Section 2.3 ALEKS

Polynomial Division
Remainder Theorem

Last time we saw how to graph a polynomial if it is in factored form.

ex: $Tom(x) = 4(x+2)^2(x-2)(x+3)$.

Q: if $Tom(x) = 4x^4 + 20x^3 + 8x^2 - 80x - 96$

how do we factor it?

1st note that if

$Tom(x) = 4(x+2)^2(x-2)(x+3)$

Then the zeros are $x = -2, 2, -3$.

so... step 1) Determine the zeros of the polynomial.

- plot its graph
- guess.... (Int. value them!)

Problem: we cannot use a graphing calc. on a test :(

⇒ We hope that we are nice enough to have easy to guess zeros.

w/ that in mind..

$Tom(0) = -96$

$Tom(1) = -144$

$Tom(2) = 0$ Bingo! First try!

This means $Tom(x) = (x+2)$ [polynomial of deg. 3].

This, in turn, implies that

$$\frac{Tom(x)}{x+2} = \text{Polynomial of deg. 3.}$$

How do we do this?

How does div. w/ numbers work?

$$\frac{5076}{42} = ?$$

$$\begin{array}{r|l} & \;\;120 \\ \hline 42 & 5076 \\ & \underline{42} \\ & \;\;87 \\ & \;\;\underline{84} \\ & \;\;\;36 \end{array}$$

remainder

nope.

5×10^3

8×10^2

3×10^1

so $5076 = 42 * 120 + 36$

Polynomial div. works the same way. Instead of factors of 10 we have factors of x $(1, x, x^2, x^3, \dots)$

so...

$$\begin{array}{r|l} & 4x^3 + 28x^2 + 64x + 48 \\ \hline x-2 & 4x^4 + 20x^3 + 8x^2 - 80x - 96 \\ & \underline{4x^4 - 8x^3} \\ & \quad 28x^3 + 8x^2 \\ & \quad \underline{28x^2 - 56x^2} \\ & \qquad 64x^2 - 80x \\ & \qquad \underline{64x^2 - 128x} \\ & \qquad\quad 48x - 96 \\ & \qquad\quad \underline{48x - 96} \\ & \qquad\qquad 0 \end{array}$$

remainder is zero.

so $4x^4 + 20x^3 + 8x^2 - 80x - 96 = (x-2)(4x^3 + 28x^2 + 64x + 48) + \underline{\underline{0}}$

If the remainder is not zero then you did <u>something</u> wrong.

Are we done? Nope... That is just the 1^{st} step.

$$Tom(x) = (x-2)\underbrace{(4x^3 + 28x^2 + 64x + 48)}_{\text{call this } F_T(x)}$$

$F_T(0) = 48$
$F_T(1) = 144$
$F_T(2) = 370$
$F_T(-1) = 8$
$F_T(-2) = 0$ bingo!

$x-(-2) = x+2$ is a factor of $F_T(x)$.

$$\begin{array}{r l}
 & 4x^2 + 20x + 24 \\
x+2 & \overline{)\,4x^3 + 28x^2 + 64x + 48} \\
 & \underline{4x^3 + 8x^2} \\
 & \quad 20x^2 + 64x \\
 & \quad \underline{20x^2 + 40x} \\
 & \qquad 24x + 48 \\
 & \qquad \underline{24x + 48} \\
 & \qquad\qquad 0 \leftarrow \text{zero!}
\end{array}$$

so
$4x^3 + 28x^2 + 64x + 48$
$= (x+2)(4x^2 + 20x + 24)$

are we there yet? Nope! $Q_T = 4x^2 + 20x + 24$

$Q_T(x) = 4(x^2 + 5x + 6)$ A quadratic!

$$\text{roots} = \frac{-5 \pm \sqrt{25-24}}{2} = -3, -2$$

so $Q_T(x) = 4(x+3)(x+2)$

and

$$Tom(x) = 4(x+3)(x+2)(x+2)(x-2)$$

are we there yet? Nope! Still need to graph it!
(do on your own)

Q/ This seems oddly familiar. Have I done this before?
A/ yes.
Q/ where?
A/ Synthetic div.
Q/ That's what we did?
A/ yes.
Q/ can I do that instead?
A/ whatever. — read the book though!

