Transpn. Res.-B Vol. 20B, No. 4, pp. 331–344, 1986
Printed in Great Britain.

0191-2615/86 $3.00+.00

BUS NETWORK DESIGN

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(Received 6 February 1984; in revised form 15 April 1985)

Abstract—This paper describes the bus network design problem, summarizes the different approaches that have been proposed for its solution and proposes a new approach incorporating some of the positive aspects of prior work. The proposed approach is intended to be easier to implement and less demanding in terms of both data requirements and analytical sophistication than previous methods. An algorithm is presented that can be used to design new bus routes taking account of both passenger and operator interests; however, this algorithm focuses on only a single component of the overall bus operations planning process described in this paper.

INTRODUCTION

This paper describes the bus network design problem, summarizes the different approaches that have been proposed for its solution and proposes a new approach incorporating some of the positive aspects of prior work. It is argued that although the redesign of existing bus networks is not an activity that should often be undertaken by bus properties, as an infrequent initiative it can have significant impacts on bus system performance. However, there are real risks that a redesign will result in poorer performance, however measured, if not carefully thought through, so it is necessary to use a design procedure that recognizes the full range of potential impacts. Although several of the approaches that have been developed have attractive features, only one approach has been applied to any significant extent. The approach proposed in this paper is intended to be easier to implement and more sensitive to the risks of making changes than the previous methods.

In the next section of this paper, the bus network design activity is placed in the context of other bus service planning functions, and the motivation for investigating the problem is developed. Following a literature review of previous approaches to bus network design, the key features of the problem are defined, and the methodological approach presented. Finally, an example illustrates the method as it applies to a simple situation.

PROBLEM CONTEXT

Figure 1 shows the bus planning process as a systematic decision sequence. The output of each activity positioned higher in the sequence becomes an important input into lower level decisions. Clearly the independence and orderliness of the separate activities exist only in the diagram, that is decisions made further down the sequence will have some effect on higher level decisions. Nevertheless, because this sequence is repeated frequently, this feedback will be incorporated over time, because the results of the last cycle will be the basis for the next iteration.

In most properties, although all five elements are included in the process, the bulk of resources is devoted to the last two steps: bus and driver scheduling. In the United States, these components are generally referred to as vehicle blocking (a block is a sequence of revenue and nonrevenue activities for an individual bus) and driver run-cutting (splitting and recombining vehicle blocks into legal driver shifts or runs).

The concentration on these elements of the bus planning process is reflected in the welter of professional papers on these topics and in the development of numerous computer programs to automate (at least partially) these steps. This concentration is understandable from the prop-

Independent Inputs	Planning Activity	Output
Demand data Supply data Route performance indices	Level A Network Design	Route changes New routes Operating strategies
Subsidy available Buses available Service policies Current patronage	Level B Setting Frequencies	Service frequencies
Demand by time of day Times for first & last trips Running times	Level C Timetable Development	Trip departure times Trip arrival times
Deadhead times Recovery times Schedule constraints Cost structure	Level D Bus Scheduling	Bus schedules
Driver work rules Run cost structure	Level E Driver Scheduling	Driver schedules

Fig. 1. Bus planning process.

erties perspective because the largest single cost of providing service is the drivers' wage and fringe benefit, and focusing on levels D and E would seem to be the best way to reduce this cost. Another argument in favor of automating this part of the process is that this scheduling process is extremely cumbersome and time-consuming to do manually, and aside from the potential for more efficient schedules, the automated process should be more controllable and more responsive. Further, the cost and complexity of manual scheduling have served to discourage adjusting services (for example, through activities at levels A and B). Only now with automated scheduling methods widely accepted is it feasible to focus on the higher levels in the planning process.

Nonetheless, a case can be made that the higher levels of the bus planning process have received short shift from both researchers and practitioners. In particular, planning bus routes and setting frequencies are both critical determinants of system performance but have received nothing like the same degree of effort towards improving current practice as have the two scheduling elements. Recently some serious attention has been paid to the problem of efficiently setting frequencies, but little of this work has yet been implemented (see Furth and Wilson, 1981; Koutsopoulos and Wilson, 1983; Ceder, 1983). It is to be hoped that as computer acceptance in the planning and scheduling departments of bus properties grows in the next decade, some of these methods for timetable development will also begin to find application.

This will leave the planning of bus routes as the single important element of the bus planning process not to have seen serious research results carried into practice. Although there have been attempts to develop methods for bus route planning (to be reviewed in the next section), none of these have had real impact on the planning units in even the largest bus properties. Planning practice in terms of bus route design focuses almost entirely on individual routes that, for one reason or another, have been identified as candidates for change (Wilson *et al.*, 1984). Occasionally sets of interacting (e.g. overlapping or connecting) routes are subject to redesign, usually after a series of incremental changes to individual routes has resulted in a confusing and inefficient local system.

