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Team 1071 - Problem B

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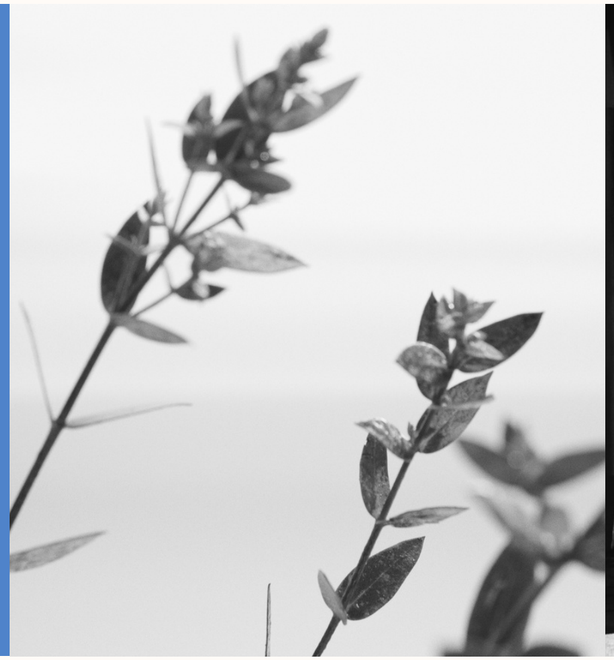
BACKGROUND

People often have a tendency to punish those who seem to harm others, as claimed in the problem and some relevant papers.

Our purpose is to build up a model to describe the interaction of each group that has the level of dealing with the situation to some different extent. And we have done 2 models for the investigation of the problem.



DEVELOPING THE MODEL

OUR APPROACH TO THE PROBLEM.

- We will build a model which describes the interaction between groups with different ways of punishing people.
- We consider only two ways of punishing people:

  + The philosophy of **Type 0** is punishing people severely, the one who has been punished sees punishment as a dangerous threat.

  + The philosophy of **Type 1** is punishment should be unpleasant but not severe. the punished people know what they did is wrong, and they have a chance to adjust their behaviors.

OUR APPROACH TO THE PROBLEM.

- We only consider the interaction among four groups:

  + Group A: The people having the propensity not to punish anyone who does wrong things.

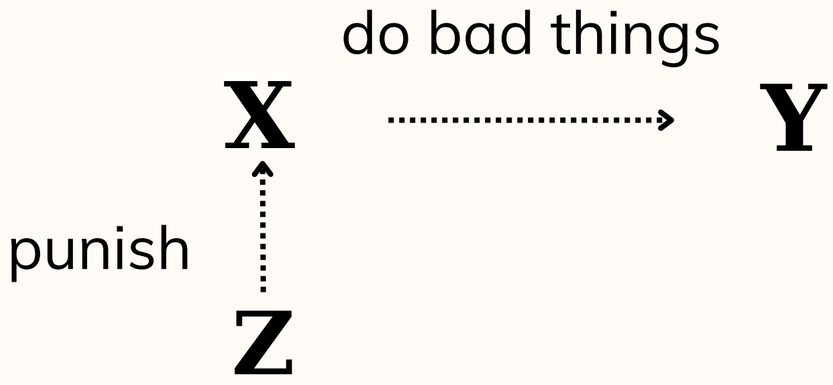
  + Group B0: The people having a propensity to punish people severely or **type 0.**

  + Group B1: The people who belong to **type 1**.

  + Group C: The people having the propensity to commit a crime, or aggressive behaviors more often than normal people.

OUR APPROACH TO THE PROBLEM.

- Our model also describes the changes among four groups.
- We consider this type of situation as the graph below always happens.



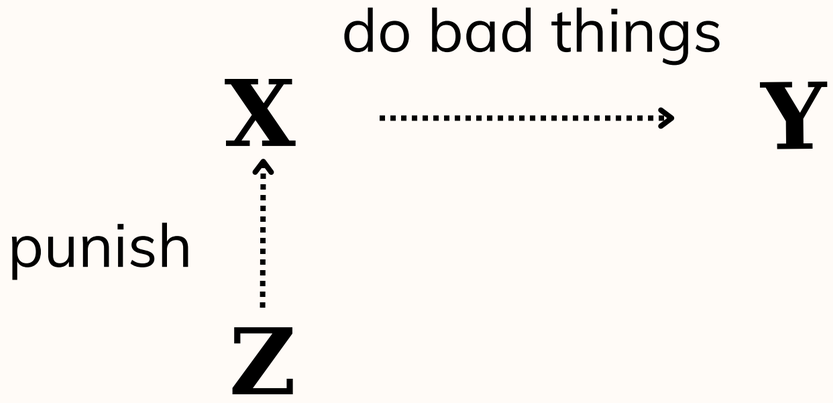
- **X** who do bad things to **Y**
- **Y** victim of **X**
- **Z** the observer, he can do nothing or punish **X**

