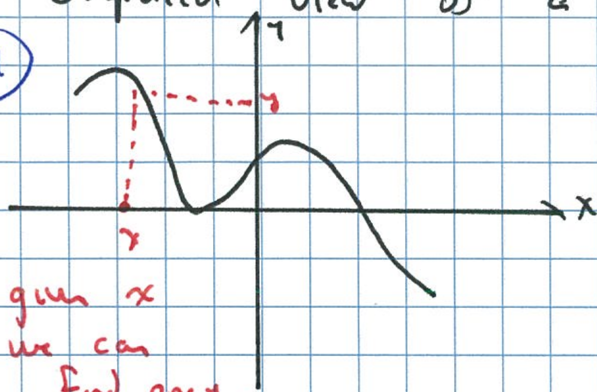
Section 1.7 — ALEKS HW

even and odd functions
piecewise defn fcns.
increasing vs decreasing
max/min.

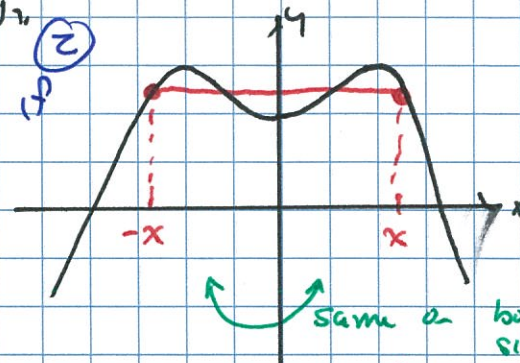
Graphical view of a function

(1)



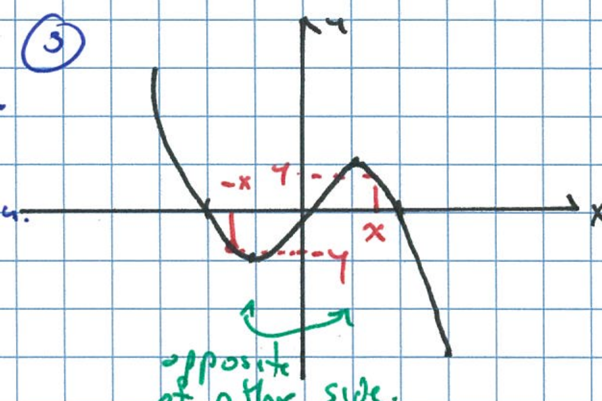
given x we can find one y.

(2) what if



i.e. $f(x) = f(-x)$
This is an "even" fcn.

(3) what if



i.e. $f(x) = -f(-x)$
This is an "odd" fcn.

(4)

Even fcn: $f(x) = f(-x)$ x in domain.
odd fcn: $f(x) = -f(-x)$ x in domain.

ex/ $f(x) = x^4 - 2x^2 + 1$

then $f(-x) = (-x)^4 - 2(-x)^2 + 1$
$= (-1)^4 (x)^4 - 2(-1)^2 (x)^2 + 1$
$= x^4 - 2x^2 + 1 = f(x)$

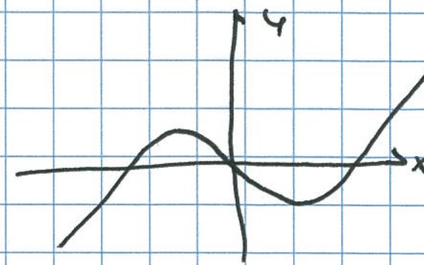
The function is even.

ex/ $\text{Fred}(x) = x^3 - 4x$

$\text{Fred}(-x) = (-x)^3 - 4(-x) = (-1)^3 x^3 + 4x = -x^3 + 4x$

$= -\text{Fred}(x)$

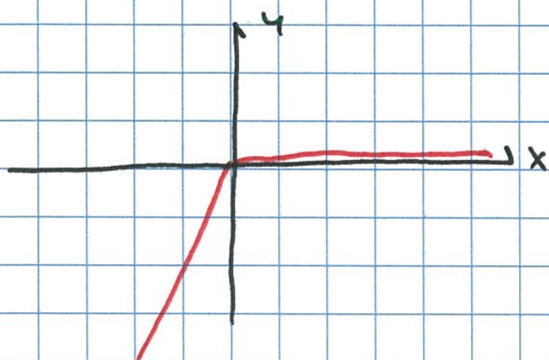
The function is odd.



ex/ $\text{Neil}(x) = x - |x|$

$\text{Neil}(-x) = -x - |-x| = -x - |(-1)x| = -x - |x|$

neither odd nor even.



not even, not odd.

