

1. Using the substitution $u = 3x - 2$, the integral $\int_0^3 \sin(3x - 2) \, dx$ is equivalent to

A) $\int_{-2}^7 \sin u \, du$

B) $\int_0^3 \sin u \, du$

C) $\frac{1}{3} \int_{-2}^7 \sin u \, du$

D) $\frac{1}{3} \int_0^3 \sin u \, du$

E) $\frac{1}{3} \int_0^7 \sin u \, du$

2. $y = (x^4 + 2)^2$. Find $\frac{dy}{dx}$.

A) $8x^3(x^4 + 2)$

B) $(x^4 + 2) \cdot 4x^3$

C) $2(4x^3 + 2)$

D) $2(x^4 + 2)$

E) $(4x^3)^2$

3. A line through the point $(2, -2)$ is tangent to function f at the point $(-2, 6)$. What is $f'(-2)$?

- A) -8 B) -2 C) $-\frac{1}{2}$ D) 6 E) Undefined

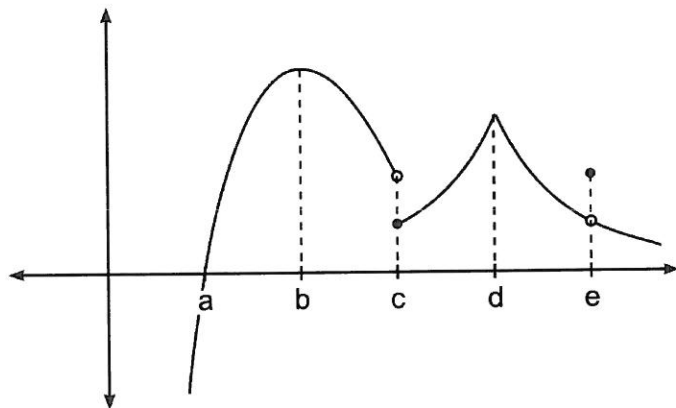
4. Grain is pouring out of an opening in the bottom of a grain silo. The rate at which the height, h , of the grain in the silo is changing with respect to time, t , is proportional to the square root of the height. Which of the following is a differential equation describing this situation?

- A) $\frac{dh}{dt} = k\sqrt{t}$
- B) $\frac{dh}{dt} = \frac{k}{\sqrt{h}}$
- C) $\frac{dh}{dt} = k\sqrt{h}$
- D) $h(t) = k\sqrt{t}$
- E) $h(t) = k\sqrt{h}$

5. Where is the graph of $f(x) = xe^{2x}$ concave up?

- A) $x > -1$ B) $x < -1$ C) $x > \frac{1}{2}$ D) $x < -\frac{1}{2}$ E) $x > 0$

6. Shown is a graph of function f .



At which value of x is f continuous but not differentiable?

- A) a B) b C) c D) d E) e

7. The table shows values of f' , the derivative of function f . Although f' is continuous over all real numbers, only selected values of f' are shown.

x	-2	-1	0	1	2	3	4	5	6
$f'(x)$	4.1	2.3	1.2	0	-0.6	-0.8	-2	0	1.5

If f' has exactly two real zeros, then f is increasing over which of the following intervals?

- A) $1 < x < 5$
- B) $x < 1$ or $x > 5$
- C) $x > 5$ only
- D) $x > 1$
- E) $x > 3$

8. $y = x^3 \sin(3x)$. $\frac{dy}{dx} =$

- A) $3x^2 \cos(3x)$
- B) $9x^2 \cos(3x)$
- C) $3x^2(\sin(3x) + \cos(3x))$
- D) $3x^2(\sin(3x) - x \cos(3x))$
- E) $3x^2(\sin(3x) + x \cos(3x))$

9. At each point (x, y) on a graph of function f , the graph has a slope equal to $4x - 4$. If the graph of f goes through the point $(2, 3)$, then $f(x) =$

