

Quadratic Sorting Algorithms - $O(n^2)$

Say we have an array of values, put these values in ascending or descending order.

A few variations:

- 1) These could be objects, not numbers. In our examples, we'll make them numbers, but in practical applications they are usually objects
- 2) For example, events, sort by timestamp

IN PLACE SORT - just change the array you are in

OUT OF PLACE SORT - allocate a new array of the same size, and put your sorted version here.

(We're going to be studying in-place today)

Stable Sort - things that are equal stay in the same order

- If you're dealing with integers - who cares, a 9 is a 9
- But if you're dealing with events that you're ordering with timestamps, maybe if things occurred at the same time you want them in the original order of the array
- Most of our examples happen to be stable (or at least have a stable version)

In order to sort, you need to have a comparison function:

$f(a, b)$ to check if $a < b$.

If they are integers or doubles, your machine comes with these "<", ">"

If they are objects, Java has a Comparable interface:

- If a, b are comparable, you can call $a.compareTo(b)$ and get an answer
- Usually it's < 0 if a is smaller, 0 if they are equal > 0 if b is smaller
- So, essentially $a - b$

Notes on comparison functions. Obvious? Perhaps, but let's put on our completionist mathematician hat here.

- Needs to be deterministic
 - You shouldn't get different answers every time you call `a.compareTo(b)` on the same objects!
- Needs to follow the transitive law: $a < b$ and $b < c$ implies $a < c$
- Needs to be a total order (either $a < b$, $a = b$, or $a > b$)
 - Partial orders (where some objects don't compare to each other) lead to a different type of sort, out of scope.

One of the most studied problems in computer science, from the advent of computers. Why?

- Back to UNIVAC, ENIAC
- They discovered that they needed to run sorts a lot; very useful
- There are many different sorting algorithms, still being invented. There's no one-size-fits all optimum, and it often depends on the situation
- Interesting theoretical properties.

Let's give away the ending:

- Today we are going to go over quadratic sorts $O(n^2)$. These are simplest sorting algorithms that work well on small sets of data
- End of semester: $O(n \cdot \log n)$ algorithms, best you can do in most circumstances, and in special circumstances, where you get close to linear time.

We need another piece, a SWAP FUNCTION

```
swap(arr, i, j):  
    temp = arr[i]  
    arr[i] = arr[j]  
    arr[j] = temp
```

Bubble Sort

```
BUBBLE_SORT(arr):  
    sorted = false  
    while(!sorted):  
        sorted = true  
        for(i from 0 to n - 2):  
            if (arr[i] > arr[i + 1]):  
                swap(arr, i, j)  
            sorted = false
```

<https://www.youtube.com/watch?v=Cq7SMsQBEUw>

SHOW VIDEO

- After each pass, the highest value “bubbles” to the top
- Worst Case, Typical Case = $O(n^2)$
- List already ordered = $O(n)$
- Note - if the smallest value is at the top, it will take n passes before it goes to the bottom
- Very inefficient with the swaps, which are more involved than comparisons

<https://www.youtube.com/watch?v=8oJS1BMKE64>

Insertion Sort

```
For i = 1 to n - 1:  
    //everything less than i is sorted at this point  
    // Bubble down arr[i] to find out where it goes in arr[0]..arr[i-1]  
    for(j = i; j > 0; j--)  
        if (arr[j - 1] > arr[j]):  
            swap(j - 1, j)
```

WORST CASE: $O(n^2)$ - reverse sorted

Selection Sort

```
MIN(arr) {  
  min = arr[0]  
  minIndex = 0  
  for(i = 1; i < arr.length; i++):  
    if (arr[i] < min):  
      min = arr[i]  
      minIndex = i  
}
```

<https://www.youtube.com/watch?v=92BfuxHn2XE>

SELECTION_SORT

For i = 0 to N-1:

// 1) Find the index of the min of all elements from arr[i]..arr[N-1]

// 2) Swap it with position i

Selection sort also $O(n^2)$ but now only $O(n)$ swaps!