive, Not Symmetric, Transitive
   $R = \{(a, a), (b, b), (c, c), (a, b), (b, c), (c, a)\}$
4. Not Reflexive, Not Symmetric, Transitive
   $\{(a, b), (b, c), (c, a)\}$

Problem 3: Prove that the divides relation on the set $\mathbb{Z}$ is reflexive, transitive, and not symmetric. Note that the divides relation is denoted as $a|b$, which means $b = ka$ for some integer k.

Proof. For a relation to be reflexive, every element must relate to itself.
We know that $a|b$ means $b = ka$ for some $k \in \mathbb{Z}$.
Assume $k = 1$, so $b = (1)a$.
Therfore the divides relation is reflexive.
For a relation to be symmetric, every pair must have a matching pair in opposite order.
So lets say $a|b$ and $b = ka$ for some $k \in \mathbb{Z}$. For this to be true, $b|a$ must also be true.
$b|a$ mean $a = kb$ for some $k \in \mathbb{Z}$
So $b = ka$ and $a = kb$. This cannot be true because solving for a in the first equation would get $b/k = a$. this does not agree with the second function

therefore it is not symmetric.
For the relation to be transitive, if $a|b$ and $b|c$, then a must divide c.
if $a|b$ then $b = ka$. so then $ka|c$ then $c = j(ka)$ for some integer j.
so for $a|c$ we can substitute $a|jka$, see that jk is some constant times a, so $a|c$

$\square$