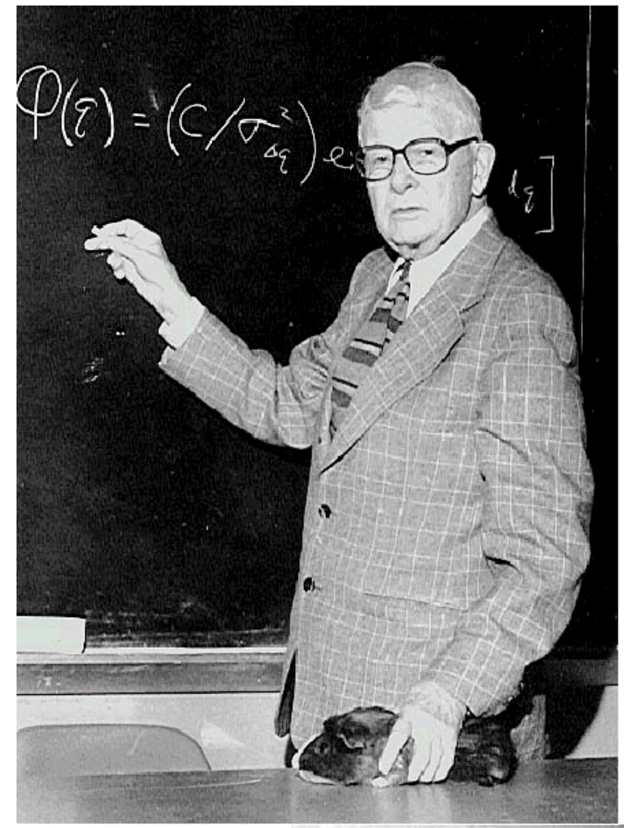
# Week 6

Simulation and Modeling Evolution

# Population genetics

Sewell Wright



J.B.S. Haldane

- Microevolution
- Change in the genetic make up of populations $p_t \xrightarrow{\text{evolution}} p_{(t+1)}$
- Evolutionary forces
- Deterministic forces
  - Selection
  - Migration
  - [Mutation]
- Stochastic forces
  - Genetic drift
  - [Recombination]
- Population genetics as a balance of interacting forces

R. A. Fisher

# What do Mendelian alleles do in a population?

Godfrey Harold Hardy
(1877 – 1947)

Hardy-Weinberg
Equilibrium

Wilhelm Weinberg
(1862 — 1937)

These two gentlemen figured it out.

# Minding Your Ps and Qs

| | | | |
|---|---|---|---|
| Genotype: | $A_1A_1$ | $A_1A_2$ | $A_2A_2$ |
| Relative Frequency: | $x_{11}$ | $x_{12}$ | $x_{22}$ |

Constraint:

$$x_{11} + x_{12} + x_{22} = 1$$

Treat genotype/allele frequencies as random variables

# Minding Your Ps and Qs

First calculate alleles freqs

Freq $A_1$ allele

$$p = x_{11} + \frac{1}{2}x_{12}$$

Freq $A_2$ allele

$$q = 1 - p = x_{22} + \frac{1}{2}x_{12}$$

Where is this coming from?

# Minding Your Ps and Qs

## Numerical Example

Sickle Cell vs. Normal Hemoglobin in Sub Saharan Village

| | | | |
|---|---|---|---|
| Genotype: | $A_1A_1$ | $A_1A_2$ | $A_2A_2$ |
| Observed Numbers: | 411 | 1404 | 185 |
| Relative Frequencies: | 0.2055 | 0.702 | 0.0925 |

Calculate the frequency of $A_1$ allele

# Minding Your Ps and Qs

## Numerical Example

Sickle Cell vs. Normal Hemoglobin in Sub Saharan Village

| Genotype: | $A_1A_1$ | $A_1A_2$ | $A_2A_2$ |
|---|---|---|---|
| Observed Numbers: | 411 | 1404 | 185 |
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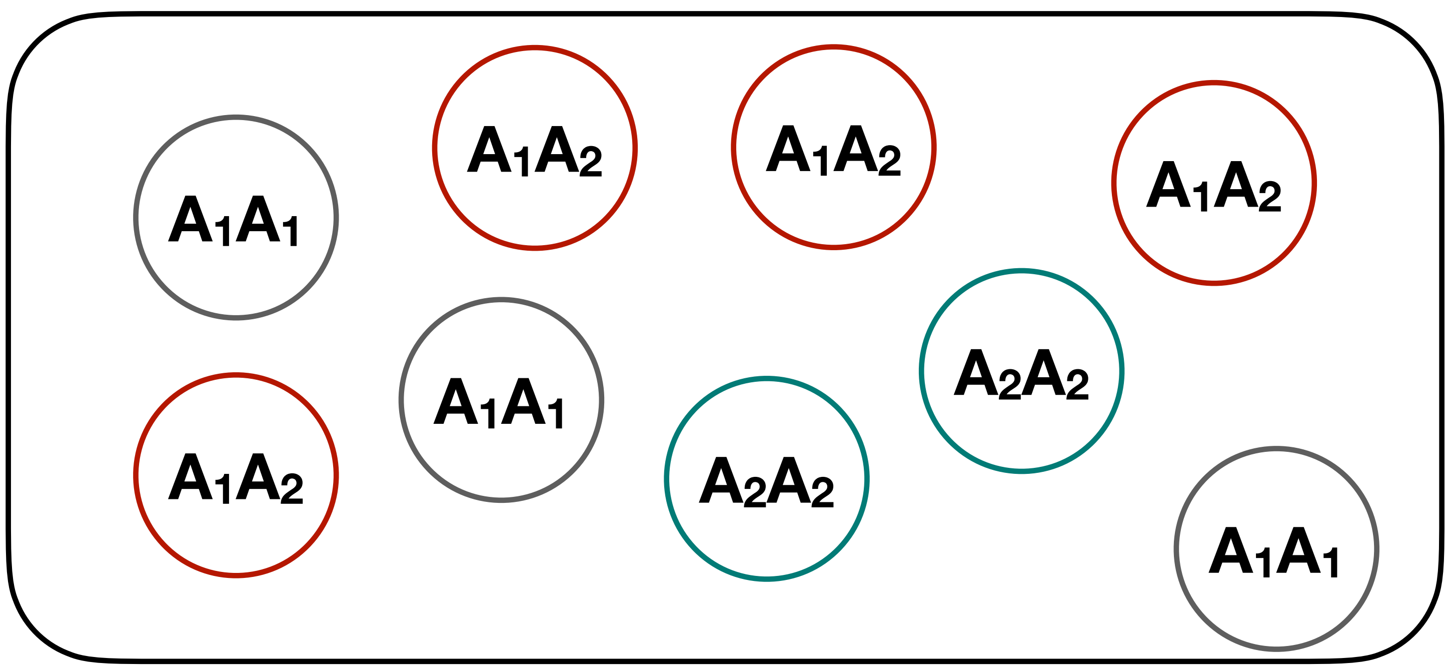
Calculate the frequency of $A_1$ allele

$$p = 0.2055 + (\frac{1}{2} \times 0.702) = 0.5565$$

# Minding Your Ps and Qs

Consider this "experiment":

1) Sample a genotype from popn
2) Sample an allele from that genotype




$$x_{11} = P(A_1A_1)$$
$$x_{12} = P(A_1A_2)$$
$$x_{22} = P(A_2A_2)$$

Probability of getting $A_1$ allele, p, is equal to

$$p = (x_{11} \times 1) + (x_{12} \times \frac{1}{2}) + (x_{22} \times 0)$$

# The Hardy-Weinberg Law

**Assumptions:**

- Diploid sexual population
- Infinite population size
- Random mating
- no selection
- no migration
- no mutation

This is a Null model. Why is this useful?