ASSUMPTIONS OF VARIABLES:

- C will be cruel and aggressive than other groups.
- When B0 punish people, they likely convert to group C.
- When B1 punish C, those in group C likely convert to group B1.
- When a person does bad things but no one cares or that behavior observed by A but A doesn’t do anything, that person is likely to convert to group C.
- **A** will belong to **type 0** or **type 1** when A is victim and B0 or B1 help A punish the criminal.
- B1 will quit his philosophy when he is the victim of C.
- Assumptions that type 0 is bad then **type 1**, we based on resources in **[1]** and **[2].**

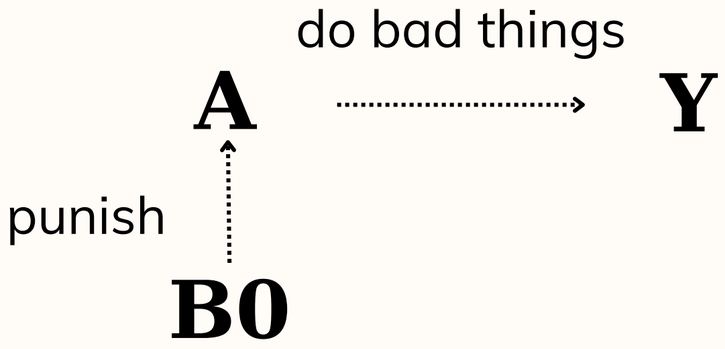
ASSUMPTIONS IN THE MODELS:

- We consider only isolated society, namely no one in or out in that society, define n = number of people in that society.
- We assume that, at any time, there always have all possible interactions **(X, Y, Z)** as below.



OUR IDEA TO CONSTRUCT THE MODEL:

Example, in situation :



Then someone in group A convert to group C because our assumsption in model that B0 punish too severe so A will likely suffers negative feelings that leads to broken moral.

Denote α_1: probability of one person in group A in that situation convert to group C

$$\frac{dA}{dt} = -\alpha_1 A \frac{B_0}{n-1}$$

$\frac{B_0}{n-1}$ is the probability of one fixed person in A meets that scenario computed by P(Z is B0 | X is A).

FORMULATING THE SYSTEM OF DIFFERENTIAL EQUATIONS

$$\frac{dA}{dt} = -\alpha_1 A\frac{B_0}{n-1} - \alpha_2 A\frac{B_1 C}{(n-1)(n-2)} - \alpha_3 A\frac{CB_0}{(n-1)(n-2)}$$

$$\frac{dB_0}{dt} = -(\beta_1+\beta_2)B_0\frac{B_0-1}{n-1} - \alpha_3 A\frac{CB_0}{(n-1)(n-2)} + \alpha_4 B_1\frac{C(B_1-1)}{(n-1)(n-2)} + \alpha_5 B_1\frac{CB_0}{(n-1)(n-2)}$$

$$\frac{dB_1}{dt} = -\alpha_6 B_1\frac{B_0}{n-1} - \alpha_4 B_1\frac{C(B_1-1)}{(n-1)(n-2)} - \alpha_5 B_1\frac{CB_0}{(n-1)(n-2)} + \beta_1 B_0\frac{B_0-1}{n-1} + \alpha_2 A\frac{B_1 C}{(n-1)(n-2)} + \alpha_7 C\frac{B_1}{n-2}$$

$$\frac{dC}{dt} = \beta_2 B_0\frac{B_0-1}{n-1} + \alpha_1 A\frac{B_0}{n-1} + \alpha_6 B_1\frac{B_0}{n-1} - \alpha_7 C\frac{B_1}{n-2}$$

GRAPHING MODEL

VISUALIZE THE MODEL

- We use the modified Euler method to plot the model. The formula for modified Euler method is

