

## Contents

### Groups and Subgroups

1. Introduction and Examples
2. Binary Operations
3. Isomorphic Binary Structures
4. Groups
5. Subgroups
6. Cyclic Groups
7. Generating Sets and Cayley Digraphs

### Permutations, Cosets and Direct Products

8. Groups of Permutations
9. Orbits, Cycles and the Alternating Groups
10. Cosets and the Theorem of Lagrange
11. Direct Products and Finitely Generated Abelian Groups
12. Plane Isometries

### Homomorphisms and Factor Groups

13. Homomorphisms
14. Factor Groups
15. Factor-Group Computations and Simple Groups
16. Group Action on a Set
17. Applications of G-Sets to Counting

### Rings and Fields

18. Rings and Fields
19. Integral Domains
20. Fermat's and Euler's Theorems
21. The Field of Quotients of an Integral Domain
22. Rings of Polynomials
23. Factorization of Polynomials over a Field
24. Noncommutative Examples
25. Ordered Rings and Fields

### Ideals and Factor Rings

26. Homomorphisms and Factor Rings
27. Prime and Maximal Ideals
28. Grobner Bases for Ideals

### Extension Fields

29. Introduction to Extension Fields
30. Vector Spaces
31. Algebraic Extensions

32. Geometric Constructions

33. Finite Fields

## **Advanced Group Theory**

34. Isomorphism Theorems

35. Series of Groups

36. Sylow Theorems

37. Applications of the Sylow Theory

38. Free Abelian Groups

39. Free Groups

40. Group Presentations

## **Groups in Topology**

41. Simplicial Complexes and Homology Groups

42. Computations of Homology Groups

43. More Homology Computations and Applications

44. Homological Algebra

## **Factorization**

45. Unique Factorization Domains

46. Euclidean Domains

47. Gaussian Integers and Multiplicative Norms

## **Automorphisms and Galois Theory**

48. Automorphisms of Fields

49. The Isomorphism Extension Theorem

50. Splitting Fields

51. Separable Extensions

52. Totally Inseparable Extensions

53. Galois Theory

54. Illustrations of Galois Theory

55. Cyclotomic Extensions

56. Insolvability of the Quintic

## **Matrix Algebra**

## Basic Concepts

### Probability and Relative Frequency

$$P(A) = \frac{N(A)}{N}$$

where  $N$  is the total number of outcomes of the experiment and  $N(A)$  is the number of outcomes leading to the occurrence of the event  $A$

The ratio  $\frac{n(A)}{n}$  is called the relative frequency of the event  $A$ . It turns out that the relative frequencies  $\frac{n(A)}{n}$  are virtually the same for large  $n$ , clustering about some constant

$$P(A) \sim \frac{n(A)}{n}$$

etc