# Hardy-Weinberg

Write down genotype freqs in terms of allele freqs

| | | | |
|---|---|---|---|
| Genotype: | $A_1A_1$ | $A_1A_2$ | $A_2A_2$ |
| H-W Frequency: | $p^2$ | $2pq$ | $q^2$ |

$$p^2 + 2pq + q^2 = 1$$

Again ask yourself, where is this coming from?

# Hardy-Weinberg

Write down genotype freqs in terms of allele freqs

| | | | |
|---|---|---|---|
| Genotype: | $A_1A_1$ | $A_1A_2$ | $A_2A_2$ |
| H-W Frequency: | $p^2$ | $2pq$ | $q^2$ |

Prob($A_1$ sperm & $A_1$ egg)

Prob($A_1$ sperm & $A_2$ egg)
+
Prob($A_2$ sperm & $A_1$ egg)

Prob($A_2$ sperm & $A_2$ egg)

# Hardy-Weinberg

## Numerical Example

Back to sickle cell anemia…

| | | | |
|---|---|---|---|
| Genotype: | $A_1A_1$ | $A_1A_2$ | $A_2A_2$ |
| H-W Frequency: | $p^2$ | $2pq$ | $q^2$ |
| Observed Frequencies: | 0.2055 | 0.702 | 0.0925 |
| H-W Expected Frequencies: | 0.309 | 0.494 | 0.197 |

# Hardy-Weinberg

## Numerical Example

Back to sickle cell anemia…

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| Genotype: | $A_1A_1$ | $A_1A_2$ | $A_2A_2$ |
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| Observed Frequencies: | 0.2055 | 0.702 | 0.0925 |
| H-W Expected Frequencies: | 0.309 | 0.494 | 0.197 |

Pretty big deviations, could assess how big using statistics!

# Heterozygosity

"temperature of a population"

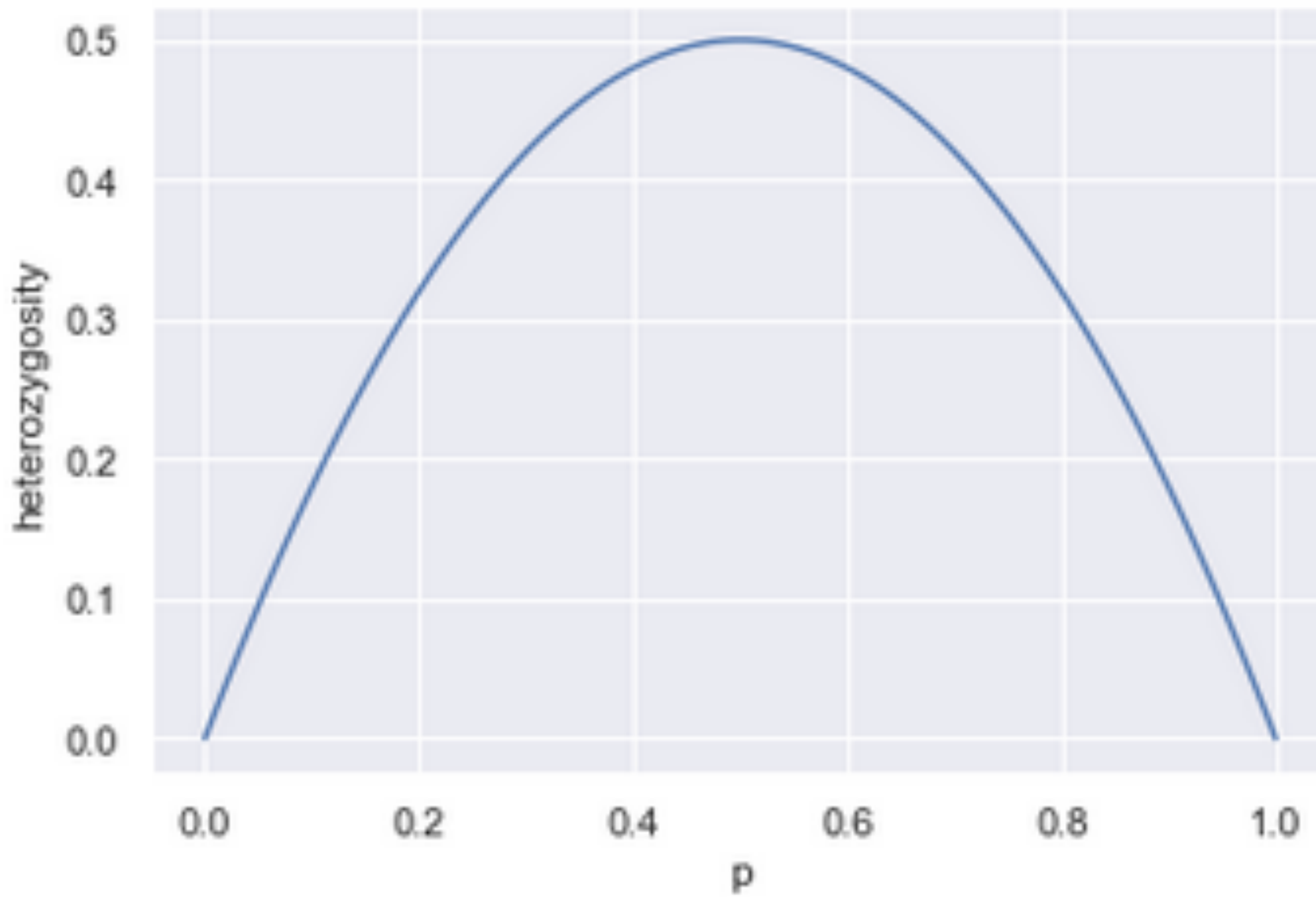
| | | | |
|---|---|---|---|
| Genotype: | $A_1A_1$ | $A_1A_2$ | $A_2A_2$ |
| H-W Frequency: | $p^2$ | $2pq$ | $q^2$ |

Call heterozygosity the probability of sampling an individual that is heterozygous

$$\hat{H} = 2pq = 2p(1-p)$$

# Heterozygosity

"temperature of a population"




$$\hat{H} = 2pq = 2p(1-p)$$

# The Hardy-Weinberg Law

**Assumptions:**

- Diploid sexual population
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- no mutation

This is a Null model. Why is this useful?