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, \hat{y}_{n+1})\right]$$

where

$$\hat{y}_{n+1} = y_n + hf(x_n, y_n)$$

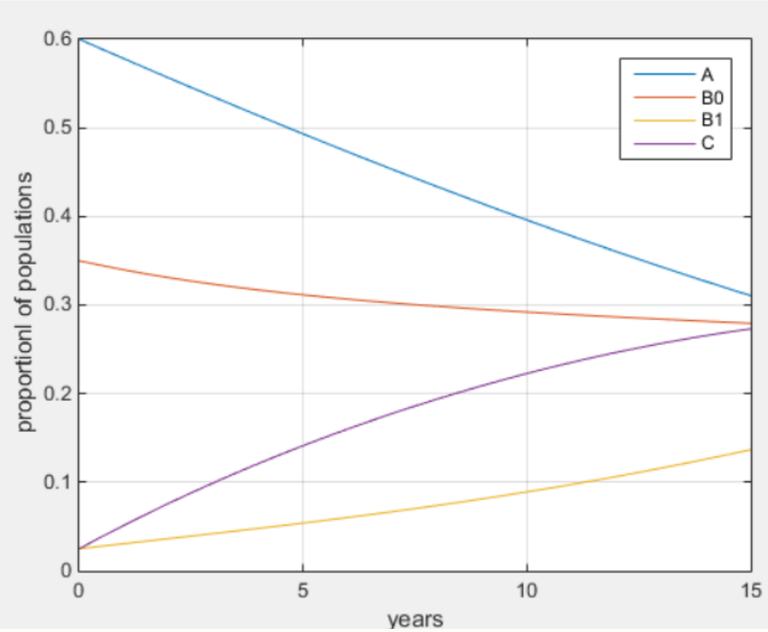
VISUALIZE THE MODEL

- We plot the changes in population of four groups over times.
- After some experiment and some assumption in estimate parameter, we set parameters as follow:

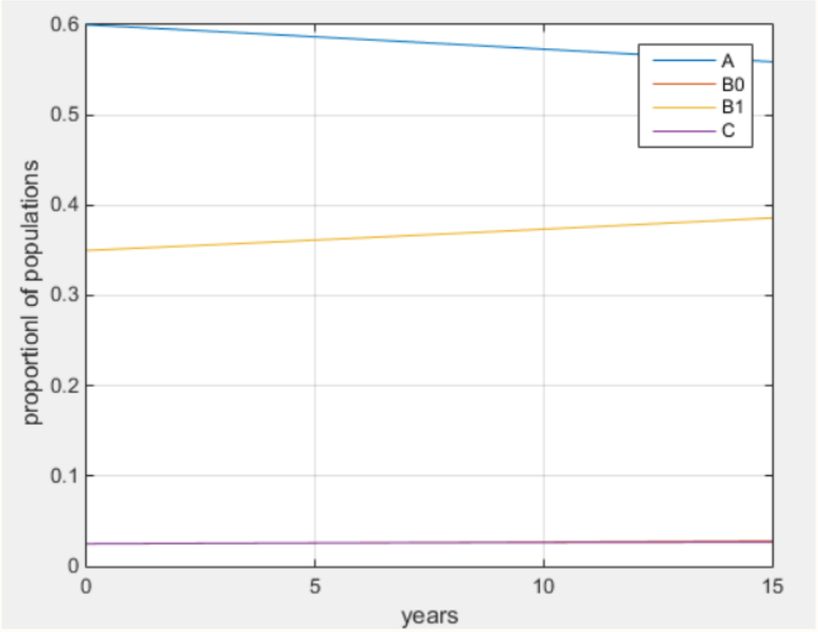
$$
\begin{aligned}
n &= 1000 \\
\alpha_1 &= 0.1, \quad \alpha_2 = 0,2, \quad \alpha_3 = 0.2 \\
\alpha_4 &= 0.05, \quad \alpha_5 = 0.05, \quad \alpha_6 = 0.05 \\
\alpha_7 &= 0.2, \quad \beta_1 = 0.05, \quad \beta_2 = 0.05
\end{aligned}
$$

