2.2 Polynomial Functions. ALEKS

Polynomial Behavior
Zeros of polynomials.
intermediate valu theorem.

Def: A polynomial fcn. is a fcn. of the form

$$p_n(x) = a_n x^n + a_n x^{n-1} + \cdots + a_1 x + a_0$$

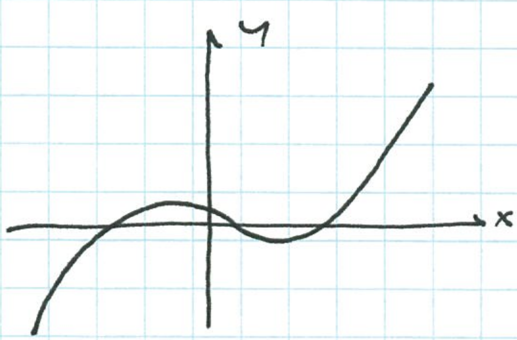
when $a_n \neq 0$, a_j are constants, and n is a non-negative integer. The order of the polynomial is n.

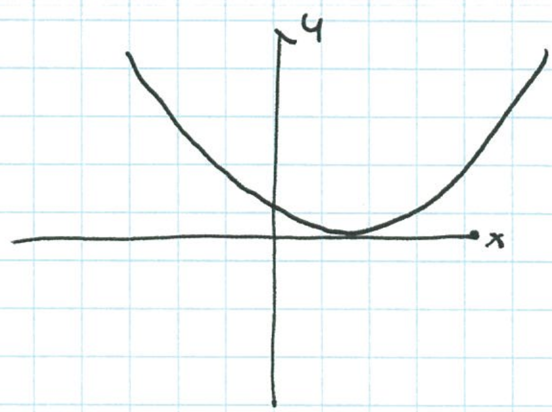
ex/ $P_3(x) = 5x^3 - 3x^2 + 1$ is a polynomial
degree $= 3$
$a_n = a_3 = 5$

ex/ $P_1(x) = -3x + 1$ is a polynomial
degree $= 1$
$a_n = a_1 = -3$

ex/ $P_2(x) = 8x^2 + 1$ is a polynomial
degree $= 2$
$a_n = a_2 = 8$

note: polynomials are "smooth"





what happens in the long run?

what is x^n as x gets "big?"

as x gets bigger:

x gets big

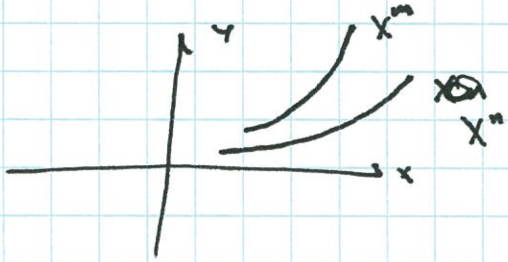
x^2 gets bigger

x^3 gets more bigger

$\vdots$

x^8 gets much more bigger

$\vdots$



$\underline{m>n}$ $\quad x^m > x^n$ for large x.

as x gets more negative:

ugh!

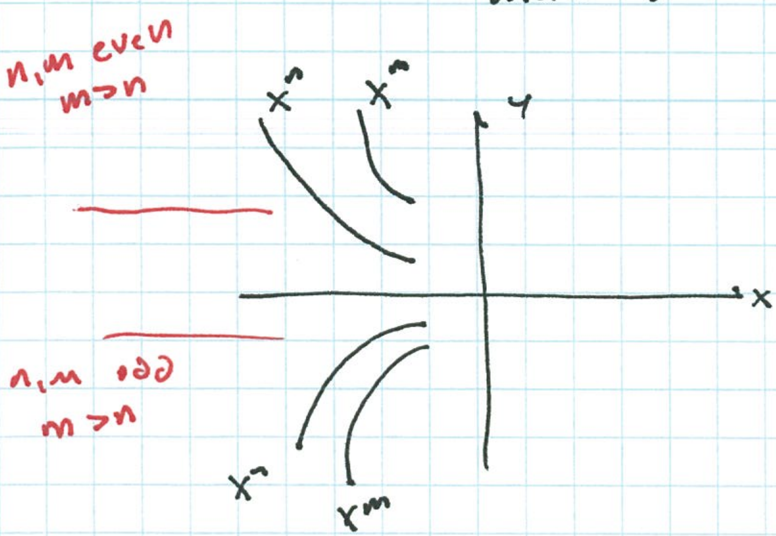
x gets more neg

x^2 gets big!

