Section 1.8 ALEKS HW

operations $+ / - / * / \div$

operations on functions

Diff Quotient

composition of functions

Nothing new here - we have been doing this. Just that we give things names now.

ex/ $Amy(x) = 3x^2 + 2x + 5$

let

$$\left.\begin{aligned} Avril(x) &= x^2 \\ Arni(x) &= x \\ Angus(x) &= 1 \end{aligned}\right\} Amy(x) = 3 * Avril(x) + 2 * Arni(x) + 5 * Angus(x)$$

We can take old functions to make new ones.

why? - In calculus we define operations on functions and need notation to make this easier to work with.

so given two functions, $f(x)$ and $g(x)$

Add: $f(x) + g(x) = (f+g)(x)$

subtract $f(x) - g(x) = (f-g)(x)$

multiply: $f(x) \cdot g(x) = (f \cdot g)(x)$

divide: $f(x) / g(x) = (f/g)(x)$

ex/ $Axel(x) = x+5$
$Ava(x) = \sqrt{x}$

Then $(Axel + Ava)(x) = x+5+\sqrt{x}$

$(Axel \cdot Ava)(x) = (x+5)\cdot\sqrt{x}$

$(Axel / Ava)(x) = \dfrac{x+5}{\sqrt{x}}$

note domain Axel is all x
Ava is $x \geq 0$

domain Axel / Ava is $x > 0$. (!)

This is trickier than it seems! be careful.

ex/ $Axel(Ava(x)) = ?$

woot???

what does $Ava(x)$ mean?
↳ give me a number and add 5

but $Axel(x) =$ give me a number and take the square root.

so $Axel(Ava(x)) = Axel(\sqrt{x})$
$= \sqrt{x} + 5$

You work from the inside out.

$Ava(Axel(x)) = Ava(x+5) = \sqrt{x+5}$

it is diff't!
(be careful)

Def: The composition of two functions, $f(x)$ and $g(x)$ is denoted

$$f \circ g(x) = f(g(x))$$

easy to confuse w/ multiplication

– warning – if range of $g(x)$ is not part of the domain of $f(x)$ then you have limit the domain.

ex1 ~~Axel ∘ Ava (x) = Axel~~ $\quad Ava \circ Axel = \sqrt{x+5}$

domain of Axel = all real #'s.

domain of Ava is #'s ≥ 0

so we need

$$Axel(x) \geq 0$$

$$\Rightarrow \quad x+5 \geq 0$$

$$\text{or} \quad x \geq -5$$

Domain $Ava \circ Axel$ is $x \geq -5$.

ex/

x	Fred
0	8
1	5
2	5
3	6

x	Gary
5	10
6	11
7	5
8	3

Fred$(1) = 5$, Fred(3) 6, ...

What is Gary $\circ$ Fred$(1) = ?$

Gary(Fred(1)) = Gary(5) = 10

Gary $\circ$ Fred (3) = Gary (Fred (3)) = Gary (6) = 11

Fred $\circ$ Gary (1) = Fred(Gary (1)) = Fred(???)

canna do it!

Fred $\circ$ Gary (6) = Fred (Gary (6)) = Fred(11)

Nope!