A) $2x^2 - 4x - 4$

B) $2x^2 - 4x$

C) $2x^2 - 1$

D) $2x^2 - 4x + 3$

E) $16x - 12$

10. The derivative of function g is given by $g'(x) = \frac{4}{x^2} - x$. When is g increasing?

A) $\left(-\infty, \sqrt[3]{4}\right)$

B) $\left(\sqrt[3]{4}, \infty\right)$

C) $(-1, 0)$

D) $(-\infty, 1)$

E) $(1, \infty)$

11. Function f is defined as $f(x) = \begin{cases} -x + 5, & x \leq 2 \\ -\frac{1}{2}x + 4, & x > 2 \end{cases}$

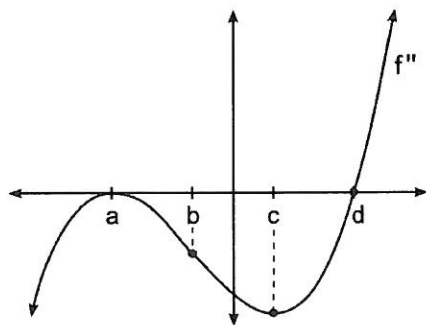
Which of the following statements is true:

- I. f is continuous at $x = 2$.
 - II. $\lim_{x \rightarrow 2} f(x)$ exists.
 - III. f is differentiable at $x = 2$
- A) None of the statements are true.
B) I only
C) II only
D) I and II only
E) I, II, and III

12. $\int_0^2 e^{-3x} dx =$

- A) $-3e^{-6}$ B) $e^{-6} - 1$ C) $\frac{1}{3} - \frac{e^{-6}}{3}$ D) $3 - 3e^{-6}$ E) $-\frac{e^{-6}}{3}$

13. Shown is a graph of f'' , the second derivative of function f . The curve is given by the equation $f'' = (x - a)^2(x - d)$. The graph of f has inflection points at which values of x ?

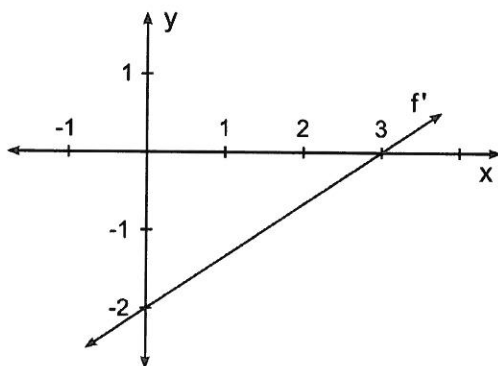


- A) b only
- B) c only
- C) a and d
- D) a and c
- E) d only

14. Function f is defined such that for all $x \geq 0$, the line $y = 3$ is a horizontal asymptote. Which of the following must be true?

- A) $f(3)$ is undefined.
- B) $f(x) \neq 3$ for all $x \geq 0$.
- C) $\lim_{x \rightarrow \infty} f(x) = 3$
- D) All of the above.
- E) None of the above.

15. Shown in the diagram is a graph of f' , the derivative of function f . If $f(3) = 3$, then $f(1) = ?$



- A) $\frac{10}{3}$ B) $\frac{13}{3}$ C) $-\frac{5}{3}$ D) 4 E) 6

16. $y = \frac{3x + 4}{4x + 3}$ $\frac{dy}{dx} =$

- A) $\frac{7}{(4x + 3)^2}$ B) $\frac{24x - 25}{(4x + 3)^2}$ C) $\frac{-7}{(4x + 3)^2}$ D) $\frac{24x + 25}{(4x + 3)^2}$ E) $\frac{3}{4}$

17. $\frac{d}{dx} \int_0^{x^3} \cos(t^2) dt =$

A. $-\sin(x^2)$

B. $\sin(x^3)$

C. $\sin(x^6)$

D. $3x^2 \cos(x^6)$

E. $3x^2 \cos(x^3)$

18. $\int_0^{\pi/6} \sin x \, dx =$

A) $-\frac{\sqrt{3}}{2} + 1$

B) $-\frac{\sqrt{3}}{2}$

C) $\frac{\sqrt{3}}{2} - 1$

D) $\frac{\sqrt{2}}{2}$

E) $-\frac{\sqrt{3}}{2} - 1$

19. Let f be the function defined by $f(x) = x^3 - 3x + 5$. The equation of the line tangent to the graph of f at $x = 2$ is

A. $y = 9x - 18$

B. $y = 3x + 1$

C. $y = 9x - 11$

D. $y = 3x - 6$

E. $y = 9x + 25$

20. $\lim_{x \rightarrow \infty} \frac{2x^3 + 3x^2 - 4x + 2}{4x^3 - x^2 + 5x - 3}$

