Sect 1.4 - Linear equations

ALEKS HW !

Lines
slopes
average rate of change
Different forms to represent a line
graphs of lines
meaning of "slope"

Def: A line is a set of coordinate pairs, (x,y), where the change in x is proportional to the change in y.

So, if (x,y) is a point on the line and (x_0,y_0) is a point on the line

Then $$\underbrace{y-y_0}_{\text{change in } y} = m \underbrace{(x-x_0)}_{\text{change in } x}$$

m ← constant of proportionality

(note: if it is a vertical line then it is different)

ex| A line has the points $(4,-1)$ and $(2,5)$.

$$\Rightarrow \quad (-1-5) = m(4-2)$$

$$\Rightarrow \quad -6 = 2m \quad \text{so} \quad m = -3$$

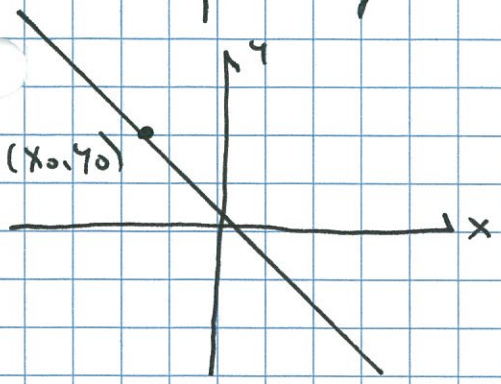
So if (x,y) is any pt. on the line then

$$y-(-1) = -3(x-4)$$

(or equivalently)

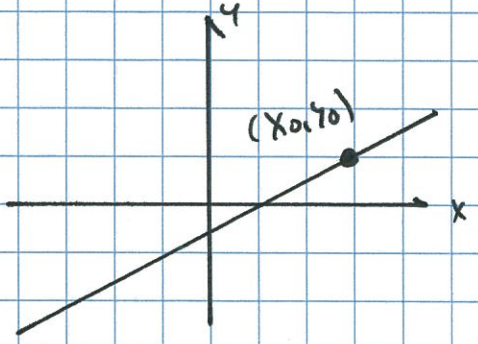
$$y-5 = -3(x-2) \qquad (1)$$

Graphically



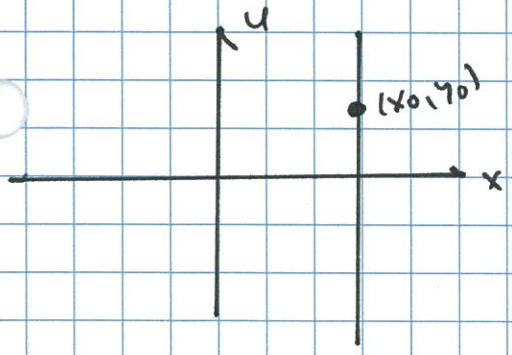
slope is negative ($m<0$)

as x inc then y dec.



slope is pos ($m>0$)

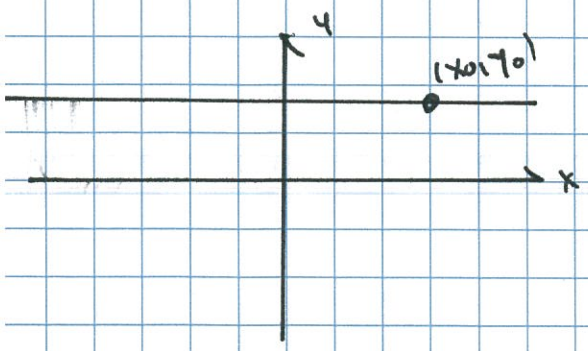
as x inc then y also inc.



x = constant

a vertical line.