Fred $\circ$ Gary (8) = Fred(3) = 6 ✓

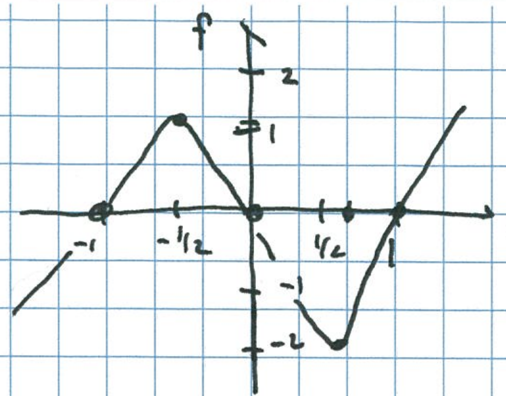
Domain of Gary is $\{5, 6, 7, 8\}$

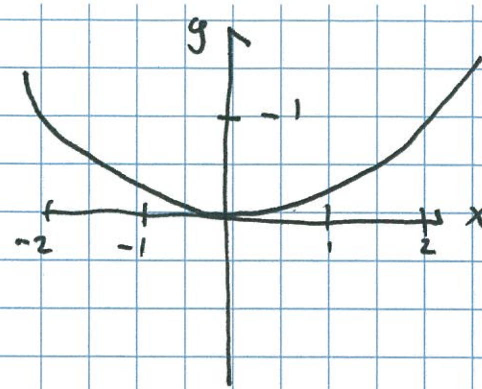
Domain of Fred $\circ$ Gary is $\{8\}$

Domain of Gary $\circ$ Fred is $\{0, 1, 2, 3\}$

range of Gary $\circ$ Fred is $\{3, 10, 11\}$

ex/





$g \circ f(x) = ?$

$g \circ f(0) = g(f(0)) = g(0) = 0.$

$g \circ f(1) = g(f(1)) = g(0) = 0$

$g \circ f(-1/2) = g(f(-1/2)) = g(1) = 1/2$

~~g o f &~~

$f \circ g(2) = f(g(2)) = f(1) = 0.$

ex/ $\text{Barb}(x) = 1/x$

$\text{Asher}(x) = x+1$

$\text{Barb} \circ \text{Asher}(x) = \text{Barb}(\text{Asher}(x))$

$= \text{Barb}(x+1)$

$= \frac{1}{x+1}$

domain is $\cancel{(x<-1) \cup (x>-1)}$ $x \neq -1$

$(-\infty, -1) \cup (-1, \infty)$

$\text{Asher} \circ \text{Barb}(x) = \text{Asher}(\text{Barb}(x))$

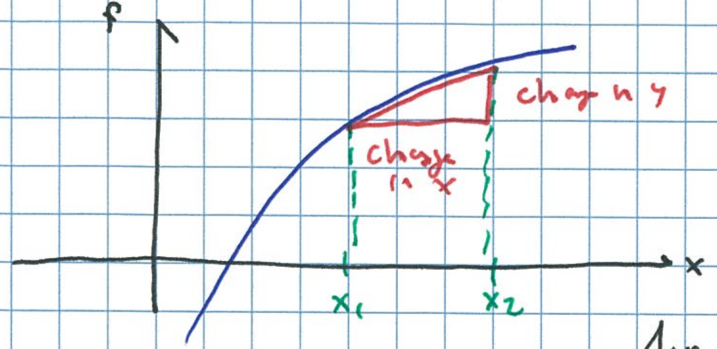
$= \text{Asher}(1/x)$

$= 1/x + 1$

domain is $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

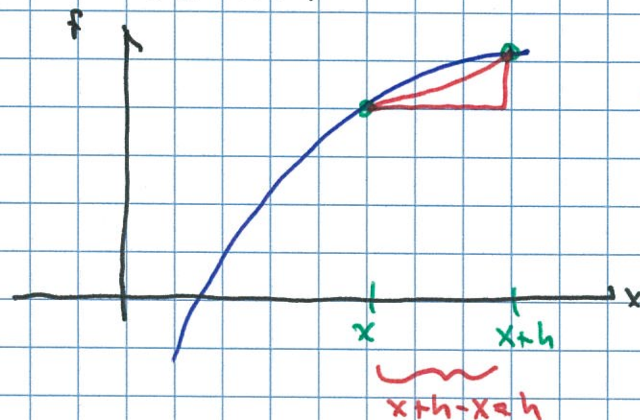
Recall Average Rate of change



$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Avg. Rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

what if $x_1 = x$, $x_2 = x+h$? A new function!



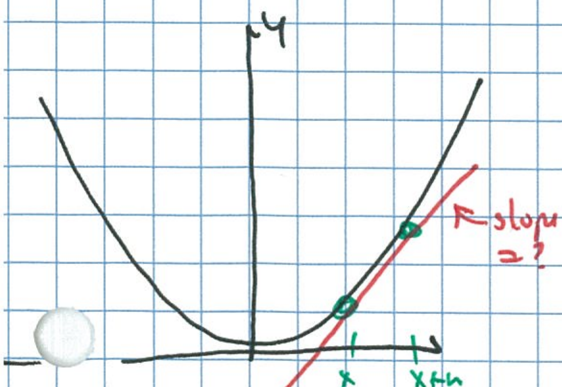
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

comp. subt.

$$\text{Diff. Quotient} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

Div.

ex/ $Chaz(x) = 3x^2$ Det. the diff quotient.



$$\frac{chaz(x+h) - chaz(x)}{h}$$

$$= \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$$

$$= \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} = 6x + 3h$$

Time Permitting

ex/ Express $April(x) = (x-1)^2+3$

as a composition of 2 functions

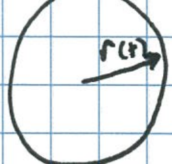
let $Bert(x) = x-1$

$Beth(x) = x^2+3$

$\Rightarrow$ $Beth \circ Bert(x) = Beth(Bert(x))$

$= Beth(x-1) = (x-1)^2+3$

ex/ A spherical balloon expands, and the radius increases at a constant rate. At t=0 seconds the radius is 5cm, and at t=3 seconds the radius is 6cm. Get the volume of the balloon @ any time.