x^3 - gets ~~more~~ even more neg than x

x^4 - gets bigger than x^2

so if x gets more neg $\quad x^n \begin{cases} \text{if } n \text{ is odd gets very neg.} \\ \text{if } n \text{ is even gets big.} \end{cases}$



so

$$P_n(x) = \boxed{\underline{a_n} x^n} + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$$

sign matters!

dominant term.

swamps all the others.

$P_n(x) = \underline{a_n} x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$

determines long term behaviour

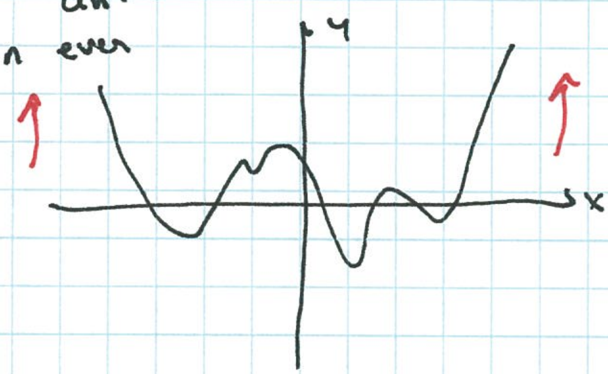
if x gets big + pos...

- a_n?
  - $a_n > 0$ — $P_n(x)$ gets "big"
  - $a_n < 0$ — $P_n(x)$ gets more negative.

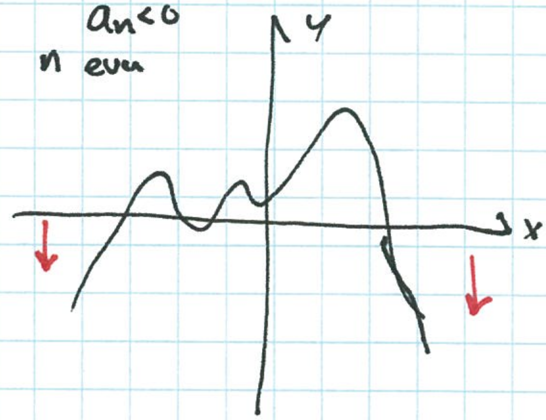
if x gets very negative...

- a_n?
  - $a_n > 0$
    - n even — gets "big" (pos)
    - n odd — gets way negative
  - $a_n < 0$
    - n even — gets way negative
    - n odd — gets "big" (pos)

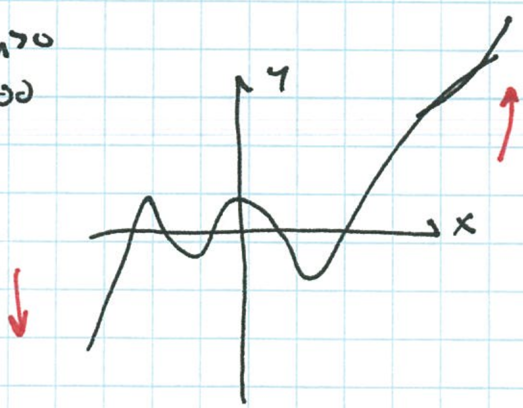
$a_n > 0$
n even



$a_n < 0$
n even



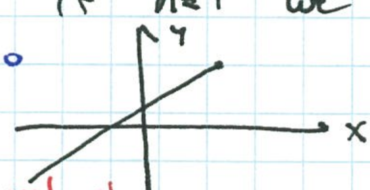
$a_n > 0$
n odd



$a_n < 0$
n odd

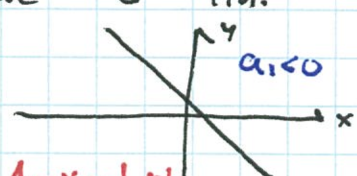
<u>Recall:</u> if n=1 we have a lin. fcn.

$a_1 > 0$



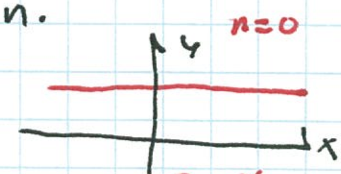
1 x-intercept.

