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CIS 315

Professor Wilson

Assignment #4

Question 1

(1)

$$W^{(1)}\quad \begin{array}{cc} & \begin{matrix}1&2&3&4&5&6\end{matrix} \\ \begin{matrix}1\\2\\3\\4\\5\\6\end{matrix} & \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{pmatrix}\end{array}$$

(2)

$$W^{(2)}\quad \begin{array}{cc} & \begin{matrix}1&2&3&4&5&6\end{matrix} \\ \begin{matrix}1\\2\\3\\4\\5\\6\end{matrix} & \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{pmatrix}\end{array}$$

(3)

$$W^{(3)}\quad \begin{array}{cc} & \begin{matrix}1&2&3&4&5&6\end{matrix} \\ \begin{matrix}1\\2\\3\\4\\5\\6\end{matrix} & \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{pmatrix}\end{array}$$

(4)

$$W^{(4)}\quad \begin{array}{cc} & \begin{matrix}1&2&3&4&5&6\end{matrix} \\ \begin{matrix}1\\2\\3\\4\\5\\6\end{matrix} & \begin{pmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{pmatrix}\end{array}$$

(5)

$$W^{(5)}\quad \begin{array}{cc} & \begin{matrix}1&2&3&4&5&6\end{matrix} \\ \begin{matrix}1\\2\\3\\4\\5\\6\end{matrix} & \begin{pmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{pmatrix}\end{array}$$

(6)

$$W^{(6)}\quad \begin{array}{cc} & \begin{matrix}1&2&3&4&5&6\end{matrix} \\ \begin{matrix}1\\2\\3\\4\\5\\6\end{matrix} & \begin{pmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{pmatrix}\end{array}$$

Question 2

In order to perform the computations in place, we must show that

$$d_{ik}^{(k)} = d_{ik}^{(k-1)}$$

as we can no longer reference the previous $D^{(k-1)}$. Due to the fact that $d_{ik}^{(k)}$ is the subpath from i to k with all vertices in {1, 2… k}, but k cannot be an intermediate vertex (because this would create a cycle), $d_{ik}^{(k)} = d_{ik}^{(k-1)}$. The same applies to the path from k to j.

Question 3

Step 1 (structure of subproblem):

P(i) = the minimum penalty required to reach hotel i

a_i = distance in miles to hotel i from starting, n is final hotel

Step 2 (recursion):

Base: P(0) = 0 (minimum penalty to reach a_o is 0)

Recurrence (for 1 <= k <= n):

P(k) = min for {0 <= j < k} of all $\{P(j) + (200 - (a_k - a_j))^2\}$

Question 4

Step 1 (structure of subproblems):

A(i) = optimal cost up to month i assuming you end up in Albany

B(i) = optimal cost up to month i assuming you end up in Beaverton

where 1 <= i <= n (n is the final month)

Step 2 (recursion):

Base: A(1) = a_1

B(1) = b_1

Recurrence (for 2 <= k <= n):

// cost of this month plus whatever would be better from last month

A(k) = a_k + min(A(k-1), B(i-1) + M)

// same principle

B(k) = b_k + min(B(k-1), A(i-1) + M)