RESULT WITH ONE DOMINATED TYPE

Initial: A=600, B0 = 350, B1 = 25
C = 25

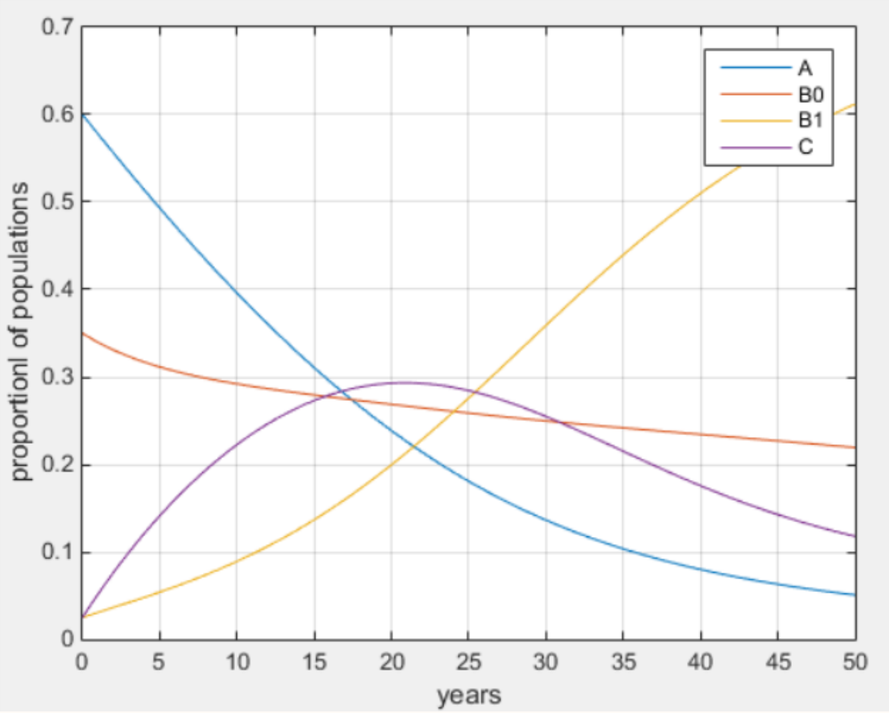


Initial: A=600, B0 = 25, B1 = 350,
C = 25

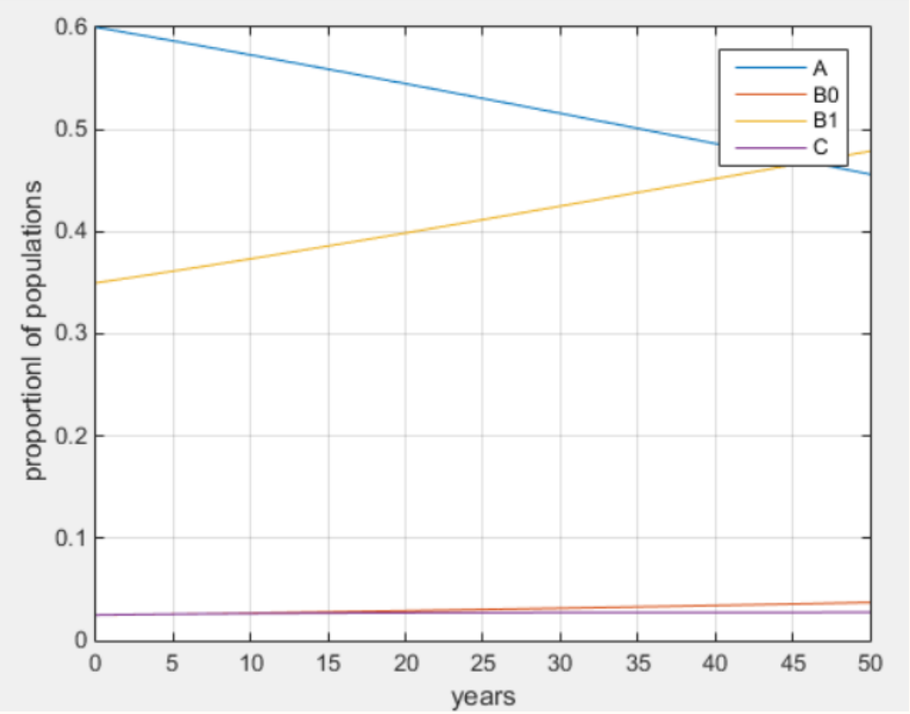


TRENDS IN OUR MODEL IN 50 YEARS LATER

Initial: A=600, B0 = 350, B1 = 25
C = 25



Initial: A=600, B0 = 25, B1 = 350,
C = 25



CONCLUSION & LIMITATIONS

CONCLUSION

- In our model, we see that type 1 will be most favourable in social.
- Social will have tend to be good, having small group with high violence, namely B0 or C is small.