A) $-\frac{1}{2}$

B) 0

C) $\frac{1}{2}$

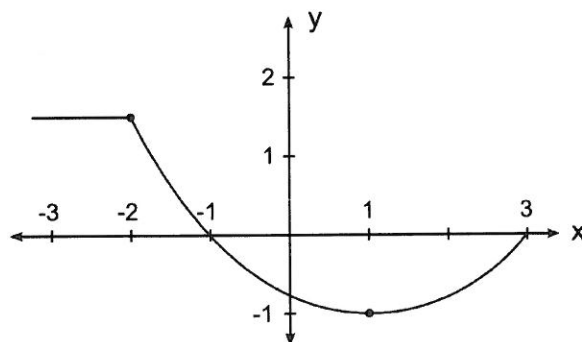
D) 1

E) 2

21. A particle's position at any time is given by the equation $x(t) = t^3 - 12t^2 + 36t - 15$. At what time is the particle at rest?

- A) $t = 3$ and $t = 6$
- B) $t = 2$ and $t = 6$
- C) $t = 2$ and $t = 4$
- D) $t = 2$ only
- E) $t = 4$ only

22. Shown is a graph of f' , the derivative of function f .



Which of the following statements is true?

- A) f is not differentiable at $x = -2$.
- B) f has a local minimum at $x = -1$.
- C) f is increasing from $x = 1$ to $x = 3$.
- D) f is increasing from $x = -3$ to $x = -1$.
- E) f is decreasing from $x = -2$ to $x = 1$.

23. What is the slope the line tangent to the curve $2x^2y - 3 = y^2 + 3x^2$ at the point $(2, 3)$?

- A) 9 B) $-\frac{3}{4}$ C) $-\frac{15}{2}$ D) $-\frac{3}{7}$ E) -6

24. $\int x^3 \cos(x^4) dx =$

- A) $-\frac{x^4}{4} \sin(x^4) + C$
- B) $\frac{x^4}{4} \sin(x^4) + C$
- C) $-\frac{1}{4} \sin(x^4) + C$
- D) $\frac{1}{4} \sin(x^4) + C$
- E) $\frac{x^4}{4} \sin\left(\frac{x^4}{4}\right) + C$

25. Function f is defined by the equation $f(x) = x^3 - 2x$. If $g(x) = f^{-1}(x)$ and $g(4) = 2$, what is $g'(4)$?

- A) $\frac{1}{10}$ B) $\frac{1}{46}$ C) 46 D) 10 E) $\frac{23}{5}$

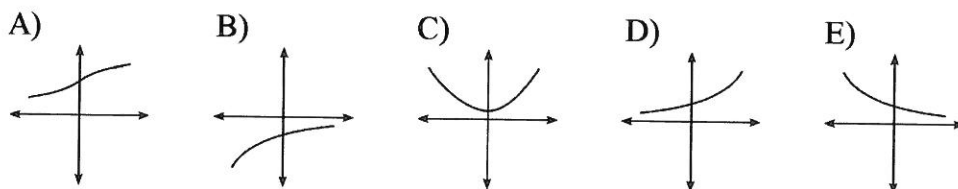
26. $f(x) = \ln(e^{-4x} + 5x + 4)$. Find $f'(0)$.

- A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $-\frac{1}{5}$ D) $-\frac{1}{4}$ E) does not exist.

27. Function f is a twice differentiable function with $f'(x) > 0$ and $f''(x) < 0$ for all real numbers x . If $f(2) = 5$ and $f(3) = 9$, what is a possible value for $f(4)$?

- A) 7 B) 9 C) 12 D) 13 E) 17

28. $f(x)$, $f'(x)$, and $f''(x)$ are all positive for any real number x . Which of the following graphs could be a graph of f ?



End of Section 1, Part A

If you finish before the time limit for this part, check your work on this part only.

Do not move on to the next part until you are told to by the test administrator.

Calculus AB
Section 1, Part B
Time - 50 minutes
Number of questions - 17

A GRAPHING CALCULATOR MAY BE REQUIRED TO SOLVE SOME QUESTIONS
ON THIS PART OF THE EXAM

Directions: Solve all of the problems that follow. Available space on the page may be used for scratch work. You may not return to the previous section of the exam.