$a_1 < 0$



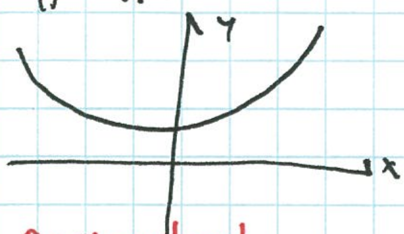
1-x intercept

n=0

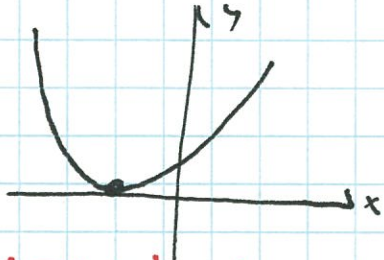


0 - x intercept.

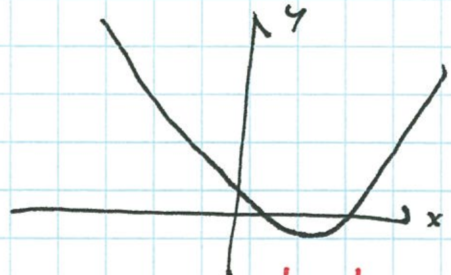
if $n=2$



0-x-intercept.



1-x-intercept



2-x-intercepts.

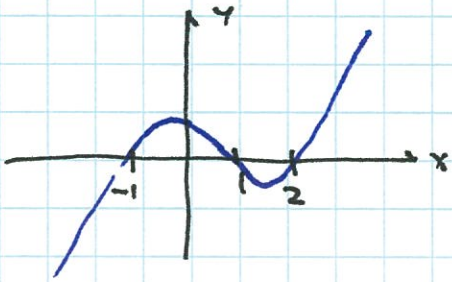
In general, if a polynomial has degree n it can have $0, 1, 2, \dots,$ or n x-intercepts.

<u>def:</u> The x-intercepts are called the <u>zeros</u> of the fcn.

ex/ determine the zeros of

$$\text{Belinda}(x) = 5(x-1)(x+1)(x-2) = ?$$

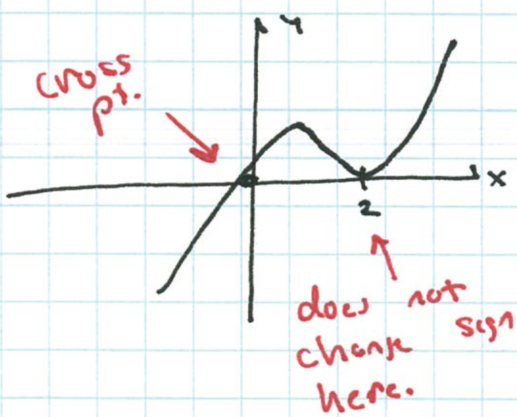
$x = 1, -1, 2.$



note: if a fcn. goes from $-$ to $+$ or $+$ to $-$ at a pt. we call that a "cross point."

ex/ determine the zeros of

$$\text{Gabriel}(x) = x(x-2)^2$$

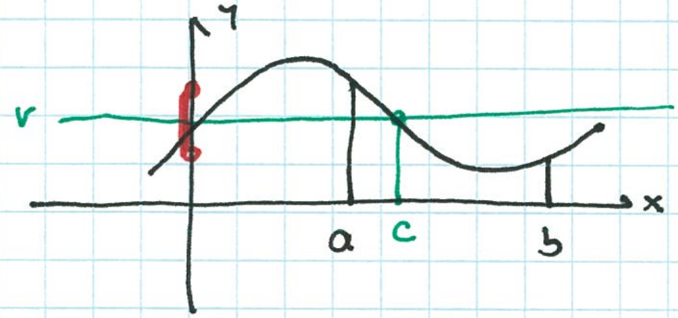


note: if a point is a zero but the function does not change sign then we call it a "touch point."

Okay, but those are contrived examples.
How do we know if a zero exists?

Well... polynomials are "smooth". They do not have jumps.