LIMITATIONS AND FUTURE DIRECTION

Limitations

- Our model really need approriate parameters, but our parameters is measured by probability so we thought that we can get it by conduct surveys and analysis.
- We not sure that our model is good in real life with “true” parameters , because parameters does really tell a different story.

Future direction

- Add more group with different philosophy in punishment.
- Add some conditions like when observer punish someone, they consider if there any risk they can afford to punish them, ….

ANOTHER APPROACH

OUR GOALS

In this model, we would like to learn about the internal nature of a society. In particular, we would like to study how the crimes and the punishments influence the development of a society. After several years, will this society disappear or grow uncontrollably? Will the criminals rule the world or the society be an utopia?

DEVELOPING THE MODEL

ASSUMPTIONS

- The number of crimes and punishments are only influenced by themself and the population.
- There are not the existences of the complex factors such as education, wealth, history, etc.
- The society is isolated and not affected by the accidents such as war, disaster, etc.
- We suppose that the aim of the development is simply the population growth.
- The environment provide immutable limited resourses for the society.
- People have a propensity to try to discipline those who they believe are hurting others.

CONSTRUCT THE MODEL

We denote the function of the number of crimes and punishments by f(t) and g(t) respectively, and the function of population will be denoted by P(t), where t represents time (years).

Then, the product fg represent the probability that a crime is punished suitably.

Because of the assumptions, f is going to decrease when the probability that a crime is punished is large and increase when the number of crimes is lagre , therefore, we obtain the first ODE

$$\frac{\mathrm{d}f}{\mathrm{d}t} = -\alpha f g + \beta f$$

CONSTRUCT THE MODEL

The number of punishments will increase when the punishments is suitable, that is, the product fg is large.

In real life, when the law is too severe, the resident will not be happy, therefore, the level of punishment of each person will decrease.

And we add a constant C to decribe the assumptions that the infants have a propensity to try to discipline those who they believe are hurting others.

In conclusion, we obtain the second ODE

$$\frac{\mathrm{d}g}{\mathrm{d}t} = \gamma f g - \delta g + C$$

CONSTRUCT THE MODEL

Finally, we introduce the third equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = rP(1 - \frac{P}{N}) + (H - \epsilon g) - \psi f$$

The first term decribe that the population grow exponentially. The constant N represent the natural resources and its term is added to construct a boundary for P.

The second term decribe the fact that if the punishment is too harsh then the life of residents will be harder, the constant H represent the boundary for the harshness.

CONSTRUCT THE MODEL

Every bad thing carried out can badly impact on other people, we express this fact by adding the third term to ODE, in which the constant represents the proportion that a crime can murder someone.

Note that the parameters’s set in the ODEs depend on the society which we would like to study and can be obtained by statistics.

FORMULATING THE SYSTEM OF DIFFERENTIAL EQUATIONS

From the above agruments, we obtain the model as the following systems of ODEs:

$$\begin{cases} \dfrac{\mathrm{d}f}{\mathrm{d}t} = -\alpha f g + \beta f \\ \dfrac{\mathrm{d}g}{\mathrm{d}t} = \gamma f g - \delta g + C \\ \dfrac{\mathrm{d}P}{\mathrm{d}t} = rP(1 - \dfrac{P}{N}) + (H - \epsilon g) - \psi f \end{cases}$$

GRAPHING MODEL

VISUALIZE THE MODEL

We again use the modified Euler method to approximate the solutions of the ODEs.

Thanks to experiments, we visualize the model with two parameters's set A and B respectively as follow to be easy to observe