① 

② $r(t)$ = rad.
t = time.

③ $Vol. = \frac{4}{3}\pi r^3$
$r(t) = m(t-t_0)+r_0$

④ Figure out $r(t)$
& compose w/ $V(r)$

⑤ line through $(0,5)$ and $(3,6)$

$m = \frac{6-5}{3-0} = 1/3$

$r(t) - 5 = 1/3\,(t-0) \Rightarrow r(t) = \frac{1}{3}t+5$

$V(r) = \frac{4}{3}\pi r^3$

$V(r(t)) = \frac{4}{3}\pi\left\{\frac{1}{3}t+5\right\}^3$

Time Permitting

ex/ The cost per item required to produce an object depends on the number of items produced. The cost is

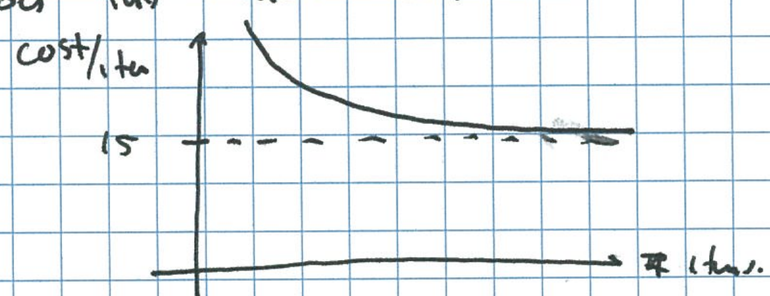
$$C(n) = 15 + \frac{12}{n} \quad \$/\text{item}$$

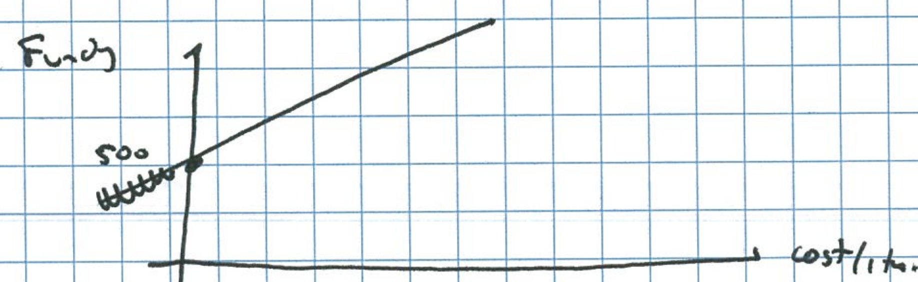
The amount of funding required to begin production depends on the cost/item of production.

$$\text{Funding} = \text{Fund}\left(\frac{\text{cost}}{\text{item}}\right) = 100 \ \text{cost}/\text{item} + 500$$

what is the required funding as a function of # of items?

Does this make sense?



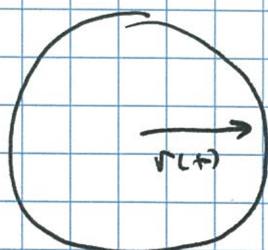


$$\begin{aligned}
\text{so} \quad & \text{Funding} \circ \text{cost}(N) \\
&= \text{Funding}(\cancel{\text{cost}(}\ \text{cost}(N)) \\
&= \text{Funding}(15 + 12/n) \\
&= 100\left(15 + \frac{12}{n}\right) + 500 \\
&= 2000 + \frac{1200}{n}
\end{aligned}$$

Time Permitting

ex/ A rock is thrown into a pond. The waves radiate outwards from the center w/ a radius that increase with a constant rate. It takes 5 seconds to get a radius of 3m. What is the area of the circle at any time?

(1)



(2) $A = \pi r^2$

t = lin. fcn.

(3) r, t

(4) Figure out $r(t) = mt + b$, Compose w/ $A = \pi r^2$.

(5) Line through $(0,0)$ and $(5,3)$

$$m = \frac{5-0}{3-0} = 5/3$$

$$r(t) - 0 = 5/3(t - 0)$$

$$r(t) = 5/3\, t$$

$$A(t) = A \circ r(t)$$
$$= A(5/3\, t)$$
$$= \pi (5/3\, t)^2$$
$$= \pi \cdot \frac{25}{9} \cdot t^2.$$