Assignment due April 5th at 11:59 p.m.

Complete the following problems. These problems are related to Chapters 12 in the Book of Proof. Found at:
https://www.people.vcu.edu/ rhammack/BookOfProof/Main.pdf

Problem 1: For each of the following functions, determine whether the function is injective, surjective, bijective or none. Also, order the functions in a manner such that if a function $f(x)$ is to the left of $g(x)$ in the ordering, then $f(x)$ is $O(g(x))$. You do not need to prove any of your conclusions. Assume the domain and co-domain of each function is $\mathbb{R}$.

- $a(x) = 2x$

  Injective, Surjective and Bijective
- $b(x) = 2^x$

  Injective, not Surjective and Bijective
- $c(x) = \log(x)$

  Injective, not Surjective and not Bijective
- $d(x) = x^x$

  not Injective, not Surjective and not Bijective
- $e(x) = \frac{1}{x}$

  Injective, Surjective and Bijective
- $f(x) = \frac{x^2}{5}$

  not Injective, not Surjective and not Bijective
- $g(x) = \frac{x^4-1}{7x+7}$
- $h(x) = 918532$

For problems 2 through 7, prove each of the statements.

Problem 2: The function $f : \mathbb{Z} \to \mathbb{N}$, $f(n) = n^2$ is not surjective and not injective.

Problem 3: The function $f : \mathbb{N} \to \mathbb{Z}$, $f(n) = 3n - 17$ is injective.

Problem 4: The function $f : \mathbb{R} \to \mathbb{Z}$, $f(x) = \lfloor x \rfloor$ is surjective. *Note that the floor function, denoted $\lfloor x \rfloor$, just rounds down.*

Problem 5: The function $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = -n + 4$, is a bijection. *Hint: first find its inverse.*

Problem 6: $\frac{x^3+17}{4x+5}$ is $O(3x^2)$.

Problem 7: $n\log(n)$ is not $O(100n)$. Assume log base 10.