Section 3.4 ALEKS

Properties of logs
relationship between the exponential + logs
using logs + exponentials together.

we have this definition

$$y = a^x \iff x = \log_a(y)$$

so it is an inverse relationship.

i.e. if $f(x) = a^x$
then $f^{-1}(y) = \log_a(y)$

and $\log_a(a^x) = x$
and $a^{\log_a(x)} = x$

We also have these properties

$$\log_a(u \cdot v) = \log_a(u) + \log_a(v)$$
$$\log_a(u^r) = r\log_a(u)$$

This assumes u, v are in domain of log!

which implies that
$$\log_a(u/v) = \log(u) + \log(v^{-1})$$
$$= \log(u) - \log(v)$$

So what?
We can break things up and bring them together!

(~~Help~~ helpful in isolating variables)

This requires practice!
You gotta do this and get familiar with it. Watching me do it is meaningless.

ex/ suppose that $3^x = 7$ what is x?

well... $\log_8(3^x) = \log_8(7)$

$$x \log_8(3) = \log_8(7)$$

$$x = \log_8(7) / \log_8(3)$$

why base 8? base 3 would be better, no?

no! it would not. Your calculator does **not** have a $\log_3$ button!

why not? coz there are not enough buttons!

we need to pick a base and just stick w/ it.

we generally use base 10 or $\underbrace{e}$, ← math dept. approved base!

↑ some people like this. Some people like vegemite, but that does not make it right!

So... $3^x = 7$

$$\ln(3^x) = \ln(7)$$

$$x \ln(3) = \ln(7) \Rightarrow x = \ln(7)/\ln(3).$$

One base to rule them all. — Tolkein.

ex/ $5^x \cdot 7^y = 4$. make a more better relationship between x & y.

$$\ln(5^x \cdot 7^y) = \ln(4)$$

$$\ln(5^x) + \ln(7^y) = \ln(4)$$

$$x \ln(5) + y \ln(7) = \ln(4)$$

$$y \ln(7) = -\ln(5) \cdot x + \ln(4)$$

$$y = -\frac{\ln(5)}{\ln(7)} x + \frac{\ln(4)}{\ln(7)}$$

↑ slope ↑ y-int.

This is a linear relationship!

logs can break apart (destroy) or bring together (unify)

Apart

$$18 \cdot 5^4 \cdot 6^r = 3$$

$$\ln(18 \cdot 5^4 \cdot 6^r) = \ln(3)$$

$$\ln(18) + 4\ln(5) + r\ln(6) = \ln(3)$$

I can solve for either variable now!

Together

$$3 + \ln(r) - \ln(s) = 5$$

$$\ln(r/s) = 2$$

$$r/s = e^2 \quad \text{or} \quad r = se^2$$

lets me tie together the other "stuff"

ex/ solve for x

$$13 \cdot 17^x = 4 \cdot 3^x$$

$$\ln(13 \cdot 17^x) = \ln(4 \cdot 3^x)$$

$$\ln(13) + \ln(17^x) = \ln(4) + \ln(3^x)$$

$$\ln(13) + x\ln(17) = \ln(4) + x\ln(3)$$

$$x\ln(17) - x\ln(3) = \ln(4) - \ln(13)$$

$$x(\ln(17) - \ln(3)) = \ln(4) - \ln(13)$$

$$x = \frac{\ln(4) - \ln(13)}{\ln(17) - \ln(13)}$$

neg #! is it okay in original relationship?

ex/ $\log_4(3x^2+1) = 2 + \log_4(x)$

$$\log_4(3x^2+1) = \log_4(4^2) + \log_4(x)$$

$$\log_4(3x^2+1) = \log_4(4^2 \cdot (x))$$

$$3x^2+1 = 4^2 \cdot x$$

$$3x^2 - 16x + 1 = 0$$

$$x = \frac{16 \pm \sqrt{16^2 - 4\cdot 3}}{2} = \frac{16 \pm \sqrt{244}}{2}$$

both pos.