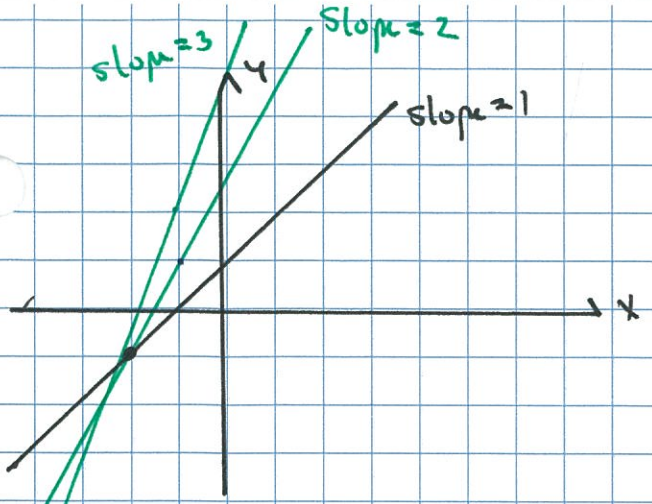
(slope not defined)



slope is zero ($m=0$)

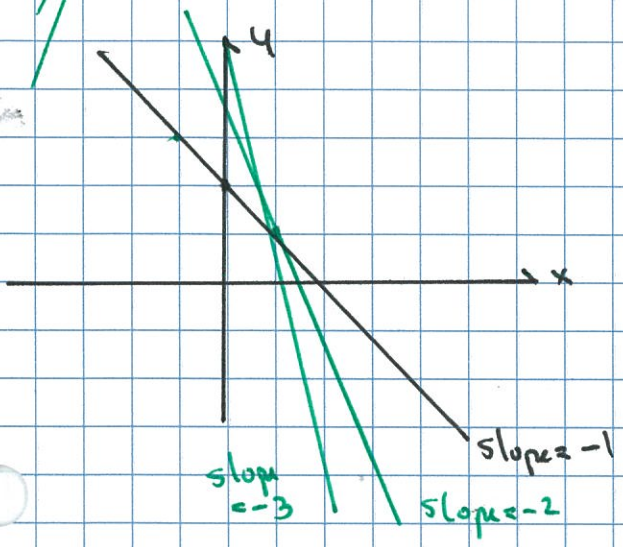
horizontal line.

no change in y.



If slope > 0

bigger slope means steeper increase.



If slope < 0

smaller slope means steeper decrease

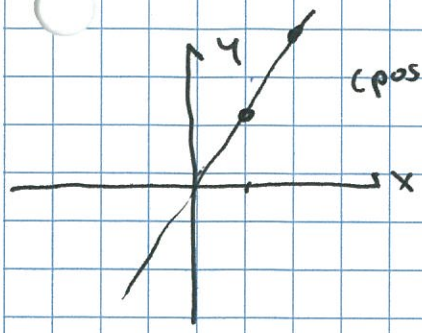
why? change in $y = m$ (change in x)

↑ bigger pos value ⇒ bigger change in y

or smaller neg value ⇒ smaller change in y.

(note $-2 < -1$) !

ex/ A line goes through the points (1, 2) and (2, 5). what is the line?



(pos slope!)

$(5-2) = m(2-1) \Rightarrow m=3$

so $y-2 = 3(x-1)$

pt. slope form for a line.

both are great!

note: we can play algebra games

$y-2 = 3x-3$

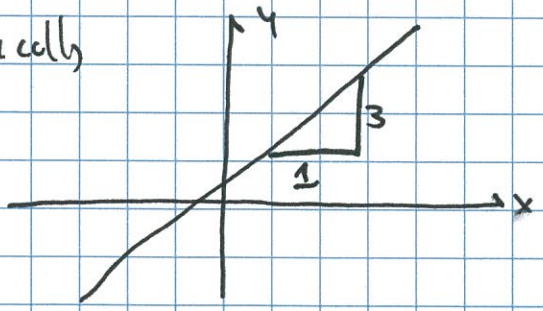
$y = 3x-3+2$

$y = 3x-1$ } slope-intercept form.

so what? if $x=0$ then $y = 3(0)-1 = -1$

↖ y-intercept!

graphically



$\text{slope} = \frac{\text{rise}}{\text{run}}$

Two forms to represent a line:

pt. slope form — $y - y_0 = m(x-x_0)$

m = slope

(x_0, y_0) is a point on the line.