In this exam:

- (1) The domain of a function in this exam is assumed to be all real numbers for which the function is defined, unless specified otherwise.
- (2) The inverse notation f^{-1} or the prefix “arc” may be used to indicate an inverse function. For example, the inverse tangent of x may be written as $\arcsin(x)$ or as $\tan^{-1}(x)$.
- (3) The exact numerical answer for a problem may not be listed as one of the given choices. When this is the case, choose the value that is the closest approximation to exact value.

76. The derivative of function f is given by $f' = \frac{2x - x^2}{e^{2x}}$. At what values of x does the graph of f have an inflection point?

- A. 0
- B. 0.382
- C. 0.775
- D. 1.136
- E. The graph has no inflection point.

77. The region in the x - y plane bounded by the lines $x = 1$, $x = 3$, $y = 2$, and $y = \ln x$ is the base of a solid. Each cross section of the solid perpendicular to the x -axis is a square. The volume of the solid is

- A. 3.85 B. 2.70 C. 2.97 D. 6.70 E. 5.41

78. Function f is differentiable and $f(3) = 6$ and $f'(3) = 4$. Function g is defined as $g(x) = \frac{f(x)}{x}$. What is the equation of the line tangent to function g at $x = 3$.

A. $y - 2 = 4(x - 3)$

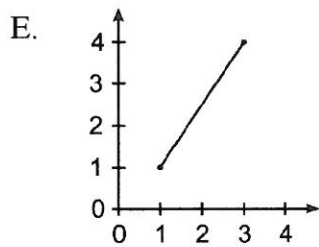
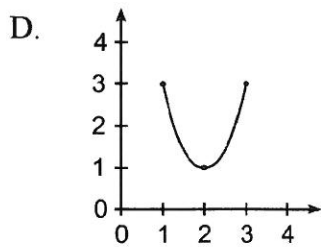
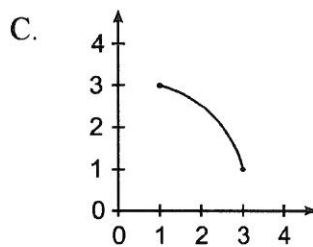
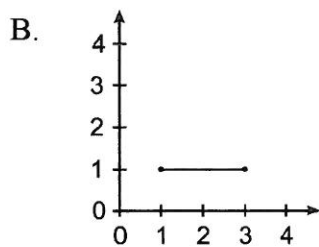
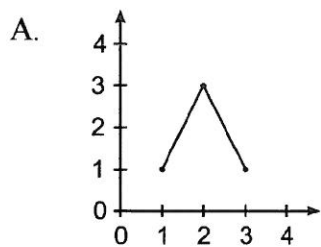
B. $y - 2 = \frac{4}{3}(x - 3)$

C. $y - 6 = 4(x - 3)$

D. $y - 2 = \frac{2}{3}(x - 3)$

E. $y - 3 = \frac{2}{3}(x - 2)$

79. Which could be a graph of f such that $\frac{1}{3-1} \int_1^3 f(x) dx = 2$?



80. The acceleration of a particle moving along the x -axis is given by $a(t) = \ln(3^t + 2)$. At time $t = 2$, the velocity of the particle is 2. What is the velocity at time $t = 3$?

- A. -0.441
- B. 3.099
- C. 3.178
- D. 4.872
- E. 5.313

81. Shown are selected values for functions f , g , h , j , and k , all of which are twice differentiable in the closed interval $[1, 4]$. Which of the functions has a negative first derivative and a positive second derivative?

A.

x	$f(x)$
1	14
2	12
3	10
4	8

B.

x	$g(x)$
1	14
2	13
3	11
4	8

C.

x	$h(x)$
1	14
2	11
3	9
4	8

D.

x	$j(x)$
1	8
2	9
3	11
4	14

E.

x	$k(x)$
1	9
2	11
3	13
4	14

82. A particle is moving such that its velocity at any time t is given by $v(t) = 2 + 2.5 \sin(0.6t)$. What is the acceleration of the particle when $t = 3$?

- A. -0.341
- B. -0.568
- C. -0.947
- D. 1.24
- E. 1.46

83. Function f is defined as $f(x) = \int_0^x \cos\left(\frac{t^2}{2}\right)$ for $0 \leq x \leq 3.8$. On what interval is f decreasing?