# Genetic Drift: the algorithm

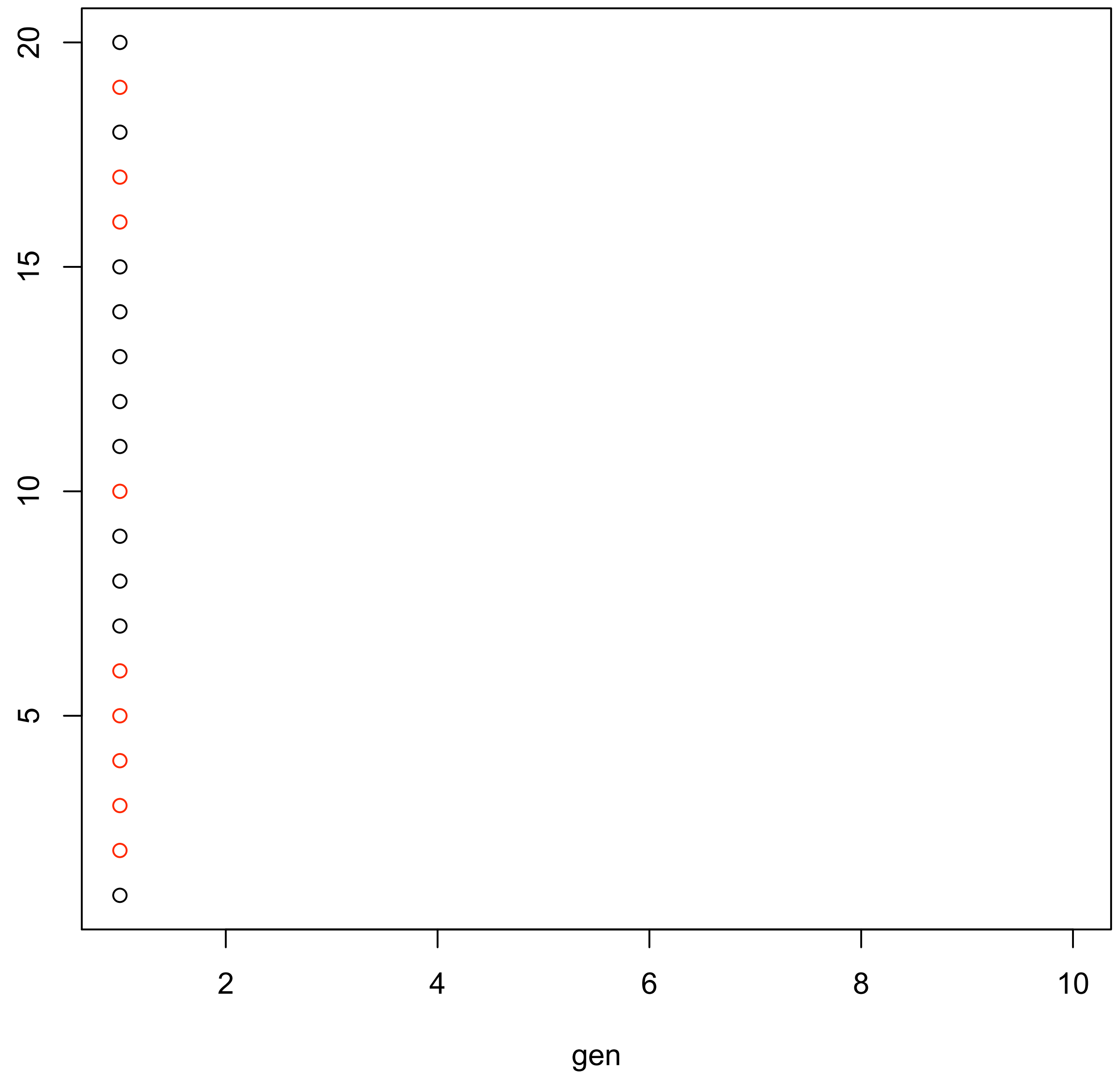
1. Choose an allele at random from the 2N alleles in the parental generation

2. Make an exact copy of the allele

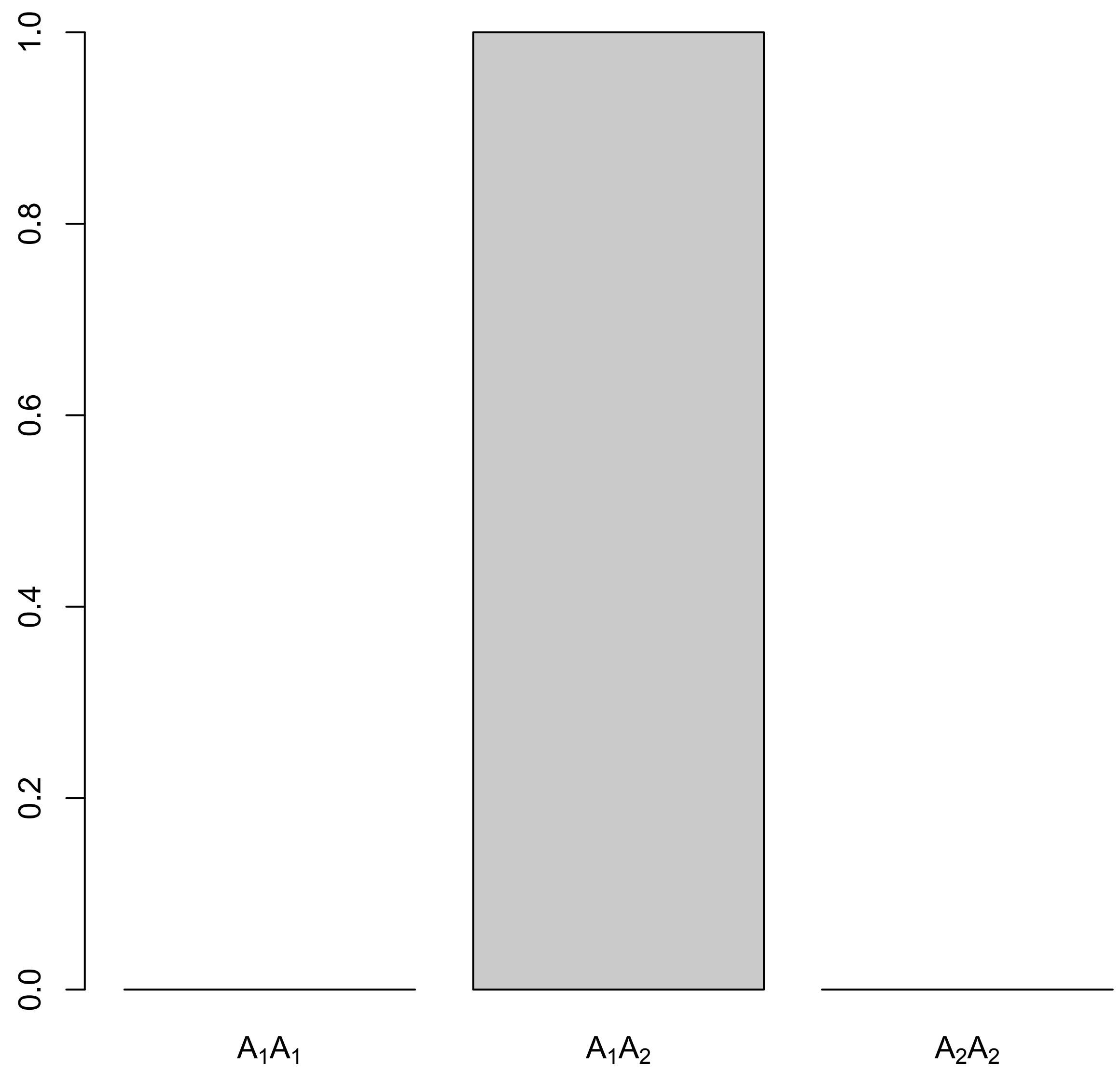
3. Place the copy of the allele in the next generation

4. Go back to 1 until next generation has 2N alleles

# Genetic Drift: the algorithm



# Drift in population of size 1




What is probability of being heterozygote in next generation?