Intermediate Value Thm: If a fcn. is smooth and $a<b$ then the function takes on every value between $f(a)$ and $f(b)$. i.e. if v is between $f(a)$ & $f(b)$ then there is a c where $v = f(c)$.



so what? go back to Belinda(x),

note Belinda$(0) = 10$ pos.
Belinda$(-2) = -60$ neg.

Then is some point, c, between 0 and -2 where Belinda$(c)=0$.

ex/ Martin$(x) = x^3 - 8$

Martin$(-1) = -9$ $-$
Martin$(3) = 19$ $+$

There is a point c between -1 and 3 where Martin$(c)=0$.

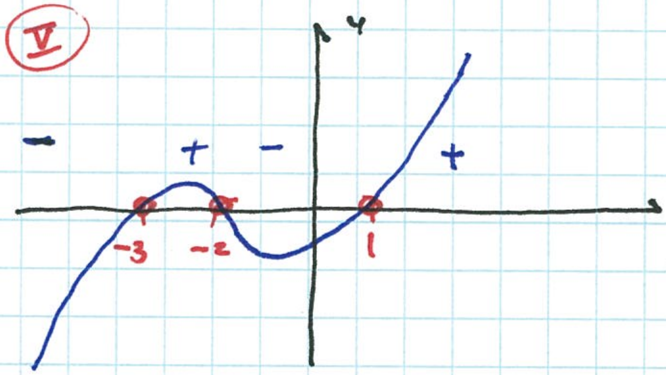
Now for something completely different:
How do we plot these things?

1. Det. the zeros.
2. Det. if $+/-$ between zeros.
3. get the long term behaviour.
4. wing it!

ex/ $\quad Jan(x) = 3(x-1)(x+2)(x+3)$

$\Rightarrow$ Jan has 3 zeros @ $x = 1, -2, -3$.

$\Rightarrow$ Jan has a a leading coef. of $3x^3$.



I if $x < -3$ $\quad (+)(-)(-)(-)$ result is neg.

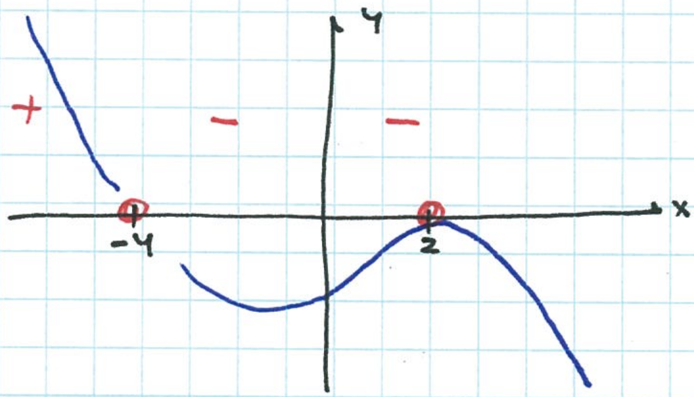
II if $-3 < x < -2$ $\quad (+)(-)(-)(+)$ result is +

III if $-2 < x < 1$ $\quad (+)(-)(+)(+)$ result is −

IV $x > 1$ $\quad (+)(+)(+)(+)$ result is +

ex/ $\quad Sally(x) = -3(x-2)^2(x+4)$

zeros @ $x = 2, 4$



$x < -4$ $\quad (-)(+)(-)$ result is pos.

$-4 < x < 2$ $\quad (-)(+)(+)$ result is ~~pos~~ neg.

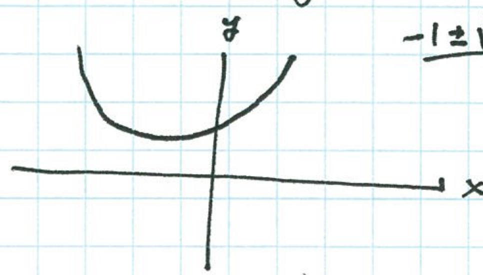
$x > 2$ $\quad (-)(+)(+)$ result. is neg.

ex/ $Kevin(x) = 7(x^2+x+8)(x-2)(x-1)$

hmm... x^2+x+8?