$$\alpha = 0.002, \beta = 0.1$$

$$\gamma = 0.0025, \delta = 0.1, C = 0$$

$$r = 0.01, N = 10000, H = 1000, \epsilon = 0.1, \psi = 0.5$$

$$\alpha = 0.002, \beta = 0.1$$

$$\gamma = 0.0025, \delta = 0.1, C = 1$$

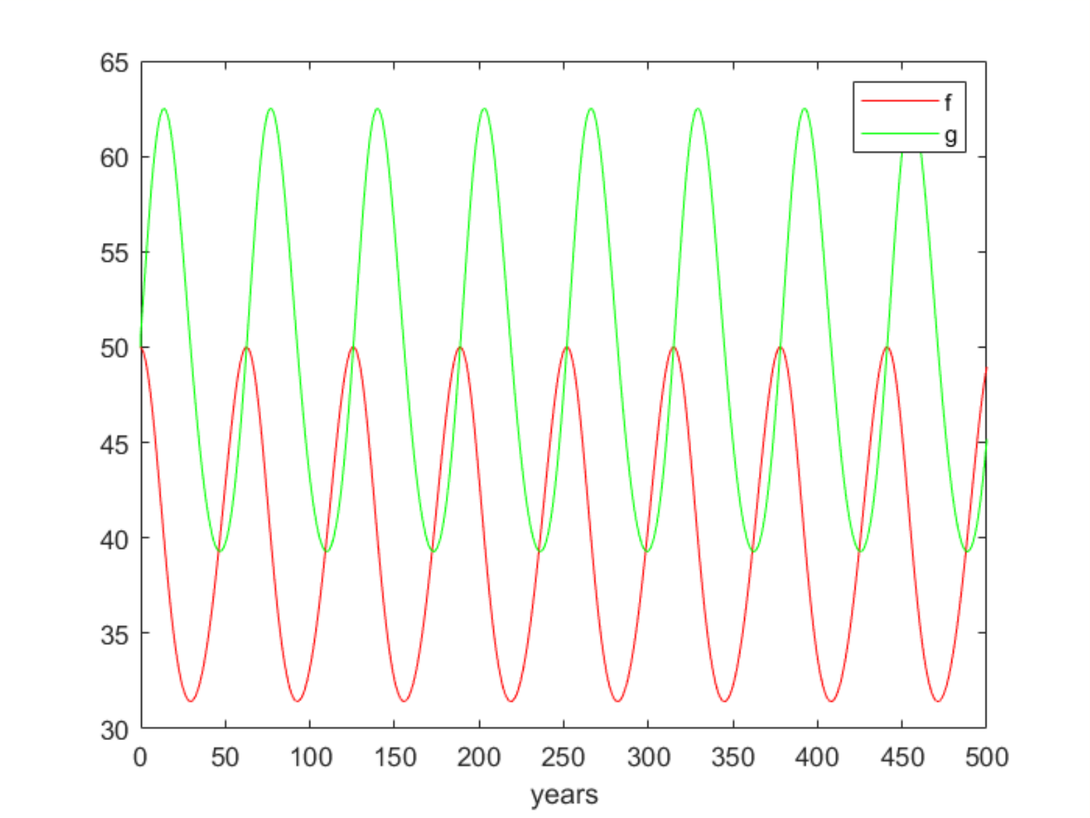
$$r = 0.01, N = 10000, H = 1000, \epsilon = 0.1, \psi = 0.5$$

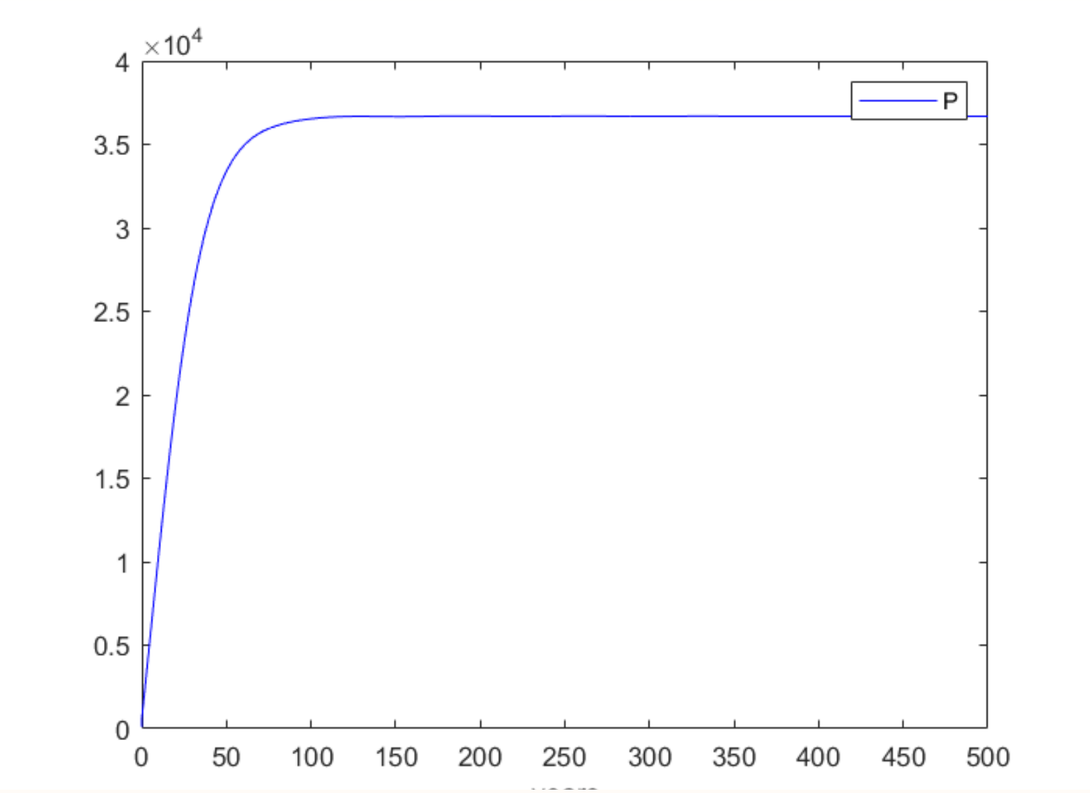
and the same initial coditions to have a comparison