y-intercept form — $y = mx+b$

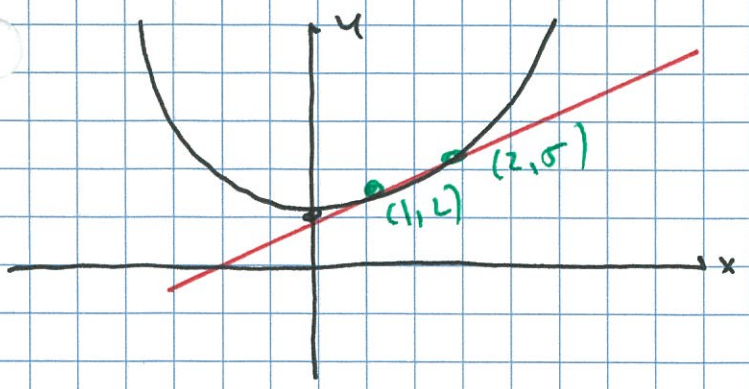
m = slope

b = y-intercept

which one is best?

whatever, man.

ex/ Suppose $f(x) = x^2 + 1$



if $x = 1$ then $y = 2$

if $x = 2$ then $y = 5$

so $(1,2)$ and $(2,5)$ are pts on the graph.

The line through the points $(1,2)$, $(2,5)$ is

$$y - 2 = 3(x - 5)$$

Definition: The line through 2 points on the graph of a function is called the secant line through the two points.

Definition: The slope of the secant line is called the average rate of change between the two points.

In prev. exmpl. the average rate of change from $x = 1$ to $x = 2$ is

$$\text{avg. rat chg} = \frac{f(2) - f(1)}{2 - 1} = \frac{5 - 2}{2 - 1} = \frac{3}{1} = 3.$$

Note: It is common to work w/ systems of equations. Life is complicated!

ex/ we have two factories.

Factory 1 produces 2 items per hour w/ 4 reads at opening. openings

$\Rightarrow \quad y - 4 = 2(\text{time} - 0)$

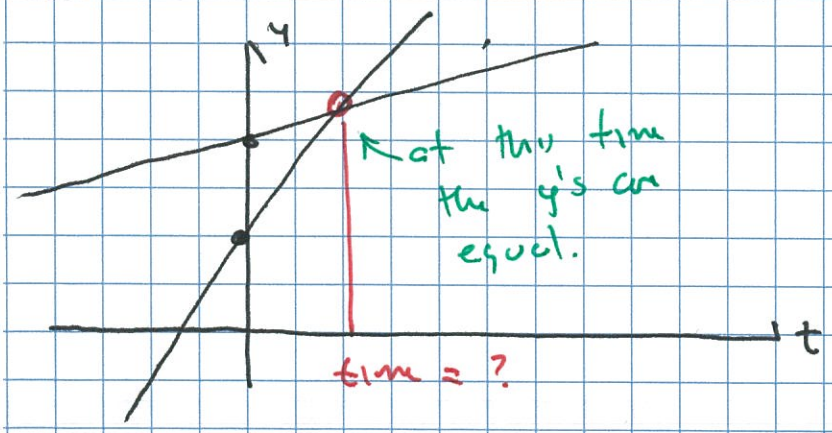
Factory 2 produces 1/3 items per hour w/ 8 reads at opening

$\Rightarrow \quad y - 8 = \frac{1}{3}(\text{time} - 0)$

at what time of the day are the productions the same?

$y - 4 = 2 \cdot (t - 0) \quad \Rightarrow \quad y = 2t + 4$

$y - 8 = \frac{1}{3}(t - 0) \quad \Rightarrow \quad y = \frac{1}{3}t + 8$



$2t + 4 = \frac{1}{3}t + 8$

$2t - \frac{1}{3}t = 8 - 4$

$\frac{5}{3}t = 4$

$t = \frac{12}{5}$ hours after opening.

ex/ ① $8x + 3y = 5$

and ② $2x - 4y = 7$ what is x and y?

solve each for y

set eqns + solve for x

subst. back to get y.