# Drift in population of size 1

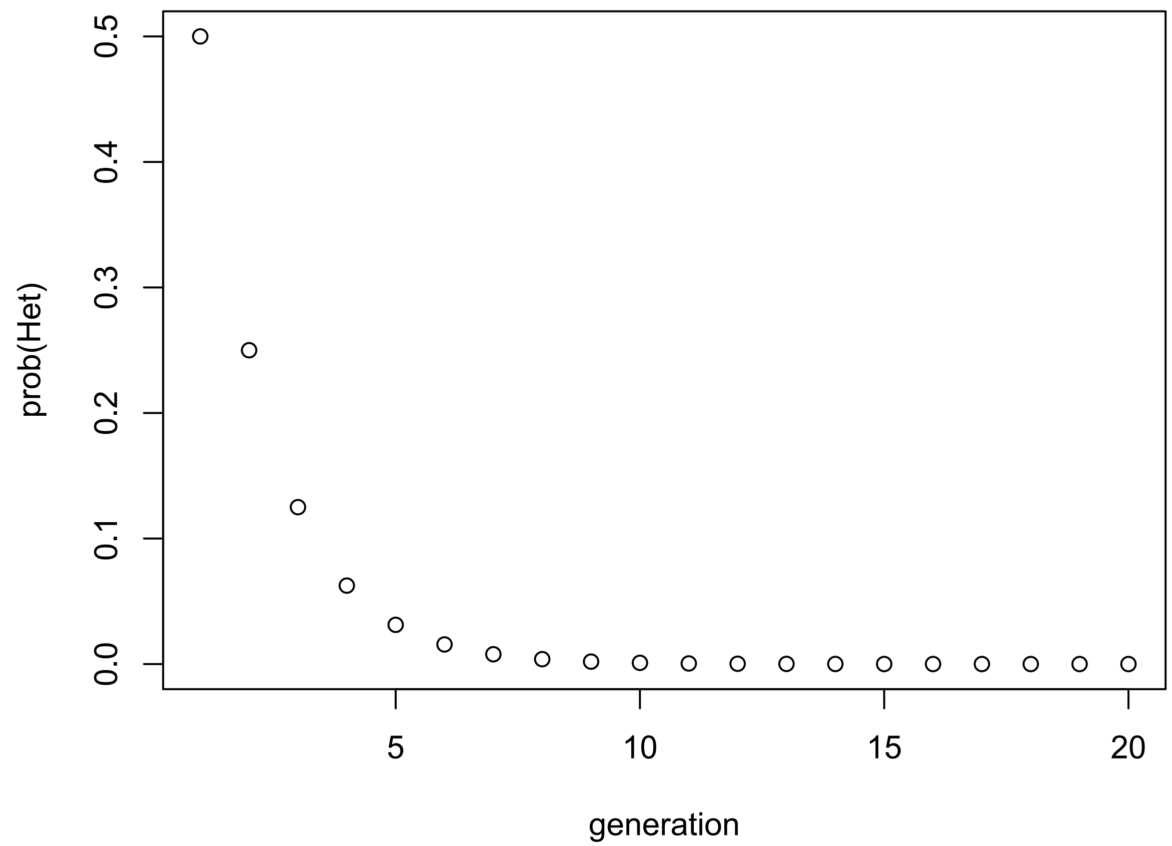
**What is probability of being heterozygote in next generation?**

$$Prob(Heterozygote) = \frac{1}{2}$$

$$Prob(Het|t) = \left(\frac{1}{2}\right)^t$$

# Decay in heterozygosity due to drift

$$Prob(Het|t) = \left(\frac{1}{2}\right)^t$$



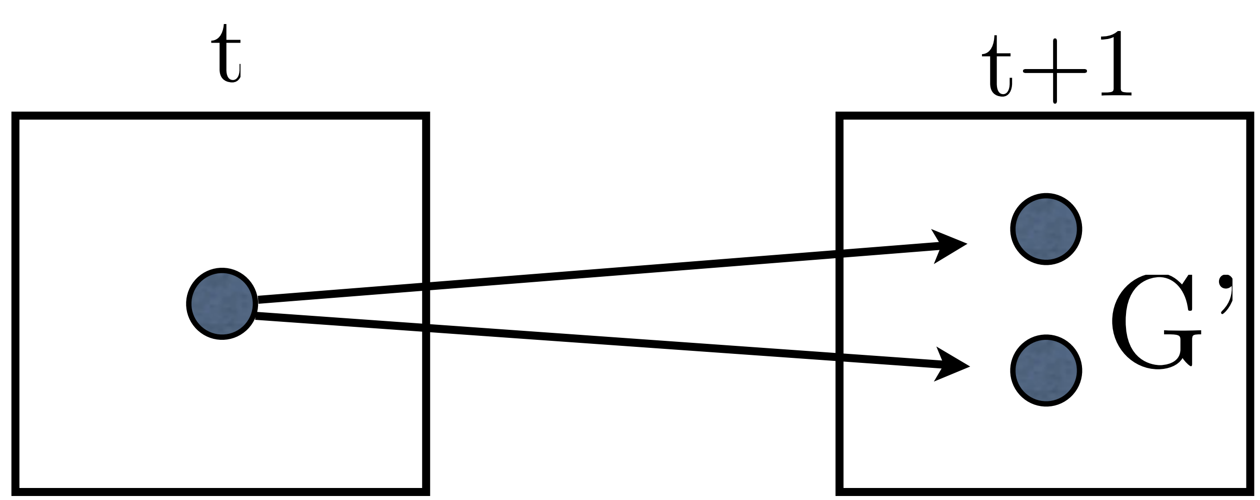

Question: How many generations until fixation on average?

# Decay in heterozygosity due to drift

Identity by origin: two alleles that are from same locus on the **same** chromosome

Identity by state: two alleles that are of the same form (e.g. same DNA sequence)