$$f(0) = 50, g(0) = 50, P(0) = 100.$$

NUMERICAL RESULTS

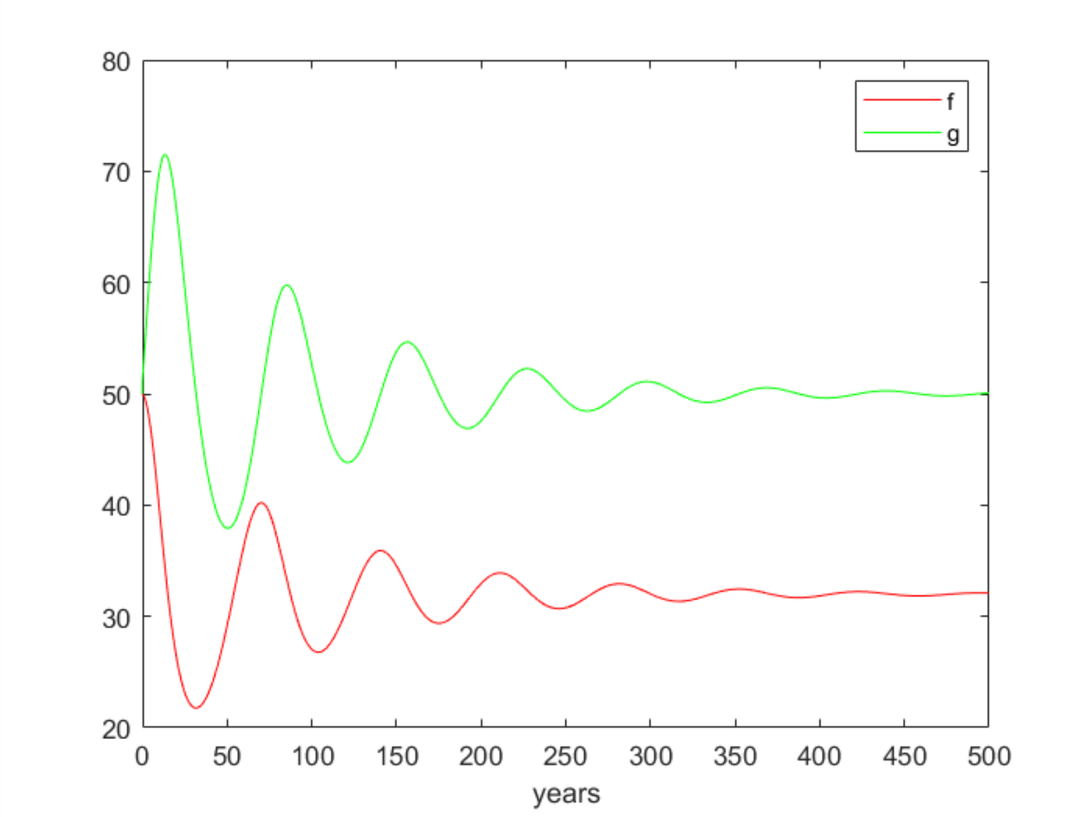
Parameters A (C = 0):