ex/ graph the function

$$\text{Venus}(x) = -3x^4 + 15x^3 - 6x^2 - 24x$$

I see that I can factor x right away!

$$\text{Venus}(x) = x\,[\underbrace{-3x^3 + 15x^2 - 6x - 24}_{C(x)}]$$

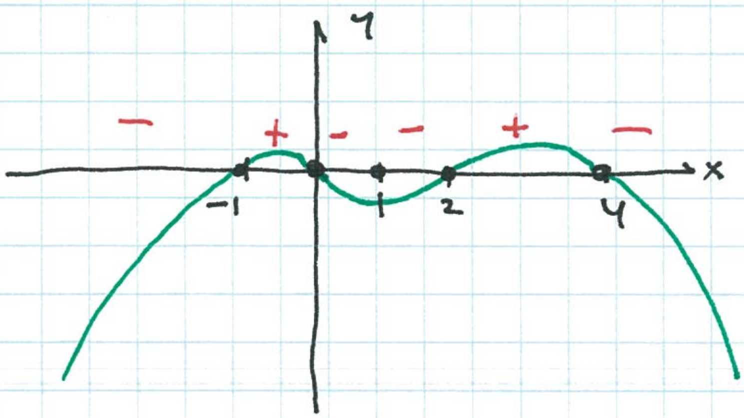
$C(0) = -24$
$C(1) = -18$
$C(2) = 0$ ✓

$$
\begin{array}{r}
-3x^2 + 9x + 12 \\
x-2\;\big)\overline{-3x^3 + 15x^2 - 6x - 24} \\
\underline{-3x^3 + 6x^2} \qquad\qquad\quad \\
9x^2 - 6x \qquad\quad \\
\underline{9x^2 - 18x} \qquad\quad \\
12x - 24 \\
\underline{12x - 24} \\
0 \checkmark
\end{array}
$$

$$\text{Venus}(x) = x(x-2)(-3x^2 + 9x + 12) = -3x(x-2)(x^2 - 3x - 4)$$

$$\frac{3 \pm \sqrt{9+16}}{2} = 4, -1$$

$$\text{Venus}(x) = (-3)(x)(x-2)(x-4)(x+1)$$



$x < -1$	$(-)(-)(-)(-)(-)$	< 0
$-1 < x < 0$	$(-)(-)(-)(-)(+)$	> 0
$0 < x < 1$	$(-)(+)(-)(-)(+)$	< 0
$2 < x < 4$	$(-)(+)(+)(-)(+)$	> 0
$x > 4$	$(-)(+)(+)(+)(+)$	< 0

ex/ graph $Wally(x) = x^3 + 6x - 7$

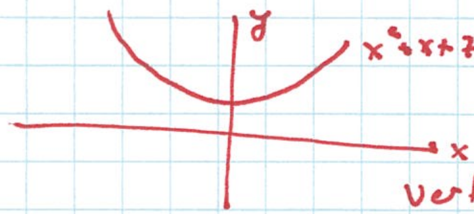
$Wally(0) = -7$

$Wally(1) = 0$ ✓

$$\begin{array}{r|l} & x^2 + x + 7 \\ \hline x-1 & x^3 + 0x^2 + 6x - 7 \\ & \underline{x^3 - x^2} \\ & \quad x^2 + 6x \\ & \quad \underline{x^2 - x} \\ & \qquad 7x - 7 \\ & \qquad \underline{7x - 7} \\ & \qquad\quad 0 \ ✓ \end{array}$$

$Wally(x) = (x-1)(x^2 + x + 7)$

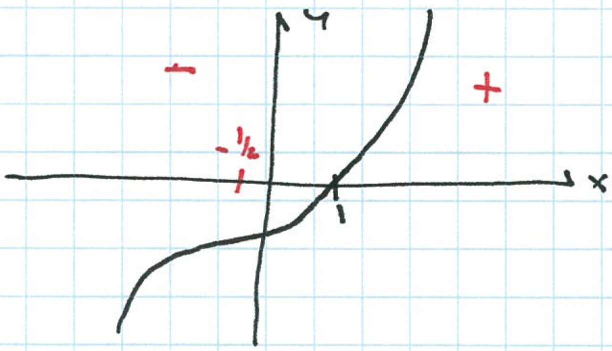
$$\frac{-1 \pm \sqrt{1 - 28}}{2}$$ Nope!