In the rare case where large North American bus properties have "taken a step back" and asked the question whether the overall bus network could be improved through restructuring, nonsystematic procedures have been used to develop the alternative route systems, typically with the help of outside consultants. One would certainly expect that this kind of major rethinking of the whole network would be appropriate very infrequently, if at all, for any property because of the major effort involved in the analysis as well as the disruption imposed on passengers if wholesale changes are made to the system. For many North American properties, however, which have not been through such a reappraisal since 1940, it is high time to consider such an undertaking.

In the past 40 years, most metropolitan areas have seen significant growth and dramatic redistribution of population, employment, retail centers and other trip generators. During the same period, total urban person trips have increased significantly, whereas transit trips have experienced a large decline. Although transit networks have changed in this period, particularly in the more rapidly growing southern and western cities, in many cases the basic network structure has remained the same. Some cities have shifted towards a grid-like network structure from a predominantly radial structure, but generally such shifts have been based on ideology rather than systematic analysis; and there has been little follow-up analysis to evaluate the effects of the change. The problem addressed in this paper is the more general one of how to design a new bus network or redesign an existing bus network, given no *a priori* specification of desirable network structure.

Although it is hard to predict the benefits that will result from redesigning any bus network without doing a detailed assessment, it is reasonable to believe that they will be large compared with the marginal benefits of additional research aimed just at the scheduling component of the problem. If one looks at the whole bus planning process in Fig. 1 as an optimization problem with levels A–E, then the domain of all feasible solutions is greatly reduced by the definition of the bus network. If one could consider the full problem domain including alternative bus networks, it is more likely that suboptimality in the final solution will be introduced by non-systematic rejection (through nonconsideration) of feasible networks than through suboptimality at stages D and E, which have already been extensively researched.

PRIOR WORK

Considering the general bus network design problem, prior approaches can be grouped into those predicated on idealization of the network and those dealing with actual routes. In the first set, there has been extensive work based on constrained optimization methods in which one or several design parameters (e.g., route spacing, route length, stop spacing, headway) are selected so as to optimize an objective reflecting benefits to the passengers and cost to the operator. Much of this work (see, for example, Byrne, 1976; Hurdle, 1973; Newell, 1979) is based on an assumption of fixed demand, limited design parameters and the objective of minimizing the sum of passenger and operator costs. Recent work by Kocur and Hendrickson (1982) has extended this approach to encompass variable demand, a broad range of design parameters and a choice of objectives reflecting user and/or operator interests. All of these methods are best suited for screening or policy analyses in which approximate design parameters are to be determined, rather than final design. As such, they are not directly applicable to the task of route design in any real situation.

Hasselström (1981) conducted a thorough review of approaches to the detailed bus network design problem in which the following six distinguishing features of each method were identified: demand, objective function, constraints, passenger behavior, solution techniques and computer time. The approaches he reviewed were by Lampkin and Saalmans (1967), Rea (1971), Silman, Barsily, and Passy (1974), Mandl (1979) and Dubois, Bell, and Llibre (1979). The only subsequent approach was that developed by Hasselström himself, which is imbedded in the Volvo transit planning package. In the following review, each of the six features mentioned above is discussed comparing the prior approaches and defining the approach most appropriate for this problem.

Demand may be treated as fixed and independent of the service quality offered between any origin–destination pair or as variable, responding to the network design. All prior approaches except Dubois *et al.* and Hasselström have assumed fixed demand. Dubois *et al.* estimated the total trip matrix and then estimated the public transport share using a diversion curve based on expected transit times. Hasselström used a direct demand model both to estimate a "desire-matrix" based on providing high-quality public transport service throughout the area and to reduce the demands as the actual design is developed providing less than ideal service between some origin–destination pairs.

Although the variable demand assumption is more appealing, there is no evidence to suggest that existing demand models are reliable for route or network changes in public transport systems (see, for example, Multisystems, 1982). In most metropolitan areas, operators are likely to be quite risk averse, have little faith in demand models and be much more concerned about the impact of changes on existing riders than about the potential for generating new ridership. For these reasons and because of the reduced complexity, a fixed demand model is preferable for the design phase, although this requires constraints in the algorithm to guarantee minimal service levels. The operator can estimate demand for the proposed network separately after the design, if desired, and perform a second iteration through the design algorithm with the new demand matrix if significantly different from the initial one. The need for a reasonably accurate public transport maxtrix for any network design algorithm already imposes a significant data collection burden on the public transport provider over and above routine data collection activities.

The objective functions used in the past have been either minimization of generalized cost (or time) or maximization of consumers' surplus. For fixed demand approaches, the objective of minimizing generalized cost (or time) is used, except for Rea who does nto use an explicit objective function, but rather seeks a solution that meets certain constraints including "service specification" developed by the operator. Hasselström uses the objective of maximizing consumers' surplus. The case of variable demand is not attractive to minimize generalized costs since the optimal solution may well be little, if any, service, few, if any, passengers, and a very low total generalized cost, even though this would not in fact be an acceptable solution. While the minimum generalized cost is a sound objective for the fixed demand formulation, careful thought needs to be given to the weights needed to collapse walk time, wait time, ride time, transfer time, number of transfers and crowding on board vehicles into a single metric.