# Decay in heterozygosity due to drift

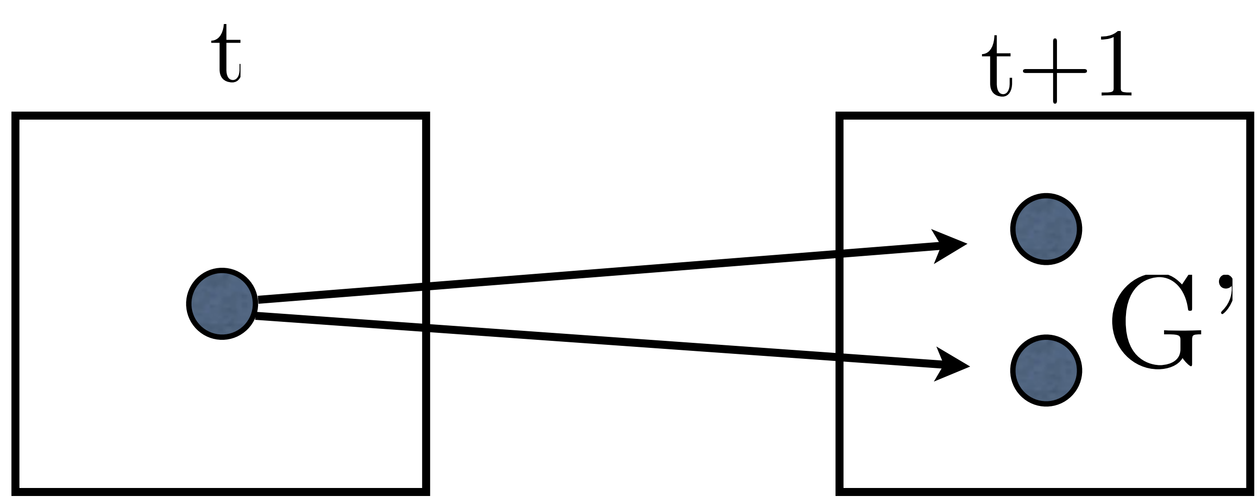



Probability that two alleles chosen at random are identical by state

No variation $G = 1$

Every allele unique $G = 0$

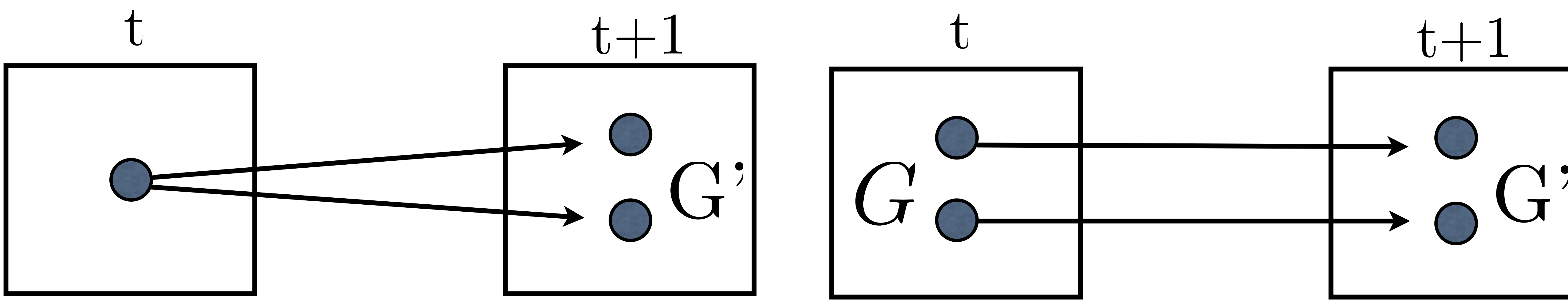
# Decay in heterozygosity due to drift




Probability that two alleles chosen at random are identical by state

$$G' = \frac{1}{2N} + \left(1 - \frac{1}{2N}\right) G$$

# Decay in heterozygosity due to drift




Probability= $\frac{1}{2N}$

Probability= $1 - \frac{1}{2N}$

$$G' = \frac{1}{2N} + \left(1 - \frac{1}{2N}\right) G$$

# Decay in heterozygosity due to drift

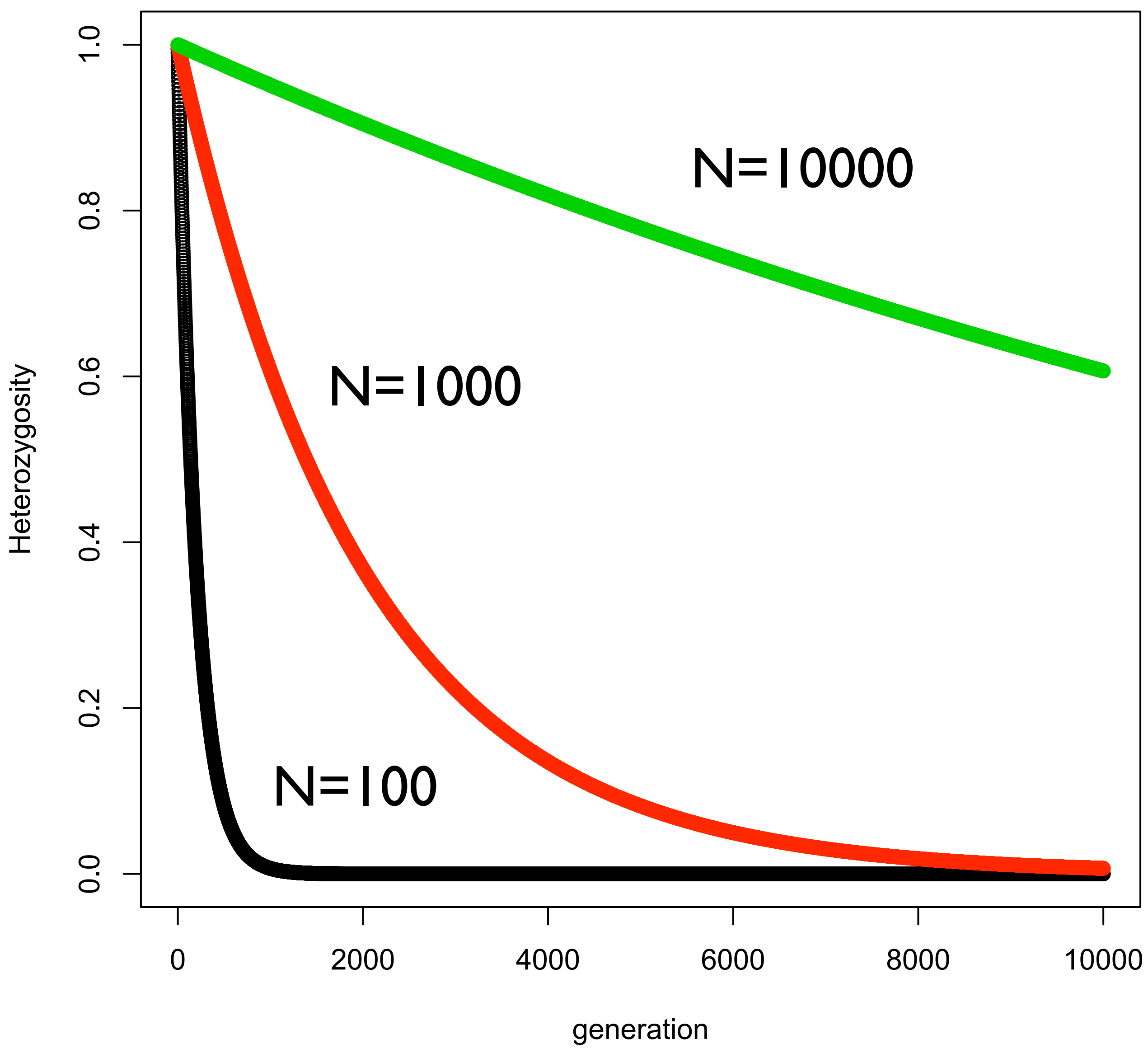
Probability that two alleles chosen at random are different by state

$$H = 1 - G$$

$$H' = 1 - G' = \left(1 - \frac{1}{2N}\right) H$$

$$\begin{aligned}
\Delta_N H &= H' - H \\
&= \left(1 - \frac{1}{2N}\right) H - H \\
&= -\frac{1}{2N} H
\end{aligned}$$

# Decay in heterozygosity due to drift




## Difference Equation

$$H_t = H_0 \left(1 - \frac{1}{2N}\right)^t$$

# Ultimately all Heterozygosity is lost!

So who fixes?

2N different alleles: $Prob(fix) = \dfrac{1}{2N}$

i copies of an allele: $Prob(fix) = \dfrac{i}{2N}$

So for neutral alleles: $Prob(fix|p) = p$

# Adaptation

Natural Selection is THE organizing force in biology

# Adaptations increase the fitness of its carrier

- Environments "challenge" the organism
- Challenges are "solved" by adaptations
- Adaptations arise by natural selection