① $8x + 3y = 5$

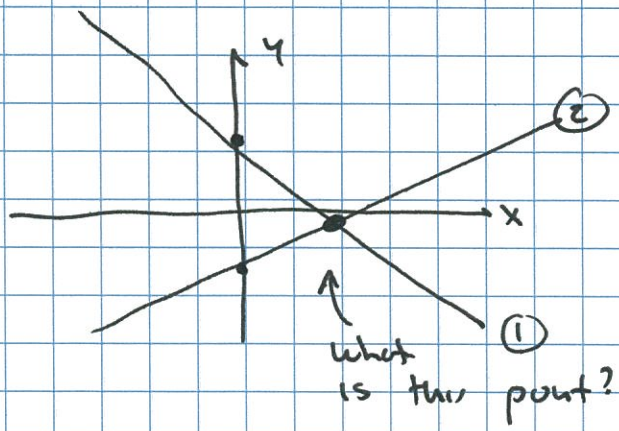
$3y = 5 - 8x$

$y = 5/3 - 8/3 x$ a line slope $= -8/3$

② $2x - 4y = 7$

$-4y = 7 - 2x$

$y = -7/4 + \frac{2}{4}x$ a line slope $= +\frac{1}{2}$!



$5/3 - 8/3 x = -7/4 + \frac{1}{2}x$

$-8/3 x - \frac{1}{2}x = -7/4 - 5/3$

$(-8/3 - 1/2)x = -41/12$

$-\frac{19}{6}x = -41/12 \Rightarrow x = -\frac{41}{2} \cdot \frac{6}{19} = \frac{41}{38}$

$y = \frac{5}{3} - \frac{8}{3} \cdot \frac{41}{38} = 23/19$

Note: There are a huge # of ways to use these ideas, and they are used in almost every discipline. You have to be prepared to deal w/ novel situations.

Time Permitting.

In equalities

- Algebra still works - but is a little more complicated.

ex/ $3x+4 < -x+1$ what does it mean?

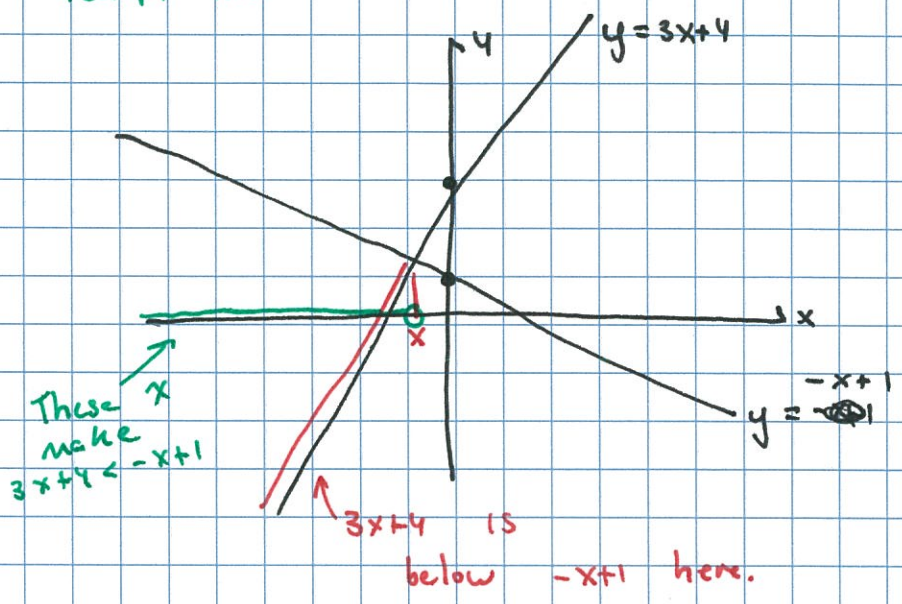
↑ a straight line!

↑ curvy line.

graphically

let $y = 3x+4$

$y = -x+1$



Algebraically

$3x+4 < -x+1$

$4x+4 < 1$

$4x < -3$

$x < -3/4$ or $(-\infty, -3/4)$