All previous formulations include a constraint on either total operator cost or number of vehicles which can be assigned to operate the system. While it is often argued that these constraints are interchangeable since the operating cost is highly correlated with the number of vehicles operated, there can be significant differences in cost depending on the vehicle requirements in different periods of the day. The formulation of this constraint also gets to the very heart of the difficulty of the bus network design problem: the impossibility of evaluating the network without defining the vehicle requirements on each route. It is the vehicle requirement on which both the cost of a design and its effectiveness critically depend. Thus when we address the bus network design problem, we must inevitably also address the bus allocation problem at the same time.

As part of the bus allocation subproblem, all formulations include constraints governing the maximum allowable passenger load expressed as a function of vehicle size. For these constraints to be effective, the demand matrix and the resulting bus allocation must be defined for the major distinct periods of operation—typically morning peak, midday (or base), afternoon peak and evening. Even if the network design does not vary across time periods, the total cost will depend on this bus allocation, as defined at a minimum by required bus hours, and the route level capacity constraints must be met in each time period. It is also probable that some routes will not operate in all time periods.

A basic distinction exists between approaches that assume that all passengers traveling between a given origin–destination pair take the same path (route or sequence of routes) and those that allow multiple paths to be selected. Rea and Mandl use single path assignment that is likely to be accurate only in cases of simple network structures offering few if any alternative paths between most origin–destination pairs. All other approaches are based on multiple path assignment of passengers with the definitions of path acceptability varying but the proportion of passengers taking each acceptable path the same as the probability that the first bus to arrive serves that path. Formulation of the multipath problem, though more realistic for most systems, is a good deal more complex, particularly if some routes are likely to be capacity constrained in the final solution.

It is in terms of solution techniques that there exists the greatest diversity among prior approaches. All methods start by partitioning the problem into two parts: the first, route design, and the second, frequency determination. This partitioning is necessitated by the complexity of the complete problem, but it imposes a burden of ensuring that the implications of frequency determination are incorporated in the route design component. Lampkin and Saalmans, Silman *et al.* and Dubois *et al.* all use a route generating procedure that builds skeleton routes consisting of a few major nodes between acceptable termini then fills in additional nodes to define each route completely. At each iteration the skeleton route that makes the largest marginal contribution to the objective function is selected for inclusion in the candidate set of routes, and the iterative procedure continues until some defined criterion is met. This criterion is based on the marginal contribution of the new route or total number of routes generated. Silman *et al.* allow for the generation of many more routes than will actually be operated, with the iterative deletion of some routes and addition of others, and relies on the frequency allocation procedure to select the final route set. Mandl and Rea each focus on the acceptability of links which are then aggregated to form routes—manually in Rea's method. Hasselström uses a complicated two-level optimization for route generation in which routes are formed from initial assignment of desired trips onto a network of all possible transit links. Routes are formed using criteria of normal practice, route length, terminal restrictions and flow fluctuations.

Of all these methods, only Hasselström's has received extensive application, in the form of the Volvo transit planning package. It is without doubt the most sophisticated of the approaches taken to date and appears to be effective in the total route redesign task. However, it is also quite expensive, both in terms of the data needed to execute the package and the direct cost and staff time required for the process.

In the remainder of this paper, we present an alternative approach that aims to be more practical, given typical data availability, and less complex than the Hasselström model in order to increase its chance of acceptance within most transit properties.

TWO-LEVEL METHODOLOGICAL APPROACH

Due to a vast number of external and operational factors that are involved in the design of a bus network (e.g., financial, socioeconomic, political, etc.), it is desirable to establish a planning process incorporating alternative levels of complexity. The acceptability of such a process by bus properties depends on its simplicity, flexibility and practicality.

A two-level methodological approach is proposed in this section, based on five main objectives:

(i) to develop an algorithm for optimal network design so that routes will provide service between all origin–destination pairs and comply with service and operational constraints on trip, transfer and waiting times;

(ii) to develop performance measures from the passenger, operator and community perspectives;

(iii) to combine other operations planning components (timetable construction and vehicle scheduling) with the network design procedures;

(iv) to develop sensitivity analyses to determine solution tolerances due to possible changes in demand and the constraints; and

(v) to develop an interactive person–machine system so that the planner can change constraints or routes during the design process.

The proposed overall methodological development is depicted in flowchart form in Fig. 2. Within the methodology, routes can be selected at two levels: Level I—considering just the passenger viewpoint, and Level II—considering both the passenger and the operator impacts. Certainly, Level II presents a more challenging developmental effort; however, it deals both with passenger and operator objectives and, therefore, increases the probability that the recommended route changes or network design will be accepted. Nevertheless, it is probable that Level II will have to rely on heuristic solution procedures, whereas Level I can be handled by an optimization program. Furthermore, for large networks and large-scale route revisions, it may be difficult, if not impossible, to solve the Level II problem— which will then require the use of a less detailed Level I procedure.