- A. $0 \leq x \leq 1.772$
- B. $0 \leq x \leq 2.506$
- C. $1.772 \leq x \leq 3.07$
- D. $0 \leq x \leq 3.07$
- E. $3.545 \leq x \leq 3.8$

84. If a circle's radius is increasing at a constant rate of $0.3 \frac{\text{m}}{\text{s}}$, what is the rate of change of the area of the circle when the circle's circumference is 6π meters?

A. $0.09\pi \frac{\text{m}^2}{\text{s}}$

B. $0.6\pi \frac{\text{m}^2}{\text{s}}$

C. $1.8\pi \frac{\text{m}^2}{\text{s}}$

D. $6\pi \frac{\text{m}^2}{\text{s}}$

E. $9\pi \frac{\text{m}^2}{\text{s}}$

85. Function f is continuous on the closed interval $[-1, 3]$ and differentiable on the open interval $(-1, 3)$. If $f(-1) = 1$ and $f(3) = 3$, which of the following statements must be true?

A. There exists a number c on $(-1, 3)$ such that $f(c) = \frac{1}{2}$.

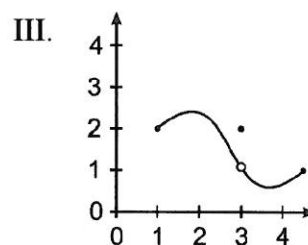
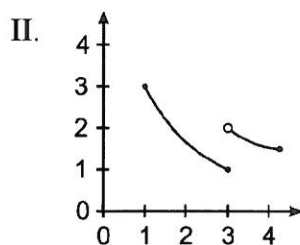
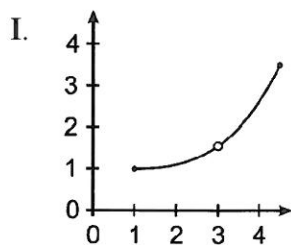
B. There exists a number c on $(-1, 3)$ such that $f'(c) = 0$

C. There exists a number c on $(-1, 3)$ such that $f(c) = 0$

D. There exists a number c on $(-1, 3)$ such that $f(c) \geq f(x)$ for all x

E. There exists a number c on $(-1, 3)$ such that $f'(c) = \frac{1}{2}$

86. For which of the graphs shown does the $\lim_{x \rightarrow 3} f(x)$ exist?



- A) I only
B) II only
C) I and II only
D) III only
E) I and III only

87. $f'(x) = \cos(2x)$. How many relative extrema does function $f(x)$ have on the interval $\frac{\pi}{2} < x < 2\pi$.

- A. 2 B. 3 C. 4 D. 5 E. 7

88. A remote control helicopter is launched. During the first 6 seconds of flight, the rate of change of the helicopter's altitude is given by the equation $r(t) = -t^3 + 5t^2 - 8$. Which of the following expressions represents the helicopter's change in altitude during the time that the altitude is increasing?

A. $\int_0^{10.518} r(t) dt$

B. $\int_0^{10.518} r'(t) dt$

C. $\int_{1.515}^{4.626} r(t) dt$

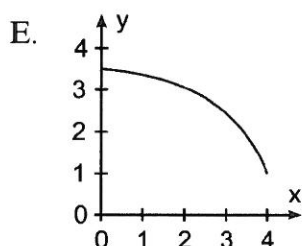
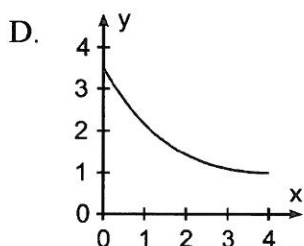
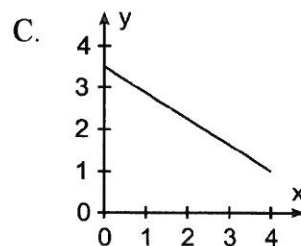
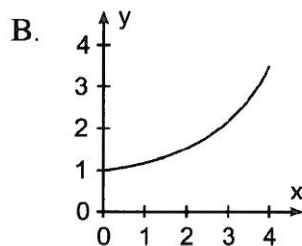
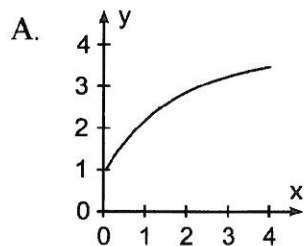
D. $\int_{1.515}^{4.626} r'(t) dt$

E. $\int_0^6 r(t) dt$

89. A potato is baked at a temp of 400°F . At time $t = 0$ it is taken out of the oven and allowed to cool in a 72°F room. The rate of change of the potato's temperature is given by the equation $r(t) = -82e^{-0.25t}$ where t is the time in minutes. What is the potato's temperature, to the nearest degree, after it has cooled for four minutes?

