Deep Learning for Computer Vision (2018 Spring)
HW1

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Problem 1

The minimum P_e decision scheme with $\mathrm{P}(\omega_1) = 3/4$ is:

$$x = \begin{cases} \omega_1\,, & \text{if } x < 5. \\ \omega_2\,, & \text{otherwise.} \end{cases}$$

The decision regions are:

$$R_1 = \{\, x \mid 0 < x < 5 \,\},$$
$$R_2 = \{\, x \mid 5 \leq x < 6 \,\}.$$

The resulting P_e is:

$$P_e = \frac{1 - P(\omega_1)}{6 - 3} \times (5 - 3) = \frac{1}{6}$$

Problem 2

(a) Plot the mean face and the first three eigenfaces.

Face	Mean Face	1^{st} Eigenface	2^{nd} Eigenface	3^{rd} Eigenface
Image				

(b) Plot the reconstructed images with corresponding MSE values.

Use first n eigenfaces	n = 3	n = 50	n = 100	n = 239
Image				
MSE (round off to 2nd decimal place)	659.41	213.26	81.95	0.00

(c) Apply the k-nearest neighbors classifier.

Validation results (accuracy)

Hyperparameters choice	Fold 1 accuracy	Fold 2 accuracy	Fold 3 accuracy	Average accuracy
k = 1, n = 3	0.7875	0.7375	0.6875	0.7375
k = 1, n = 50	0.9500	0.8875	0.9625	0.9333
k = 1, n = 159	0.9500	0.9000	0.9500	0.9333
k = 3, n = 3	0.6375	0.6250	0.5250	0.5958
k = 3, n = 50	0.8750	0.8250	0.8125	0.8375
k = 3, n = 159	0.8750	0.8250	0.8125	0.8375
k = 5, n = 3	0.5625	0.5500	0.4750	0.5292
k = 5, n = 50	0.8125	0.7625	0.7500	0.7750
k = 5, n = 159	0.8000	0.7500	0.7125	0.7542

Since there are two choices of hypermeters resulting in the same average accuracy, I use both of the hyperparameters choices to report the recognition rate on the test set.

Test results (recognition rate)

Hyperparameters choice	Recognition rate (accuracy)
k = 1, n = 50	0.96250
k = 1, n = 159	0.94375

Problem 3

In order to derive the first eigenvector of a $d \times d$ positive semi-definite symmetric matrix (denoted as A) with d distinct eigenvalues, we can use the method called "power iteration".

Here is the procedure to achieve our goal via power iteration:

First, let a vector x_0 be a random vector, which has a non-zero component in the direction of an eigenvector associated with the first eigenvalue.

Second, multiply the vector x_k $(k = 0, 1, \dots)$ with the matrix A and normalize.

$$x_{k+1} = \frac{Ax_k}{||Ax_k||}$$

The subsequence (x_k) will converge if A has a unique greatest eigenvalue, and we can repeat the second step many times to obtain the approximation of the first eigenvector.

If the symmetric matrix A is not constrained to be positive semi-definite, the vector x_k will converge to the eigenvector with the greatest value of absolute eigenvalue, not the greatest eigenvalue. This is why we need to have the symmetric matrix to be positive semi-definite.

[Reference]: https://en.wikipedia.org/wiki/Power_iteration