Math 175: Elementary Number Theory

Problem set: Week 3

Congruences and modular arithmetic

1. Exercices 3.1, 3.4, 3.7, 3.8, 3.9 of [Jones-Jones]

2. Let $\sigma(n) = \sum_{d|n}$ denote the sum of divisors of a natural number $n \in \mathbf{N}$. Prove that $\sigma(mn) = \sigma(m)\sigma(n)$ if $\gcd(m, n) = 1$.

3. A number $n \in \mathbf{N}$ is said to be perfect if $\sigma(n) = 2n$. Prove that $2^{p-1}(2^p - 1)$ is a perfect number if $2^p - 1$ is a Mersennes prime.

4. Prove that all even perfect number $n \in \mathbf{N}$ is of the form $n = 2^{p-1}(2^p - 1)$ where p is a perfect prime. (Euclid-Euler theorem).

5. Exercices 4.15, 4.16 of [Jones-Jones]