At both Levels I and II, as shown in Fig. 2, the planner can insert desirable network characteristics such as topographic factors and network configuration in terms of route classification (radial, crosstown, feeder, etc.). These network characteristics may also include guidance on area coverage, accessibility measures and transfer policy. The remaining parts of Fig. 2 include the major components identified in the prior problem context discussion.

We will now present possible initial Level I and Level II formulations. In these formulations and in the route construction algorithm in the next section, the following notation is used with the simplifying assumption that the design is for a single uniform time period.

$G = \{N, A\}$ a road network with $|N|$ nodes and $|A|$ arcs, where N and A are the sets of all nodes and arcs in the network, $D = \{D(i, j)\}$ demand matrix in terms of expected number of passengers from node i to $j \,\forall\, i, j \in N$, $R = \{j\}$ set of all potential transfer nodes, $j \in N$, $T = \{i\}$ set of all possible route terminii, $i \in N$, $T \in R$, $\{r(i, j)\}$ total passenger transfer time between i and $j \,\forall\, i, j \in N$ (in passenger-hours), $\{t(i, j)\}$ matrix of travel times from i to $j \,\forall\, i, j \in N$, $\{t_m(i, j)\}$ matrix of shortest travel times from i to $j \,\forall\, i, j \in N$, $\{pH(i, j)\} = \{D(i, j) \cdot t_m(i, j)\}$ direct passenger-hours matrix from i to $j \,\forall\, i, j \in N$, $\{\Delta pH(i, j)\}$ matrix of the difference in passenger-hours between the route travel time and the shortest travel time from i to $j \,\forall\, i, j \in N$; i.e. $\Delta pH(i,j) = D(i,j)[t(i,j) - t_m(i,j)]$, L_k the length of route k (expressed as running time), L_1, L_2 lower and upper bounds on route length: $L_1 \leqq L_k \leqq L_2$, n the number of routes, m upper bound on number of routes, Z number of vehicles required to operate the system, F_k service frequency on route k, F_m minimum frequency (the reciprocal of policy (max) headway), C_1 value of one passenger-hour of travel time, C_2 cost of one vehicle (including capital and operating cost over the time period), $\{t'(i, j)\}$ matrix of estimated wait time (in passenger-hours) for the demand from i to $j \,\forall\, i, j \in N$, $\{\alpha(i, j)\}$ matrix of allowable excess travel time (difference between actual and minimum travel time) expressed as a percentage of minimum travel time for route(s) connecting i to $j \,\forall\, i, j \in N$; $\alpha(i, j) \geqq 0$.

INITIAL LEVEL I FORMULATION

For different sets of routes, such that each set accommodates the demand matrix D, objective function

