# 6.1

**Question**: Find the orders of the elements of $U_9$ and of $U_10$.
**Solution**:
Answer

# 6.2

**Question**: Show that if l and m are positive integers with highest common factor h, then $gcd(2^l-1, 2^m-1)$ divides $2^h - 1$.
**Solution**:
Answer

# 6.3

**Question**: The groups $U_10$ and $U_12$ both have order 4; show that exactly one of them is cyclic.
**Solution**:
Answer

# 6.4

**Question**: Find primitive roots in $U_n$ for n = 18,23,27 and 31.
**Solution**:
Answer

# 6.5

**Question**: Show that if $U_n$ has a primitive root then it has $\phi(\phi(n))$ of them.
**Solution**:
Answer

# 6.6

**Question**: Verify that the element 5 is a generator of $U_7$
(answer to problem)

# 6.7

**Question**: Find the elements of order d in $U_11$, for each d dividing 10; which elements are generators?
**Solution**:
Answer

# 6.8

**Question**: Verify that 2 is a primitive root mod(25) by calculating its powers.
**Solution**:
Answer

# 6.9

**Question**: Show that 2 is a primitive root mod $(3^e)$ for all $e \geqslant 1$.
**Solution**:
Answer

## 6.10

**Question**: Find an integer which is a primitive root $mod(7^e)$ for all $e \geqslant 1$.
**Solution**:
Answer

## Problem 2

**Question**: Check that 3 is a primitive root modulo 17 by constructing an explicit isomorphism between $Z/16Z$ and $(Z/17Z)^x$ mapping the class of 1 on the class of 3. Use this map to solve the congruence equations
**Solution**:
Answer

## (a)

$z^{12} \equiv 16 \bmod 17$
**Solution**:
Answer

## (b)

$x^{20} \equiv 13 \bmod 17$
**Solution**:
Answer

## (c)

$x^{48} \equiv 9 \bmod 17$
**Solution**:
Answer

## (d)

$x^{11} \equiv 9 \bmod 17$
**Solution**:
Answer

## 7.1

**Question**: Find all solutions in $Z_{15}$ of the congruence $x^2 - 3x + 2 \equiv 0 \bmod (15)$.
**Solution**:
Answer

## 7.2

**Question**: What square roots do the elements 5 and 16 have in $Z_{21}$? Hence find all solutions of the congruences $x^2 + 3x + 1 \equiv 0 \bmod (21)$ and $x^2 + 2x - 3 \equiv 0 \bmod (21)$.
**Solution**:
Answer