Just weird. :-)

Another way to describe $\text{Neil}(x)$

if x is negative $\text{Neil}(x) = +2x$ (<0)

if x is positive then $\text{Neil}(x) = 0$.

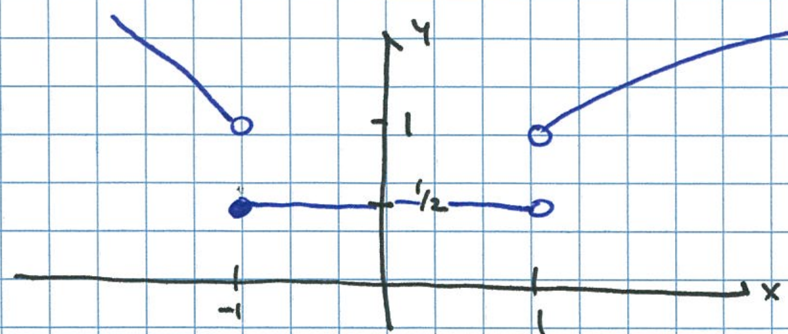
Well, that is awkward. How about this:

$$\text{Neil}(x) = \begin{cases} 2x & \text{for } x < 0 \\ 0 & \text{for } x \geq 0 \end{cases}$$

sometimes just a comma.

ex/ graph the function

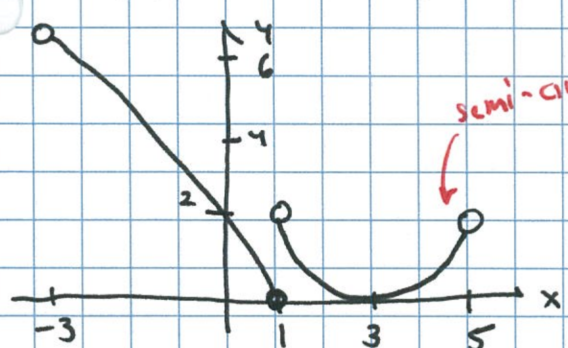
$$\text{Audrey}(x) = \begin{cases} -x & x < -1 \\ 1/2 & -1 \le x < 1 \\ \sqrt{x} & x > 1 \end{cases}$$



what is Audrey(1)?
not defn.
so domain is
$(-\infty, 1) \cup (1, \infty)$

Range of Audrey is $\{1/2\} \cup \{y > 1\}$
or $\{1/2\} \cup (1, \infty)$

ex/
what is the function in the plot below?



$$\text{April}(x) = \begin{cases} \frac{6}{4}(x-1) & -3 < x \le 1 \\ 2 - \sqrt{4 - (x-3)^2} & 1 < x < 5 \end{cases}$$

Domain of April is $(-3, 5)$ or $-3 < x < 5$

range of April is $[0, 6)$ or $0 \le y < 6$.

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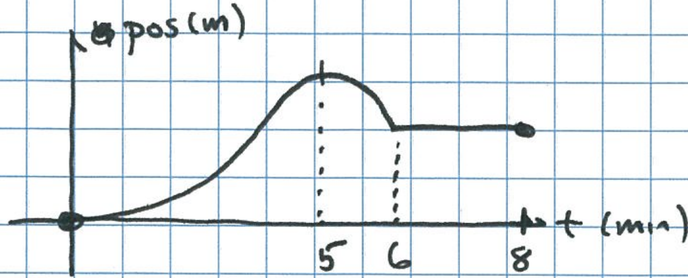
so, we have talked a lot about how to define a function. what about its general behaviour?

ex/ A car's position is give in the plot below:



what did the car do? (what is the story?)

- For the time from 0 to 5 min. it moved from left to right.
  position increased
- For the time fr 6 to 8 min. it mov from right to left.
  position decreased
- For the time fr 6 to 8 min. it stood still.
  position unchanged (constant)

what does "~~increase~~" "increase" mean?

$f(x)$ is inc. on the interval (a,b) if
$$f(x_1) > f(x_2) \quad \text{when } b > x_1 > x_2 > a$$

what about "decrease?"

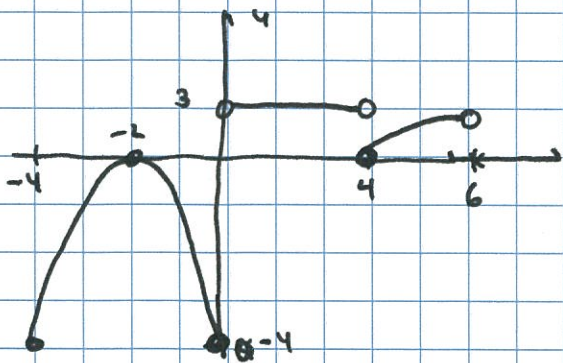
$f(x)$ is dec. on the interval (a,b) if
$$f(x_1) < f(x_2) \quad \text{when } b > x_1 > x_2 > a.$$

and constant...