$$\min \quad \{\sum_{(i,j)\in N} \Delta pH(i,j) + \sum_{(i,j)\in N} r(i,j)\}$$

$$\text{s.t.} \quad \text{(a)}\ \frac{t(i,j)}{t_m(i,j)} \leqq 1 + \alpha(i,j),$$

$$\text{(b)}\ L_1 \leqq L_k \leqq L_2,$$

$$\text{(c)}\ n \leqq m.$$

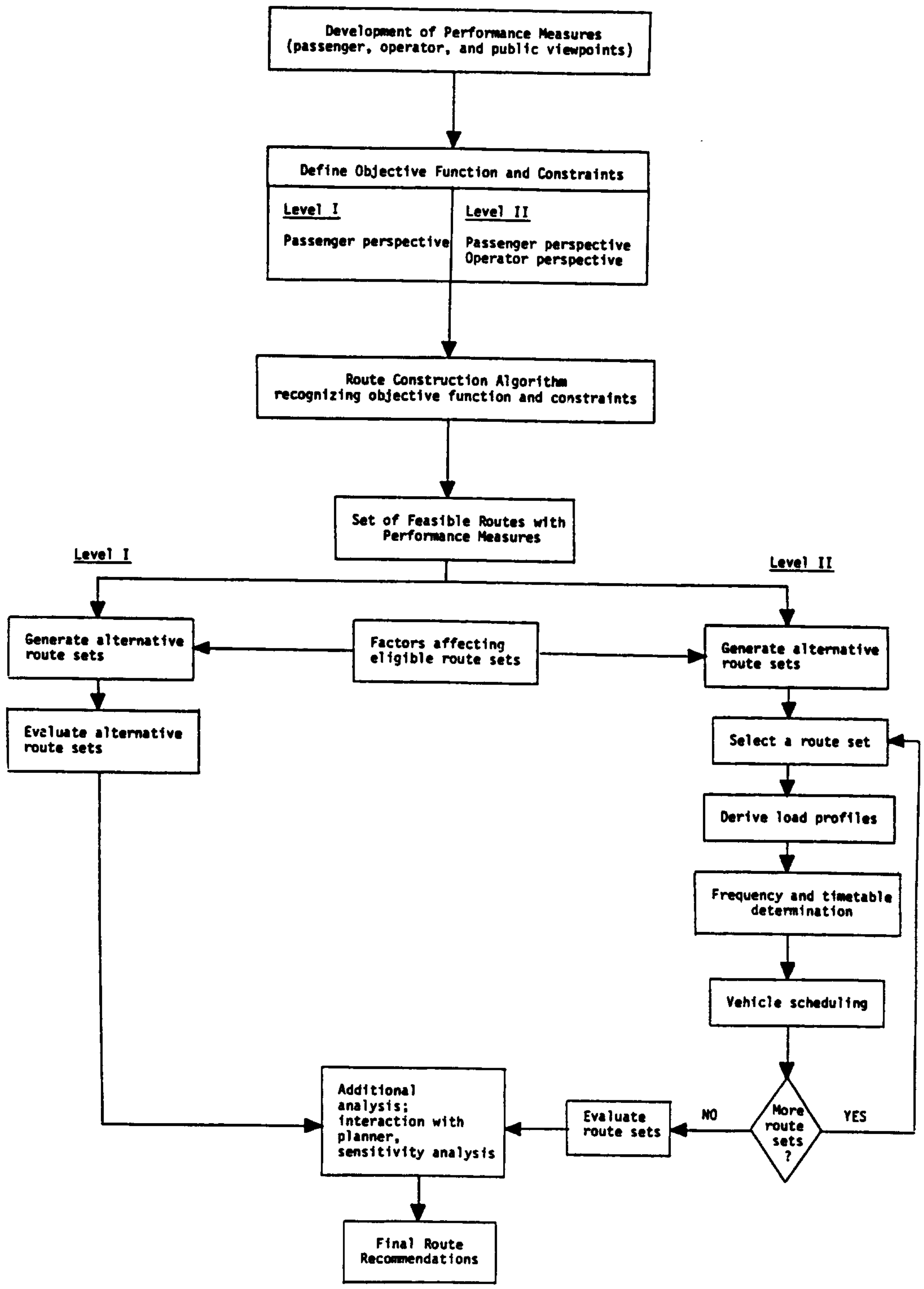

Fig. 2. Methodological framework.

INITIAL LEVEL II FORMULATION

For different sets of routes (each satisfies D):
objective function

$$\min \quad \{C_1[\sum_{(i,j)\in D} \Delta pH(i, j) + \sum_{(i,j)\in D} r(i, j) + \sum_{(i,j)\in D} t'(i, j)] + C_2 Z$$

s.t. (a), (b), and (c) above plus:
minimum frequency

$$F_k \geqq F_m \quad \text{all routes } k,$$

fleet size constraints

$$\sum_k (L_k F_k) \leqq W, \quad W = \text{maximum (or available) fleet size.}$$

These simple formulations are based on a single performance measure given in constraint (a). Alternative performance measures could readily be incorporated through either the objective function or constraints. For example, if passengers value wait or transfer time differently from in vehicle time, or if the transfer itself is onerous beyond just the added time, the objective function could be modified accordingly.

Generally speaking, the public perceives a good bus route network to be one that does not have too many routes, long (they are likely to be unreliable in terms of schedule adherence), or circuitous routes, or require many transfers. The bus operator envisions a good route network as one that is perceived favorably by the public and at the same time does not require excessive resources that might be associated with complicated schedules or operational complexities. These criteria have been taken into consideration in the two-level methodological approach and in the initial formulations.

Another point to note in the approach is that sensitivity analysis allows the examination of possible changes both in demand and in service level. It is worth noting that new routes are likely to induce changes in demand which will serve (in the next cycle) as a new input to the process. It is desirable to have two stages to the sensitivity analysis: (i) prior to route revision (including examination of vehicle frequency and scheduling elements); and (ii) after implementation of the new or revised routes. Because route revisions will be infrequent, the sensitivity analysis will include reference to threshold values. That is, if the objective function value does not increase above a threshold level, the routes will remain the same, and changes will be made only in terms of vehicle frequency and scheduling.

ROUTE CONSTRUCTION ALGORITHM: ONE COMPONENT IN THE APPROACH

The matrix $\{t_m(i, j)\}$ represents the shortest possible travel time from i to j, whereas $\{t(i, j)\}$ is the time required to travel from i to j on the proposed bus network. The suggested route construction algorithm is based on minimizing the difference between indirect and direct passenger-hours. This procedure attempts to reduce the extensive computer time required for an optimization program when trying to map all possible routes in the network as followed, for example, by Hasselström (1981).

The following algorithm screens out routes according to the values specified in the $\{\alpha(i, j)\}$ matrix. In other words, a given demand (ij) cannot be assigned to a bus route if its travel time $t(i, j)$ exceeds the minimum travel time from i to j, $t_m(i, j)$, by more than the given $\alpha(i, j)$ percentage. Mathematically,

$$t(i, j) \leqq \left[1 + \frac{\alpha(i, j)}{100}\right] t_m(i, j) \text{ for all } i, j \in N.$$

The algorithm consists of the following eight steps:

Step 0. Initialization: set $q = 1$.