http://www.members.carol.net/~jmaloney/animal76.htm

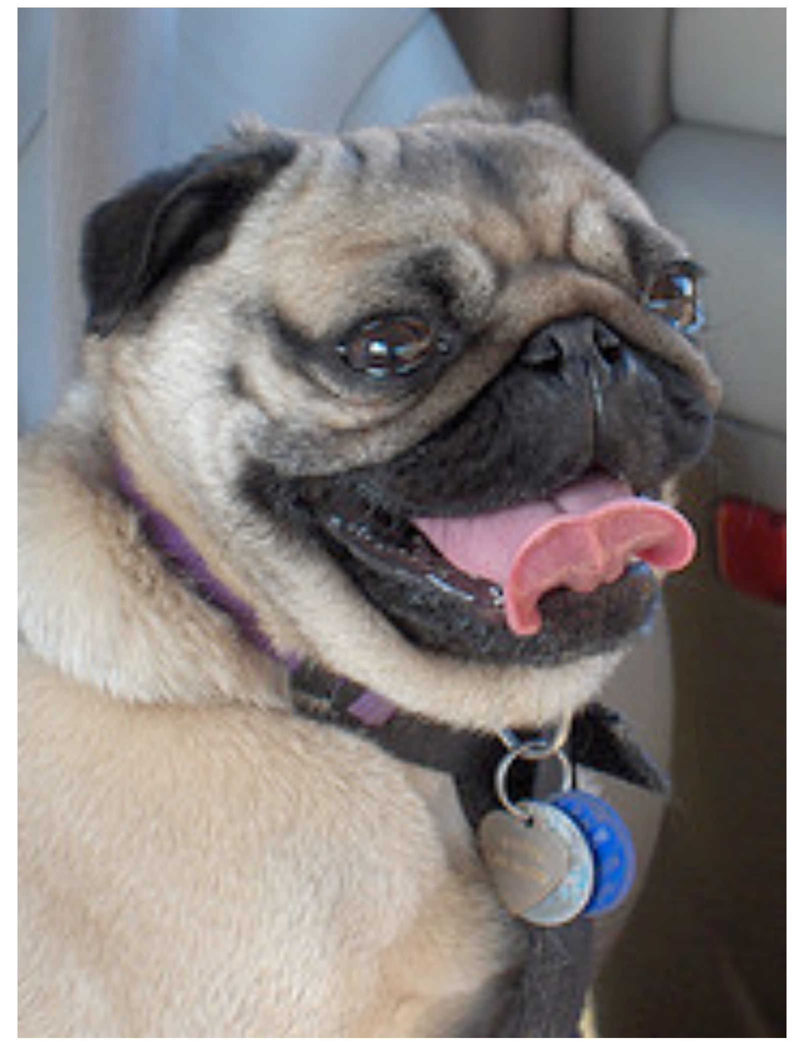
# Good fit to environment

Leafy sea dragon

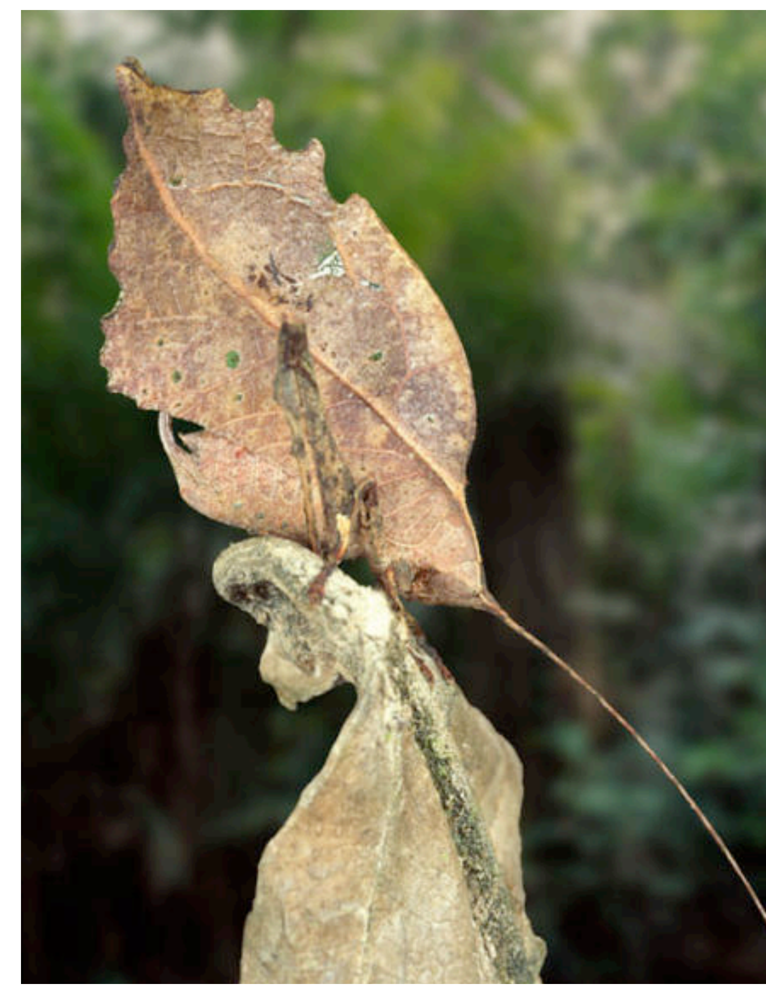
http://en.wikipedia.org/wiki/File:Leafydragon.jpg

# Kinds of Adaptation

- Morphological/Physiological
  - If you are hot you sweat
- Behavioral
  - If you are hot you move into the shade
- Genetic
  - Evolution of biochemical systems to deal with thermal stress



# Adaptation: crypsis



Leaf Mimics: Katydids

http://conservationreport.com/2008/11/08/can-you-see-me-animal-camouflage-leaf-mimics/

# Adaptation: crypsis

http://www.youtube.com/watch?v=MGM3sPCZI94

# Adaptation: crypsis

Leaf Mimics: Verts

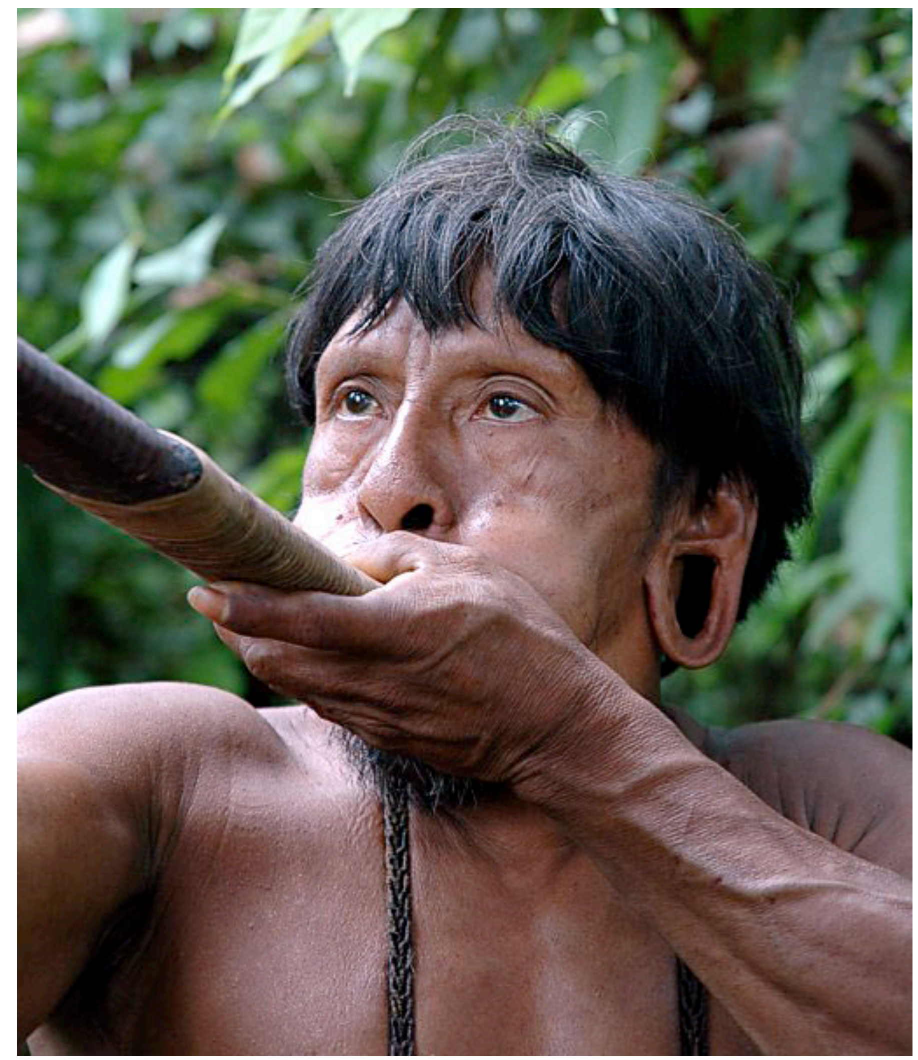
http://conservationreport.com/2008/11/08/can-you-see-me-animal-camouflage-leaf-mimics/

# Adaptation: warning coloration


These frogs stand out in any environment- why?