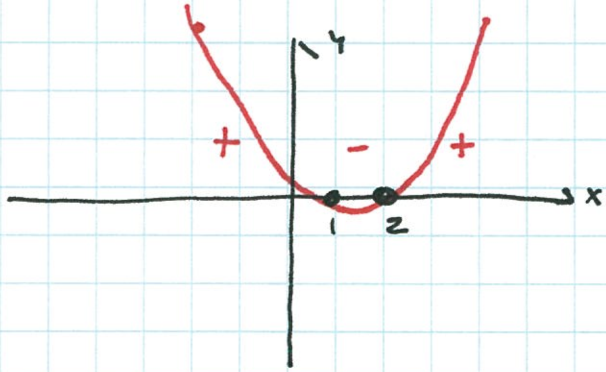
$y = x^2+x+8 = 0$

$\frac{-1 \pm \sqrt{1-32}}{2}$ Nope!



always pos!

so back to Kevin.



$x<1$	$(+)(+)(-)(-)$	pos.
$1<x<2$	$(+)(+)(-)(+)$	neg.
$x>2$	$(+)(+)(+)(+)$	pos.

zeros @ $x=2,1$

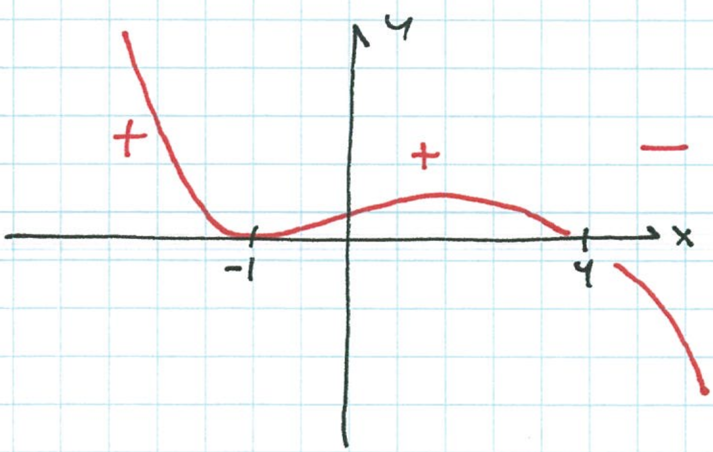
Time permitting

ex/ $Tom(x) = -2(\underbrace{x^2-3x-4}_{?})(x+1)$

$\frac{3 \pm \sqrt{9+16}}{2} = -1, 4$

$Tom(x) = -2(x+1)(x-4)(x+1)$

zeros @ $4, -1$



$x<-1$	$(-)(-)(-)(-)$	pos.
$-1<x<4$	$(-)(+)(-)(+)$	pos.
$x>4$	$(-)(+)(+)(+)$	neg

Time permitting

ex/

Velma$(x) = 4 - (x-2)^6$

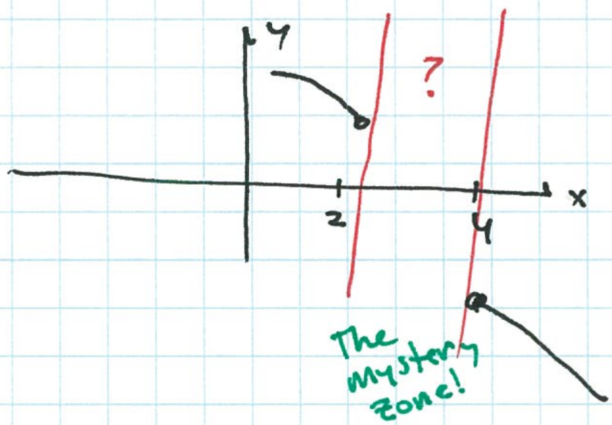
Does this fcn. have any zeros?

hmmm... well Velma$(2) = 4 - (2-2)^6 = 4$

Velma$(4) = 4 - (4-2)^6 = -60$

yes!

By the intermediate value thm. the fcn. has at least one zero between $x=2$ and $x=4$.



How do I know it is "smooth?"

It is a polynomial! why?

$$(x-2)^6 = (x-2)(x-2)(x-2)(x-2)(x-2)(x-2)$$
$$= x^6 - (\ \ )x^5 + (\ \ \ )x^4 + \dots + (-2)^6$$

so $4 - (\quad)$ is a polynomial of degree 6

and leading coef. is $-x^6$.

So the int. value thm. can be applied here.