$f(x)$ is constant on the interval (a,b) if
$$f(x_1) = f(x_2) \qquad b > x_1 \geq x_2 > a$$
↑!

see top of page 206.

ex/ $\text{Buzz}(x) = \begin{cases} -(x+2)^2 & -4 \le x \le 0 \\ 3 & 0 < x < 4 \\ \sqrt{x-4} & 4 \le x < 6 \end{cases}$



Buzz is...

inc. on $(-4,-2) \cup (4,6)$

dec. on $(-2, 0)$

const. on $(0,4)$

Note we are not so picky about endpoints.

note - at $x=-2$ it is neither inc nor dec.

If there is a point where the function goes from inc. to dec. we say that it is a relative maximum. at that point.

See page 207 for more pedantic definition.

- I can find an open interval containing $x=-2$ where $f(-2) \ge f(x)$ for all x in the interval.

by this definition there is a rel. max of 3 for any x between 0 and 4.

Formal Definitions:

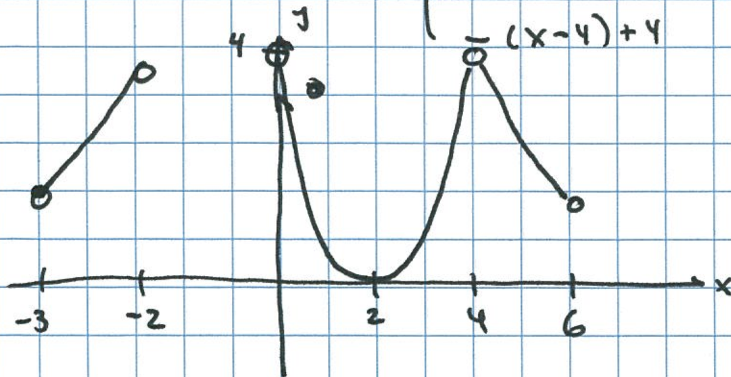
- $f(a)$ is a relative max if you can find an open interval containing a where $f(a) \ge f(x)$ for all x in the interval.

- $f(a)$ is a relative min if you can find an open interval containing a where $f(a) \le f(x)$ for all x in the interval.

side note - if $f(x)$ is const. over an interval it is both a rel max and rel min on the interval!

ex/ Det. the rel. max's and rel min's of the fcn.

$$Alfred(x) = \begin{cases} 2(x+3)+2 & -3 < x < -2 \\ (x-2)^2 & 0 < x < 4 \\ -(x-4)+4 & 4 < x < 6 \end{cases}$$



inc: $(-3,-2) \cup (2,4)$

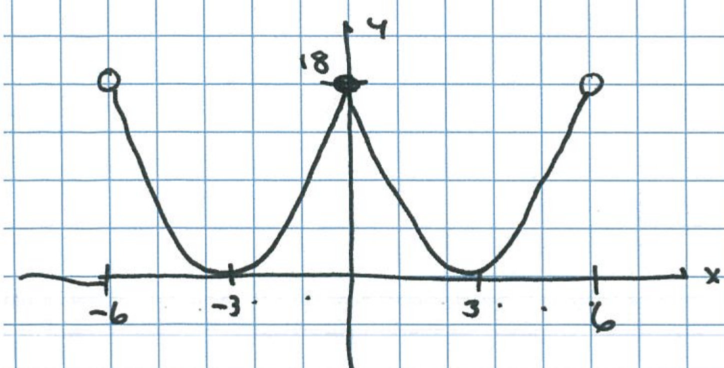
dec: $(0,2) \cup (4,6)$

rel. min @ $x=2$ + the min is 0.

<u>no</u> <u>rel. max!</u>

ex/

$$Bruce(x) = \begin{cases} 2(x+3)^2 & -6 < x < 0 \\ 2(x-3)^2 & 0 \le x < 6 \end{cases}$$



increasing ~~fcn~~ on $(-3,0) \cup (3,6)$

decreasing on $(-6,-3) \cup (0,3)$

Rel min. @ $x=-3$ when Bruce $=0$

<u>and</u> $x=3$ when Bruce $=0$.

Rel. max @ $x=0$ which is 18.