- A. 104 B. 193 C. 207 D. 333 E. 370

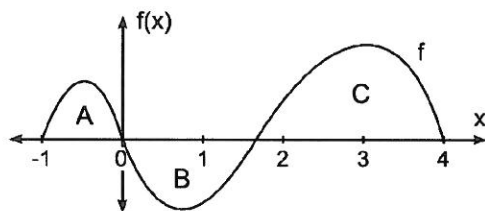
90. The integral $\int_0^5 g(x) dx$ is under approximated by a trapezoidal sum and over approximated by a left Riemann sum. Which of the following could be a graph of $y = g(x)$?



91. An object is moving along the x -axis such that its velocity in m/s is given by $v(t) = 2e^{-t} + t \sin(t)$. What is the average velocity of the object from time $t = 0$ to time $t = 4$?

- A. 25.95 B. 26.04 C. 27.26 D. 54.08 E. 104.17

92. A graph of function $f(x)$ on the interval $[-1, 4]$ is shown. Regions A, B, and C have areas of 1, 2, and 3 respectively. What is $\int_{-1}^4 (f(x) + 2) dx$?



- A. 2 B. 4 C. 10 D. 12 E. 16

End of Section 1, Part B

If you finish before the time limit for this part, check your work on this part only.

Do not move on to the next part until you are told to by the test administrator.

Calculus AB
Section 2, Part A
Time - 30 minutes
Number of problems - 2

A graphing calculator is required for these problems

1. A hot air balloon is launched at time $t = 0$. Its altitude in meters is modeled by a twice differentiable function of time, t . For $0 \leq t \leq 10$ min, the altitude h at various times is shown in the table.

t (min)	0	2	3	5	6	9	10
h (meters)	0	280	330	240	270	420	340

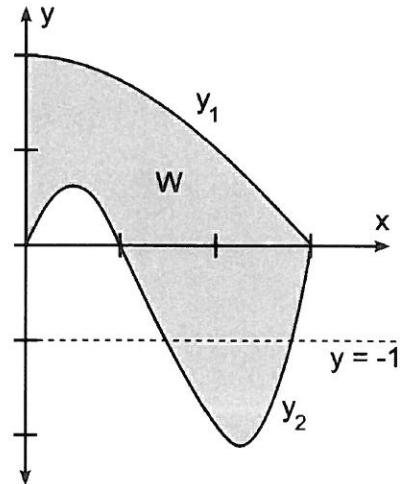
- a) From the data shown, estimate the rate at which h is changing at $t = 7.5$. Show your work. Indicate units of measure.
- b) Use a trapezoidal approximation with three subintervals to estimate the average height of the balloon during the first five minutes of its flight.
- c) During the time interval $0 \leq t \leq 10$ min, what is the least number of times that $h'(t)$ must be zero? Justify your answer.
- d) A pressurized propane tank supplies propane to the burner. The rate at which propane is dispensed is given by the function $8t \cdot e^{-0.4t}$ liters per minute. How many liters of propane are dispensed in the first five minutes?

2. The region W in the x - y plane is bound by

$$y_1 = 2 \cos\left(\frac{\pi x}{6}\right) \text{ and}$$

$$y_2 = x^3 - 4x^2 + 3x \text{ and}$$

the y -axis, as shown in the diagram.



- Find the area of region W .
- The horizontal line $y = -1$ divides region W into two sections. Write an integral expression for the area of the lower section. Do not evaluate the integral.
- Consider the region W to be the base of a solid S . The cross sections of the solid perpendicular to the x -axis are squares. Find the volume of S .
- The region W is a top view of an experimental aircraft wing. The thickness of the wing at any distance x from the y -axis is given by the function $h(x) = 1 - \frac{x}{4}$. Find the volume of the wing.

End of Section 2, Part A

If you finish before the time limit for this part, check your work on this part only.

Do not move on to the next part until you are told to by the test administrator.