no roots!

vertex @ $-\frac{1}{2}$.

so $Wally(x) = (x-1)(x^2 + x + 7)$



$x < 1$	$(-)(+)$	< 0
$x > 1$	$(+)(+)$	> 0

ex/ Rewrite the expression below so it is a polynomial plus a simple expression.

$$\frac{3x^3 + 4x^2 - 7}{x+1}$$

$$\begin{array}{r|l} & 3x^2 + x - 1 \\ \hline x+1 & 3x^3 + 4x^2 + 0x - 7 \\ & \underline{3x^3 + 3x^2} \\ & \quad x^2 + 0x \\ & \quad \underline{x^2 + x} \\ & \qquad -x - 7 \\ & \qquad \underline{-x - 1} \\ & \qquad\quad -6 \end{array}$$

so $(3x^2 + x - 1)(x+1) - 6 = 3x^3 + 4x^2 - 7$

$\Rightarrow$ $\boxed{3x^2 + x - 1 - \frac{6}{x+1}} = \frac{(3x^3 + 4x^2 - 7)}{x+1}$

Time permitting

ex/ Rewrite the expression below so it is in a simpler form

$$\frac{8x^4 + 3x^2 + 1}{x + 2}$$

$$\begin{array}{r|l} & 8x^3 - 16x^2 + 35x - 70 \\ \hline x+2 & 8x^4 + 0x^3 + 3x^2 + 0x + 1 \\ & 8x^4 + 16x^3 \\ \hline & \quad -16x^3 + 3x^2 \\ & \quad -16x^3 - 32x^2 \\ \hline & \qquad 35x^2 + 0x \\ & \qquad 35x^2 + 70x \\ \hline & \qquad\quad -70x + 1 \\ & \qquad\quad -70x - 140 \\ \hline & \qquad\qquad 141 \end{array}$$

so

$$8x^4 + 3x^2 + 1 = (x+2)(8x^3 - 16x^2 + 35x - 70) + 141$$

or

$$\frac{8x^4 + 3x^2 + 1}{x+2} = 8x^3 - 16x^2 + 35x - 70 + \frac{141}{x+2}.$$

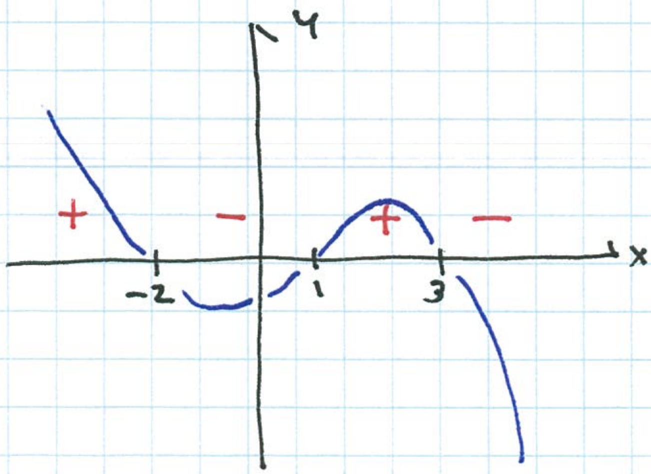
Time permitting

ex/ Plot the function $Uma(x) = -2x^3 + 4x^2 + 10x - 12$

$Uma(1) = 0$ (!) Hole in one!

$$\begin{array}{r|l} & -2x^2 + 2x + 12 \\ \hline x-1 & -2x^3 + 4x^2 + 10x - 12 \\ & -2x^3 + 2x^2 \\ \hline & \qquad 2x^2 + 10x \\ & \qquad 2x^2 - 2x \\ \hline & \qquad\quad 12x - 12 \\ & \qquad\quad 12x - 12 \\ \hline & \qquad\qquad 0 \ \checkmark \end{array}$$

$$\Rightarrow Uma(x) = (x-1)(-2x^2 + 2x + 12)$$
$$= -2(x-1)(x^2 - x - 6)$$
$$= -2(x-1)(x-3)(x+2)$$



$x < -2$	$(-)(-)(-)(-)$	> 0
$-2 < x < 1$	$(-)(-)(-)(+)$	< 0
$1 < x < 3$	$(-)(+)(-)(+)$	> 0
$x > 3$	$(-)(+)(+)(+)$	< 0