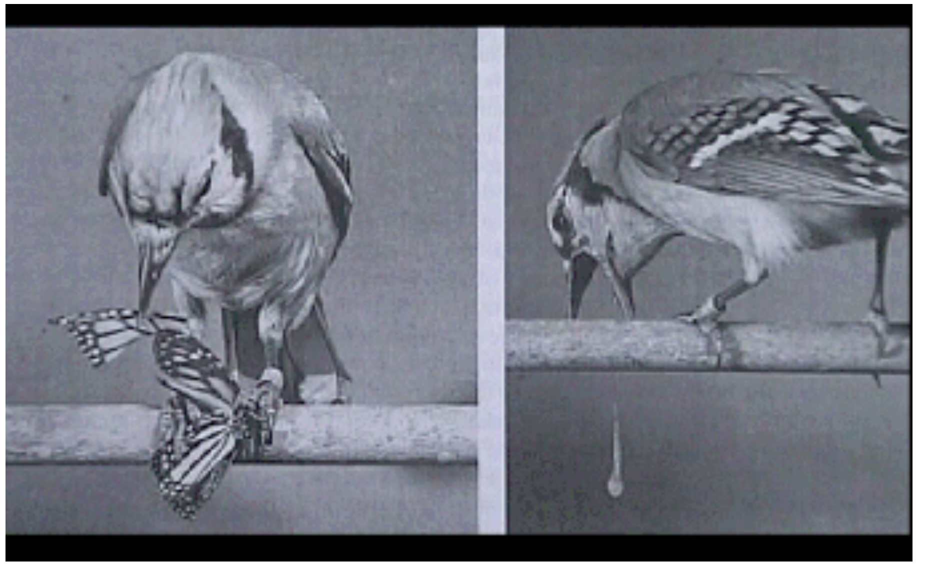
http://www.amnh.org/exhibitions/frogs/vivarium/blue.php
http://advocacy.britannica.com/blog/advocacy/2008/10/2008-—-the-year-of-the-frog/
http://faculty.ucr.edu/%7Echappell/INW/herps/dendrobates.html

# Adaptation: warning coloration



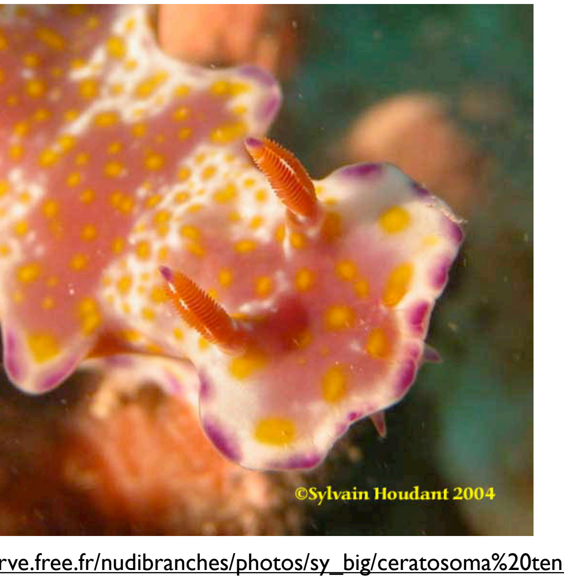
http://www.amazon-tribes.com/Amazonian-Tribes-Blowpipe.html
http://advocacy.britannica.com/blog/advocacy/2008/10/2008-—-the-year-of-the-frog/

# Adaptation: aposomatic coloration


http://slugsite.us/bow/nudwk402.htm

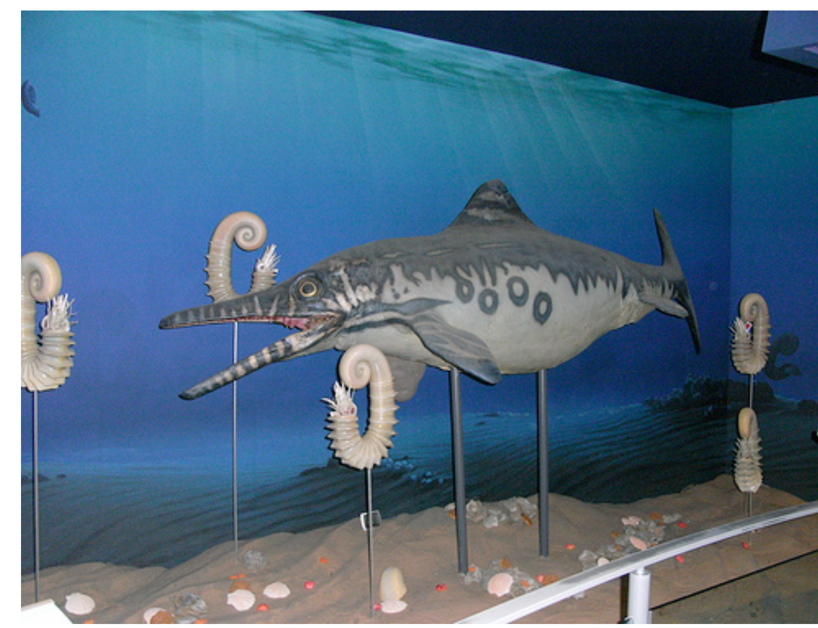

http://slugsite.us/bow/nudwk438.htm




http://jfherve.free.fr/nudibranches/photos/sy_big/ceratosoma%20tenue.jpg

http://www.geocities.com/melanitis2001/AsABird.html

# Adaptation: functional morphology

Fusiform Body shape

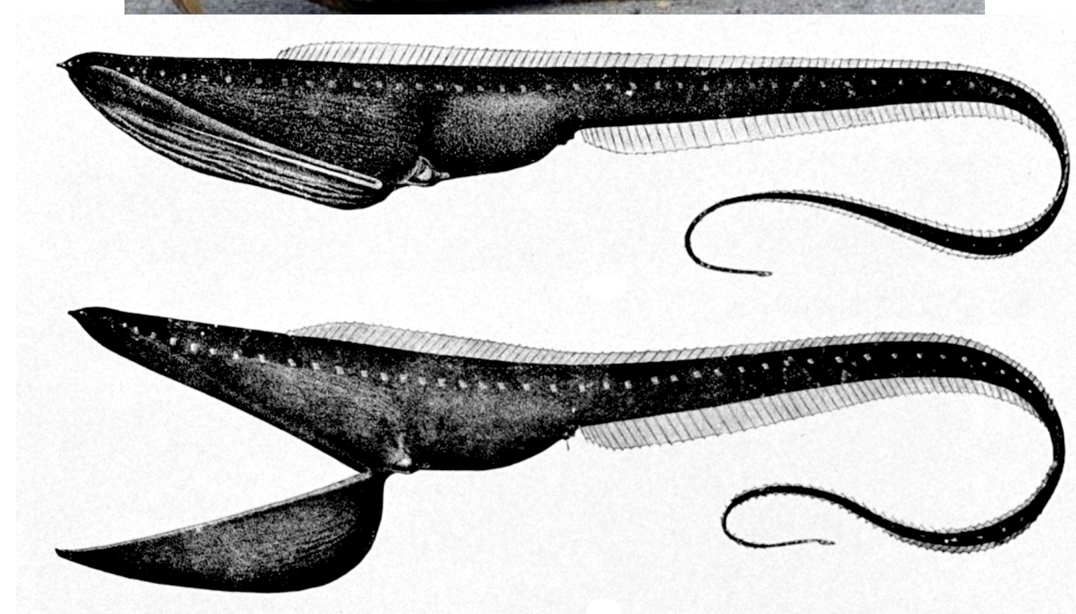
http://www.flickr.com/photos/janewilliams/1997634766/