OK. 2 solutions.

ex/ $\ln(x+1) - \ln(x-1) = 3$ OK!

$$\ln(x+1) + \ln\left(\frac{1}{x-1}\right) = 3$$

$$\ln\left(\frac{x+1}{x-1}\right) = 3$$

$$\frac{x+1}{x-1} = e^3 \Rightarrow x+1 = (x-1)e^3 = xe^3 - e^3$$

$$x - xe^3 = -e^3 - 1$$

$$x(1-e^3) = -(e^3+1)$$

$$x = -\frac{e^3+1}{1-e^3} \approx 1.105$$

ex/ $4^{x+1} \cdot 7^{x+6} \cdot 3^x \cdot 5^{x+1} = 4$ yuk: — screw it - log everything

$$\ln\left(4^{x+1} \cdot 7^{x+6} \cdot 3^x \cdot 5^{x+1}\right) = \ln(4)$$

$$\ln(4^{x+1}) + \ln(7^{x+6}) + \ln(3^x) + \ln(5^{x+1}) = \ln(4)$$

$$(x+1)\ln(4) + (x+6)\ln(7) + x\ln(3) + (x+1)\ln(5) = \ln(4)$$

$$x[\ln(4) + \ln(7) + \ln(3) + \ln(5)] + \ln(4) + 6\ln(7) + \ln(5) = \ln(4)$$

$$x = \frac{\cancel{\ln(4)} - \cancel{\ln(4)} - 6\ln(7) - \ln(5)}{\ln(4) + \ln(7) + \ln(3) + \ln(5)}$$

ex/ $\log_3(x+4) + \log_3(x+2) = 7$

$$\log_3((x+4)(x+2)) = 7$$

$$(x+4)(x+2) = 3^7$$

$$x^2 + 6x + 8 = 3^7$$

$$x^2 + 6x + 8 - 3^7 = 0$$

$$x = \frac{-6 \pm \sqrt{36 - 4(8-3^7)}}{2}$$

$x \approx 43.78$ or $x \approx \underline{-49.78}$ Nope!

so $x \approx 43.78$

ex/ $\log_2(2x+1) = \underline{\log_4(3x+1)}$ Wot? who does this?

yuck!

let $u = \log_4(3x+1)$ — I want $\underline{\log_2}$ $\underline{\text{somehow}}$

$\Rightarrow 4^u = 3x+1$

$\log_2(4^u) = \log_2(3x+1)$

$u \log_2(4) = \log_2(3x+1) \Rightarrow u = \dfrac{\log_2(3x+1)}{\log_2(4)}$

original eqn:

$$\log_2(2x+1) = \frac{\log_2(3x+1)}{\log_2(4)} = \frac{\log_2(3x+1)}{2}$$

$= 2$ (!)

$$2\log_2(2x+1) = \log_2(3x+1)$$

$$\log_2((2x+1)^2) = \log_2(3x+1)$$

$$(2x+1)^2 = (3x+1)$$

$$4x^2 + 4x + 1 = 3x+1$$

$$4x^2 + x = 0$$

$$x(4x+1) = 0$$

$x = 0$ or $x = -1/4$. both ok in orig'l eqn.

ex/ $\log(x+1) - \log(x+3) = 4$ $\log = \log_{10}$ by convention.

$$\log\left(\frac{x+1}{x+3}\right) = 4$$

$$\frac{x+1}{x+3} = 10^4$$

$$x+1 = (x+3)10^4 = x\cdot 10^4 + 3\cdot 10^4$$

$$x - x\cdot 10^4 = 3\cdot 10^4 - 1$$

$$x(1-10^4) = 3\cdot 10^4 - 1$$

$$x = \frac{3\cdot 10^4 - 1}{1-10^4} \approx -3.0002 \quad \text{Nope!}$$

No solution!

