STAT 24410 ASSIGNMENT 5

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Hulek 1.1. Prove that for any ideal J, the radical $\sqrt{J}$ is also an ideal.

Proof. Let $a, b \in \sqrt{J}$ with $a^m, b^n \in J$ for integers $m, n \geq 2$. Then

$$(a+b)^{m+n} = \sum_{j=0}^{m+n} \binom{m+n}{j} a^j b^{m+n-j}.$$

Each term in the sum must have a factor of either a^m or b^n, and since J is an ideal each term is thus in J, and in turn so is the sum. Thus $\sqrt{J}$ is an additive group.

Let $a \in \sqrt{J}$ with $a^m \in J$ and let $g \in A$. Then $(ag)^m = a^m g^m$ contains a factor of a^m and is thus in J. Then J is an ideal. $\square$

Hulek 1.2. Let $V := V(I) \subset \mathbb{A}_k^3$ be the algebraic set corresponding to the ideal $I := (x^2 - yz, xz - x)$. Decompose V into its irreducible components.