http://bronzetuna.com/

http://www.whale-images.com/facts_about_dolphins.jsp

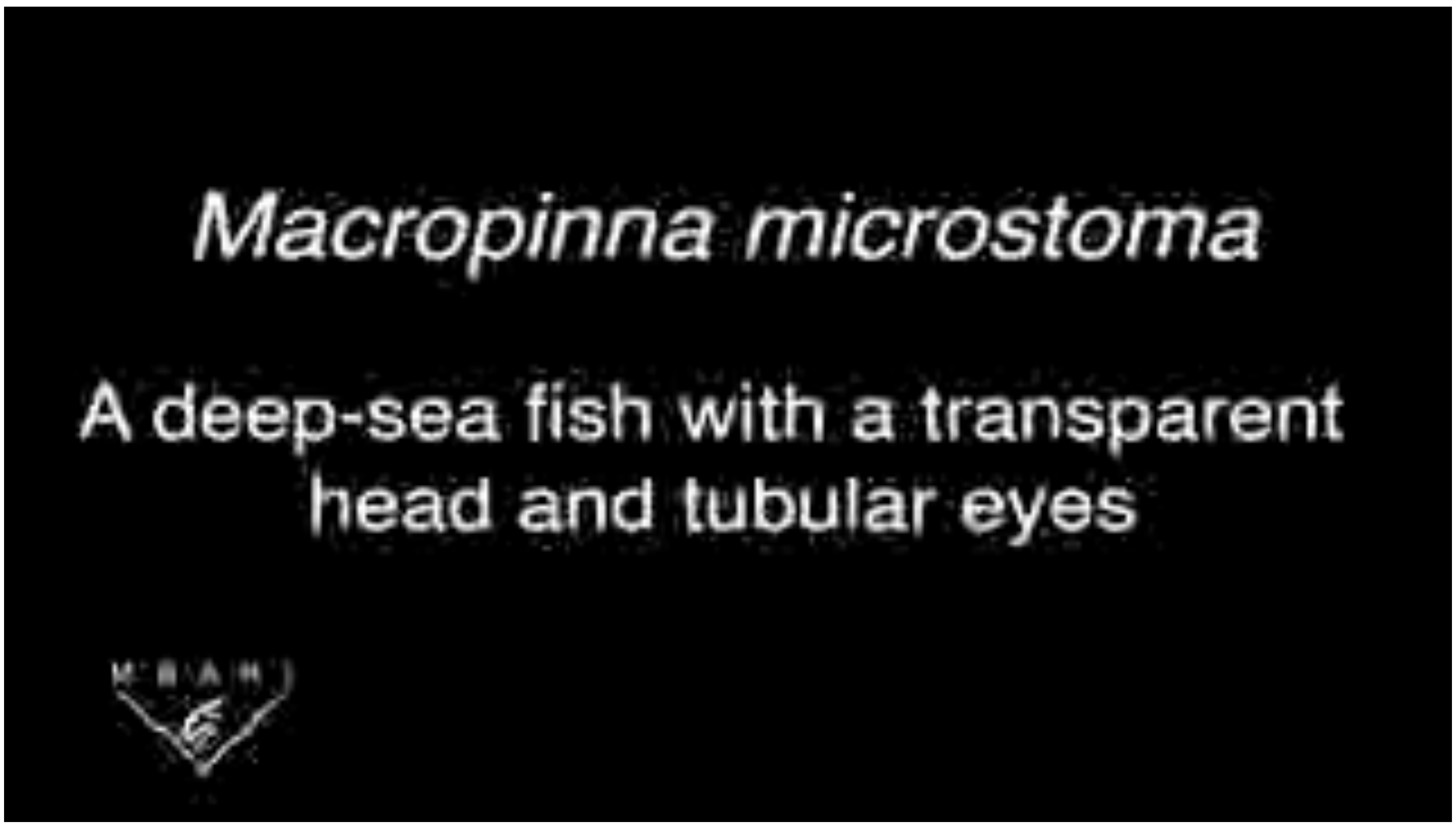
# Adaptation: freaky morphology





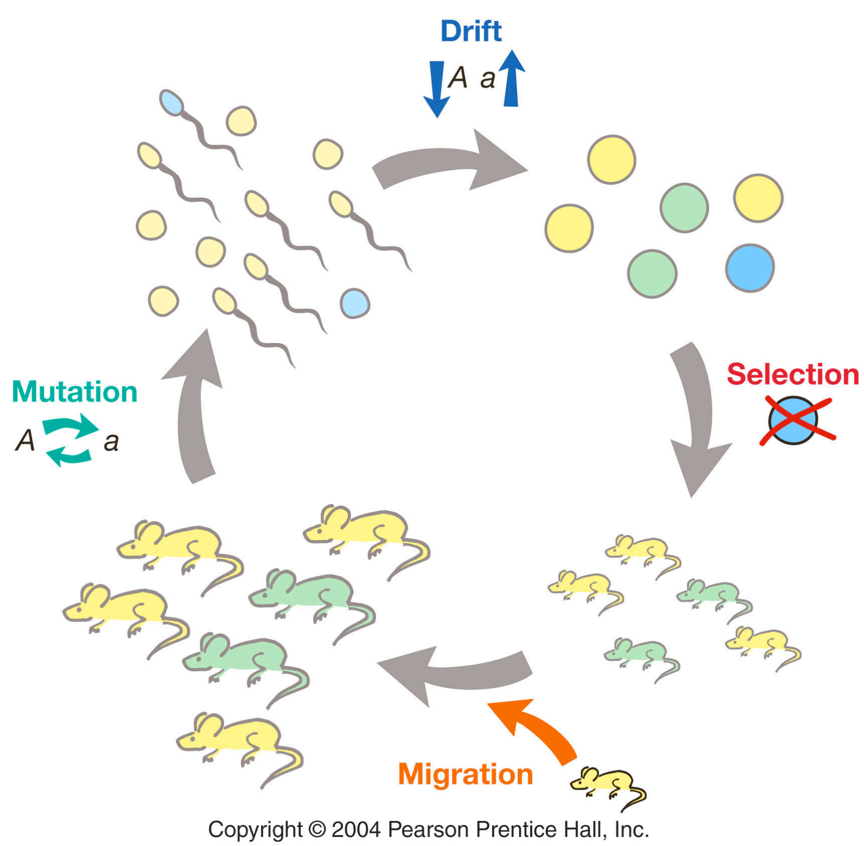
http://dic.academic.ru/dic.nsf/enwiki/910935
http://thephoenix.com/BLOGS/outsidetheframe/archive/2007/01/19/rocky-start.aspx
http://www.canpages.ca/blog/?cat=40

# Adaptation: freaky morphology




http://www.youtube.com/watch?v=RM9o4VnfHJU

# So how does this all come about?




This is what we call Darwinian Fitness

Components:

- Survival
- Reproduction
- Representation in next generation
- If the design "works" its ok.