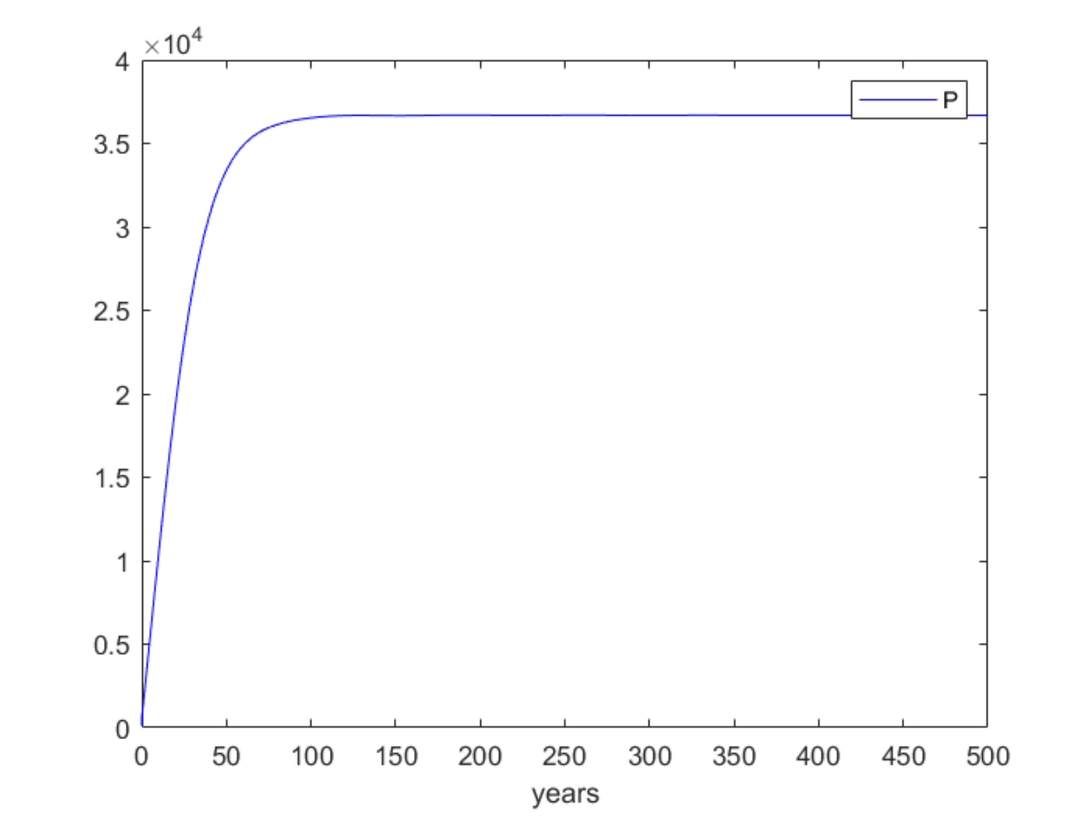




NUMERICAL RESULTS

Parameters B (C = 1):





CONCLUSION & LIMITATIONS

CONCLUSION

The graphs show us that:

- The number of crimes and punishments changes in a kind of periodical way.
- When the number of crimes increases, the number of punishments tends to decrease and vice versa.
- If our assumptions are true, then the amplitude of the fluctuation of the graphs of f and g will decrease and converge to a balance point in the future.
- The population increases sharply in the beginning and tends toward an equilibrium in the after years.

CONCLUSION

We draw some conclusions about the societies:

- An isolated society can not kill itself unless the initial conditions is too extreme.
- It is hard for the number of crimes and punishments to vanish, but they tend to converge to an exact point which is optimal for the society.
- There is a high possibility for the assumption mentioned in Problem B because of the fact that the number of crimes in many countries change little every years.
- A society tends to reach an equilibrium of development if there does not exist alien factors. Then society makes the best use of natural resources and the situation of crime and punishment is balanced.

LIMITATIONS AND FUTURE DIRECTION

Limitations

- This model is too simple with many constants. It is difficult for us to estimate the parameters and we need a huge amount of data for this work.

- Some assumptions are not actual in modern life. The meaning of f and g is coarse to estimate the characteristics of people.

Future direction

- Modelize complex factors such as education, religion, etc. Add more parameters or change them by some functions to be closer to the reality.

- We suggest using the system of PDEs, even, SPDEs for modelizing to observe more variables and situations. The functions f and g will be more natural if they can be considers as distribution functions.

REFERENCES

1. Crime and Punishment: Does Punishment Work? David J. Cherrington
2. An Examination of Deterrence Theory: Where Do We Stand?


THANK
YOU
VNUHCM - University of Science