Step 1. Start to construct routes from the terminal (r_q) having the largest originating demand expressed in passenger hours; select $r_q \in T$ such that $\Sigma_{j \in N} pH(r_q, j) = \max_{i \in T} \Sigma_{j \in N} pH(i, j)$.

Step 2. Set the selected terminal as a starting node ($a = r_q$), and indicate the first tree search attempt ($l = 1$) from r_q.

Step 3. Denote a path between the selected terminal and node j, $P(r_q, j)$, and define the set of nodes, j_a^l, which can be reached from the starting point (a) excluding the nodes belonging to the path from the selected terminal (a) via the previously considered starting points.

$$p(r_q, j) = \{(r_q, i_1)(i_1, i_2), \quad (i_2, i_3), \ . \ . \ . \ , (i_m, j)\} = \{(r_q, j)\}$$

define the set $j_a^l(r_q, j) = \{j:(a, j) \in A\} - \{i:i \in p(r_q, a)\}$.

Step 4. Check if the travel time from the selected terminal to each node j in the set J_a^l satisfies the constraint in eqn. (3). For each $j \in J_a^l(r_q, j)$, check:

$$t(r_q, i_1, i_2, \ . \ . \ . \ , i_m, j) \leqq \left[1 + \frac{\alpha(r_q, j)}{100}\right] t_m(r_q, j),$$

where $t(r_q, i_1, i_2, \ . \ . \ . \ , i_m, j) = t(r_q, i_1) + t(i_1, i_2) + \ . \ . \ . + t(i_m, j)$, if no, $J_a^l(r_q, j) := [J_a^l(r_q, j) = j]$; otherwise continue, if $J_a^l(r_q, j) = \phi$, go to Step 6.

Step 5. Calculate for the path from the selected terminal to node j, $p(r_q, j)$, the difference in passenger-hours between that path and the minimum travel time path (ΔpH).

Step 6. Select the next node j which belongs to the set J_a^l, and set it as a starting node ($a = j$) for possible extension of the path. Increment the attempt index ($l = l + 1$). If there are no more nodes to select, continue; otherwise return to Step 3.

Step 7. Consider a new set of nodes J_a^{l+1} with length (in terms of number of arcs) from the selected terminal r_q greater by one than the previously considered set J_a^l. If there is no such set, continue; otherwise return to Step 6.

Step 8. Delete the considered terminal from the set of terminals T in order to consider the next terminal ($q = q + 1$) from which bus routes can be constructed, and go to Step 1; if all terminals have been examined, stop.

This algorithm produces all feasible routes along with their directness measures. These routes can then be combined and evaluated, either at Level I or at Level II using the approach outlined in Fig. 2.

EXAMPLE AND DISCUSSION

In this section, we illustrate the route construction algorithm by applying it to the simple five-node network depicted in Fig. 3. The value on each arc (i, j) refers to the travel time $t(i, j)$. In this simple case, one can derive (by inspection) the shortest travel time matrix, $\{t_m(i, j)\}$, shown in Table 1.

The demand matrix, $\{D(i, j)\}$, is given in Table 2, and hence the direct passenger-hour matrix, $\{pH(i, j)\}$, shown in Table 3, can be obtained. Assume that only nodes 1 and 4 are possible route termini with total originating demand or 2780 and 4660 passenger-hours, re-

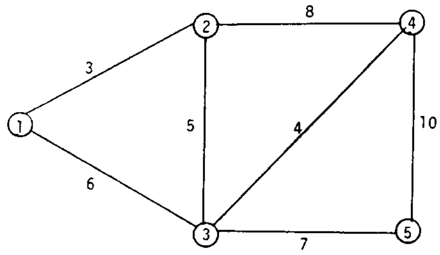

Fig. 3. Example network.

i \ j	1	2	3	4	5
1	0	3	6	10	13
2	3	0	5	8	12
3	6	5	0	4	7
4	10	8	4	0	10
5	13	12	7	10	0

Table 1. Shortest time matrix

i \ j	1	2	3	4	5
1	0	100	150	80	60
2	100	0	250	170	90
3	150	250	0	300	280
4	80	170	300	0	130
5	60	90	280	130	0

Table 2. Demand matrix

i \ j	1	2	3	4	5
1	0	300	900	800	780
2	300	0	1250	1360	1080
3	900	1250	0	1200	1960
4	800	1360	1200	0	1300
5	780	1080	1960	1300	0

Table 3. Direct passenger-hour matrix

	1	2	3	4	5
1	0	4.2	8.4	14.0	18.2
2	4.2	0	7.0	11.2	16.8
3	8.4	7.0	0	5.6	9.8
4	14.0	11.2	5.6	0	14.0
5	18.2	16.8	9.8	14.0	0

Table 4. Maximum time matrix

spectively. In this example, let $\alpha(i, j)$, the circuity limit, be 40%, resulting in the maximum allowable travel time matrix given in Table 4.

The algorithm can now be used to construct the feasible routes using the step-by-step algorithm as described in detail in the appendix.