Low energy.

ex/ $\log(x+2) = -2 + \log(x+3)$

$$10^{\log(x+2)} = 10^{-2+\log(x+3)} = 10^{-2} \cdot 10^{\log(x+3)}$$

$$(x+2) = 10^{-2}(x+3) = 10^{-2}x + 3 \cdot 10^{-2}$$

$$x - 10^{-2} \cdot x = 3 \cdot 10^{-2} - 2$$

$$x(1 - 10^{-2}) = 3 \cdot 10^{-2} - 2$$

$$x = \frac{3 \cdot 10^{-2} - 2}{1 - 10^{-2}} \approx -1.99$$ ok!

ex/ $e^{2x^2} = 2e^{3x}$

$$\ln(e^{2x^2}) = \ln(2e^{3x}) = \ln(2) + \ln(e^{3x})$$

$$2x^2 = \ln(2) + 3x$$

$$2x^2 - 3x - \ln(2) = 0$$

$$x = \frac{3 \pm \sqrt{9 + 8\ln(2)}}{4}$$ two solutions.

ex/ $2 \cdot 4^x = 8 \cdot 7^x$

$$\ln(2 \cdot 4^x) = \ln(8 \cdot 7^x)$$

$$\ln(2) + \ln(4^x) = \ln(8) + \ln(7^x)$$

$$\ln(2) + x\ln(4) = \ln(8) + x\ln(7)$$

$$x\ln(4) - x\ln(7) = \ln(8) - \ln(2)$$

$$x(\ln(4) - \ln(7)) = \ln(8) - \ln(2)$$

$$x = \frac{\ln(8) - \ln(2)}{\ln(4) - \ln(7)}$$ <0 but ok.

ex/ $e^{5x} = 10 \cdot 7^{2x}$

$$\ln(e^{5x}) = \ln(10 \cdot 7^{2x}) = \ln(10) + \ln(7^{2x})$$

$$5x = \ln(10) + 2x\ln(7)$$

$$5x - 2x\ln(7) = \ln(10)$$

$$x(5 - 2\ln(7)) = \ln(10)$$

$$x = \frac{\ln(10)}{5 - 2\ln(7)}$$

ex/ $\log_6(100/x) = \log_4(3/x)$

let $u = \log_6(100/x) \Rightarrow 6^u = 100/x$

$$\ln(6^u) = \ln(100/x)$$
$$u \ln(6) = \ln(100/x)$$
$$u = \frac{\ln(100/x)}{\ln(6)}$$

let $v = \log_4(3/x)$

$$\Rightarrow 4^v = 3/x$$
$$\ln(4^v) = \ln(3/x)$$
$$v \ln(4) = \ln(3/x) \Rightarrow v = \frac{\ln(3/x)}{\ln(4)}$$

$$\frac{\ln(100/x)}{\ln(6)} = \frac{\ln(3/x)}{\ln(4)}$$

$$\frac{\ln(100) - \ln(x)}{\ln(6)} = \frac{\ln(3) - \ln(x)}{\ln(4)}$$

$$\ln(4)\left[\ln(100) - \ln(x)\right] = \ln(6)\left[\ln(3) - \ln(x)\right]$$
$$\ln(4)\ln(100) - \ln(4)\cdot\ln(x) = \ln(6)\cdot\ln(3) - \ln(6)\ln(x)$$
$$\ln(6)\ln(x) - \ln(4)\ln(x) = \ln(6)\ln(3) - \ln(4)\ln(100)$$
$$x\left[\ln(6) - \ln(4)\right] = \ln(6)\ln(3) - \ln(4)\ln(100)$$
$$x = \frac{\ln(6)\ln(3) - \ln(4)\ln(100)}{\ln(6) - \ln(4)}.$$

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ex/ $2^{x^2}/9^{x+1} = 42$

$$2^{x^2} = 42 \cdot 9^{x+1} \Rightarrow \ln(x^2) = \ln(42 \cdot 9^{x+1})$$
$$\vdots$$
$$x = \frac{\ln(9) \pm \sqrt{\ln(9)^2 + 4\ln(2)(\ln 9 + \ln 42)}}{2\ln(2)}$$

which one? both!