The results of this example are shown in Table 5. As can be seen, none of the routes cover all five nodes, and therefore, the minimum number of routes in this problem is two.

Referring to the two-level methodological approach in Fig. 2, the outcome of the route construction algorithm can be fed into either a Level I or Level II analysis process. Further analysis may be required to include practical considerations that cannot be quantified. With such a small network, this analysis can be carried out manually or with the aid of a simple microcomputer program. For relatively large networks, it will be necessary to use either math-

Starting terminal	feasible routes	directness measure (excess passenger--hours)
1	1-2	0
	1-3	0
	1-2-4	80
	1-2-3	300
	1-3-4	0
	1-3-5	0
	1-2-3-4	630
	1-2-3-5	420
4	4-2	0
	4-3	0
	4-5	0
	4-2-1	80
	4-3-1	0
	4-3-2	170
	4-3-5	130
	4-3-2-1	630

Table 5. Feasible routes.

ematical programming or heuristics to solve the problem. The extension of this study is presently being performed by Israeli (1985) with respect to problems encountered in large networks. In addition, an effort can be made to incorporate the planners' own strategies for bus network design within the analysis approach. The two-level methodological approach can serve as a guideline for this purpose.

CONCLUDING REMARKS

This paper has presented an approach to the bus network design problem that should be easier to implement and more sensitive to downside risks of making route changes than previous proposed procedures. It is recognized that the main advantage of bus transit over other transit modes is the possibility to make route changes when necessary to better match the passengers' desires and at the same time to improve system performance. Nonetheless, the redesign of routes is not an activity frequently undertaken by bus properties. Careful consideration, therefore, should be given to achieve an approach that will be effective, practical and flexible.

The proposed approach presented in condensed form in Fig. 2 includes most of the bus planning process elements as well as consideration of the operational elements in order to recognize the full range of potential impacts. This paper has focused on three blocks in the flowchart in Fig. 2: (i) Level I formulation (objective function and constraints) considering only the passengers' viewpoint, (ii) Level II formulation considering both the passenger and operator impacts, and (iii) development of an algorithm to construct feasible bus routes. It is understood that the overall approach covers a large set of bus operations planning components and that the parts emphasized are only initial steps to fulfill all the approach objectives. Furthermore, the formulations and algorithms suggested for these three components are quite simple and do not include all constraints and objectives that would be needed for an operational procedure.

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APPENDIX

SOLUTION OF THE EXAMPLE

① $r_1 = 4 \Leftarrow \left[\sum_{j=1}^{5} pH(4, j) > \sum_{j=1}^{5} pH(1, j)\right], \quad a = 4, l = 1.$

② $J_4^1(4, j) = \{2, 3, 5\}.$

③ Check: $t(4, 2) < 11.2$ minimal $\Rightarrow \Delta pH(4, 2) = 0$,
$t(4, 3) < 5.6$ minimal $\Rightarrow \Delta pH(4, 3) = 0$,
$5(4, 5) < 14.0$ minimal $\Rightarrow \Delta pH(4, 5) = 0$.

④ $a = 2, l = 2, J_2^2(4, j) = \{1, 3, 4\} - \{4, 2\} = \{1, 3\}.$
Check: $t(4, 2, 1) = t(4, 2) + t(2, 1) = 11 < 14$ but not minimal
$t(4, 2, 3) = t(4, 2) + t(2, 3) = 13 > 5.6$.
Hence, $J_2^2(4, j) = \{1\}$,
$\Delta pH(4, 2, 1) = 80$ (this is observed only along the path between node 4 and 1).

⑤ $a = 3, l = 3, J_3^3(4, j) = \{1, 2, 4, 5\} - \{4, 3\} = \{1, 2, 5\}.$
Check: $t(4, 3, 1) = 5(4, 3) + t(3, 1) = 10 < 14$ minimal,
$t(4, 3, 2) = 5(4, 3) + t(3, 2) = 9 < 11.2$ not minimal,
$t(4, 3, 5) = t(4, 3) + t(3, 5) = 11 < 14$ not minimal.
Hence, $\Delta pH(4, 3, 1) = 0$,
$\Delta pH(4, 3, 2) = 170$,
$\Delta pH(4, 3, 5) = 130$.

⑥ $a = 5, l = 4, J_5^4(4, j) = \{3, 4\} - \{4, 5\} = \{3\}.$
Check: $t(4, 5, 3) = t(4, 5) + t(5, 3) = 17 > 4$.
Hence, $J_5^4(4, j) = \phi$.

⑦ Consider the next $J_a^l(r_q, i) = J_2^2(4, j) = \{1\}$, set $a = 1$, $l = 5$, $J_1^5(4, j) = \{2, 3\} - \{4, 2, 1\} = \{3\}$.
Check: $t(4, 2, 1, 3) = t(4, 2) + t(2, 1) + t(1, 3) = 8 + 3 + 6 = 17 > 5.6$.
Hence, $J_1^5(4, j) = \phi$.

⑧ Consider the next $J_3^3(4, j) = \{1, 2, 5\}$, set $a = 1, l = 6$, $J_1^6(4, j) = \{2, 3\} - \{4, 3, 1\} = \{2\}$.
Check: $t(4, 3, 1, 2) = t(3, 1) + t(1, 2) = 13 > 11.2$.
Hence, $J_1^6(4, j) = \phi$.
Set $a = 2, l = 7, J_2^7(4, j) = \{1, 3, 4\} - \{4, 3, 2\} = \{1\}$.
Check: $t(4, 3, 2, 1) = t(4, 3) + t(3, 2) + t(2, 1) = 12 < 14$ but not minimal.
Hence, $\Delta pH(4, 3, 2, 1) = 170 + 160 + 300 = 630$ regarding the demand between (4, 2), (4, 1), (3, 1), respectively.
Set $a = 5, l = 8, J_5^8(4, j) = \{4, 3\} - \{4, 3, 5\} = \phi$.

⑨ Select the next terminal as a candidate for route departure point $r_2 = 1, a = 1, l = 1$.

⑩ $J_1^1(1, j) = \{2, 3\}$.

⑪ Check: $t(1, 2) < 4.2 \Rightarrow \Delta pH(1, 2) = 0$,
$t(1, 3) < 8.4 \Rightarrow \Delta pH(1, 3) = 0$.

⑫ $a = 2, l = 2, J_2^2(1, j) = \{1, 3, 4\} - \{1, 2\} = \{3, 4\}$.
Check: $t(1, 2, 4) = t(1, 2) + t(2, 4) = 11 < 14$ not minimal,
$t(1, 2, 3) = 5(1, 2) + t(2, 3) = 8 < 8.4$ not minimal.
Hence, $\Delta pH(1, 2, 4) = 80$,
$\Delta pH(1, 2, 3) = 300$.

⑬ $a = 3, l = 3, J_3^3(1, j) = \{1, 2, 4, 5\} - \{1, 3\} = \{2, 4, 5\}$.
Check: $t(1, 3, 2) = t(1, 3) + t(3, 2) = 11 > 4.2$,
$t(1, 3, 4) = t(1, 3) + t(3, 4) = 10 < 14$ minimal,
$t(1, 3, 5) = t(1, 3) + t(3, 5) = 13 < 18.2$ minimal.
Hence, $J_3^3(1, j) = \{4, 5\}$, and
$\Delta pH(1, 3, 4) = 0, \Delta pH(1, 3, 5) = 0$.

⑭ Consider next $J_a^l(r_q, j) = J_2^2(1, j) = \{3, 4\}$, set $a = 3, l = 4$,
$J_3^4(1, j) = \{1, 2, 4, 5\} - \{1, 2, 3\} = \{4, 5\}$.
Check: $t(1, 2, 3, 4) = t(1, 2) + t(2, 3) + t(3, 4) = 12 < 14$ not minimal,
$t(1, 2, 3, 5) = t(1, 2) + t(2, 3) + t(3, 5) = 15 < 18.2$ not minimal.
Hence, $\Delta pH(1, 2, 3, 4) = 160 + 170 + 300 = 630$,
$\Delta pH(1, 2, 3, 5) = 120 + 300 = 420$.
Set $a = 4, l = 5, J_4^5(1, j) = \{2, 3, 5\} = \{1, 2, 4\} = \{3, 5\}$.
Check: $\left.\begin{array}{l} t(1, 2, 4, 3) = 15 > 8.4 \\ t(1, 2, 4, 5) = 21 > 18.2 \end{array}\right\} \Rightarrow J_4^5(1, j) = \phi$.

⑮ Consider next $J_3^3(1, j) = \{4, 5\}$, set $a = 4$, $l = 6$,

$J_4^6(1, j) = \{2, 3, 5\} - \{1, 3, 4\} = \{2, 5\}$.

Check: $\left.\begin{array}{l} t\,(1, 3, 4, 2) = 18 > 4.2 \\ t(1, 3, 4, 5) = 20 > 18.2 \end{array}\right. \Rightarrow J_4^6(1, j) = \phi$.

Set $a = 5$, $l = 7$, $J_5^7(1, j) = \{3, 4\} - \{1, 3, 5\} = \{4\}$.

Check: $t(1, 3, 5, 4) = 23 > 14 \Rightarrow J_5^7(1, j) = \phi$.

⑯ Consider next $J_3^4(1, 2, 3) = \{4, 5\}$, set $a = 4$, $l = 8$,

$J_4^8(1, j) = \{2, 3, 5\} - \{1, 2, 3, 4\} = \{5\}$.

Check: $t(1, 2, 3, 4, 5) = 22 > 18.2 \Rightarrow J_4^8(1, j) = \phi$.

Set $a = 5$, $l = 9$, $J_5^9(1, j) = \{3, 4\} - \{1, 2, 3, 5\} = \{4\}$.

Check: $t(1, 2, 3, 5, 4) = 25 > 14 \Rightarrow J_5^9(1, j) = \phi$.

⑰ STOP. Prepare the table with the feasible routes along with their directness measure.