Section 1.5

ALEKS H.W.

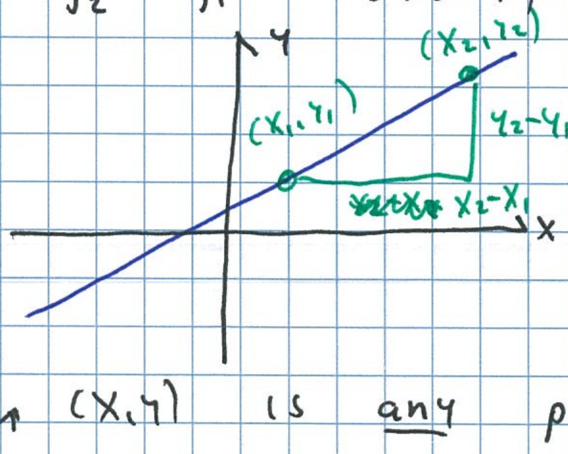
pt. slope formula
parallel + perpendicular lines
modeling w/ lin. equations

- ~~There~~ There are many ways to express a linear relationship. ☺
- There are **many** ways to state a situation that has a linear relationship.

⇒ You have to be flexible and be able to adapt to novel situations!

Recall if (x_1, y_1) and (x_2, y_2) are on a line then

$$y_2 - y_1 = m(x_2 - x_1)$$



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

If (x, y) is **any** point on the line:

$$y - y_1 = m(x - x_1)$$ pt slope form

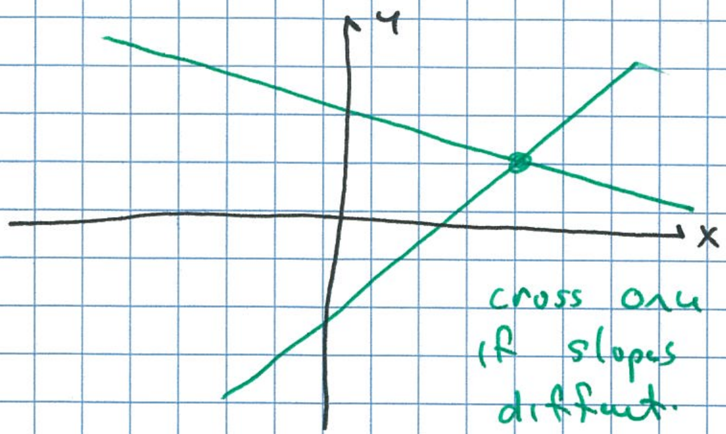
or

$$y = y_1 + m(x - x_1) = y_1 + mx - mx_1$$
$$= mx + \underbrace{(y_1 - mx_1)}_{\text{call this } b}$$

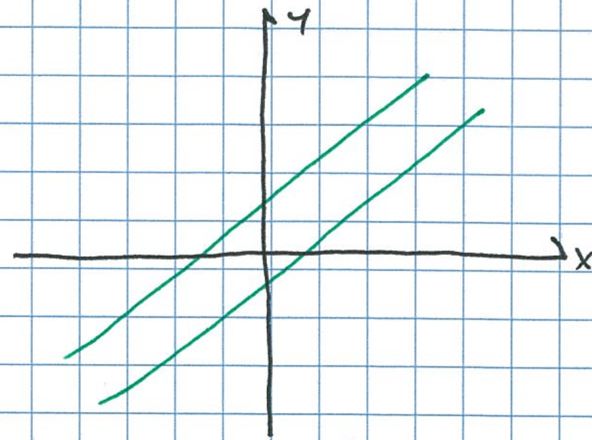
slope intercept form

Q/ Do two lines cross?

A/ maybe!



cross once if slopes different.



never cross - slopes same

BUT can cross inf. many times if overlapping.

<u>So:</u> Two lines can cross

once - slopes different

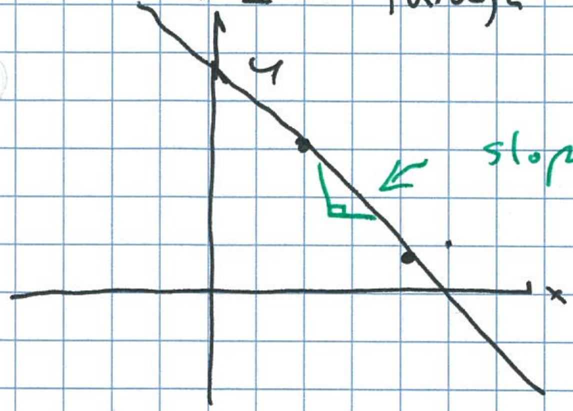
none - slopes same and not overlapping

infinitely many - slopes same and overlapping.

<u>Def:</u> If two lines have the same slope we say that they are parallel.

ex/ Determine the line through the point $(3,1)$ that is parallel through the line that goes through the points $(2,4)$ and $(4,1)$.

Line 1 – through $(2,4)$ and $(4,1)$



$$\text{slope} = \frac{4-1}{2-4} = \frac{3}{-2} = -3/2$$

Line 2 – has slope – $-3/2$

goes through $(3,1)$

so...

$$\underline{y-1 = -3/2\,(x-3)}$$

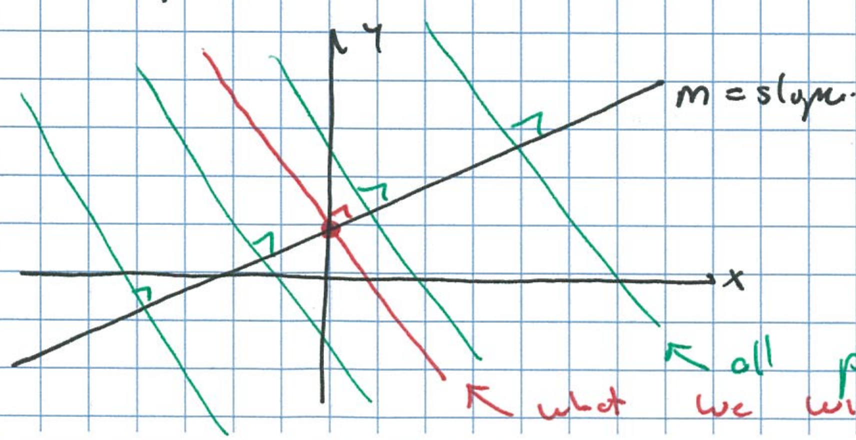
Q/ what about perpendicular lines?

(Glad you asked!)

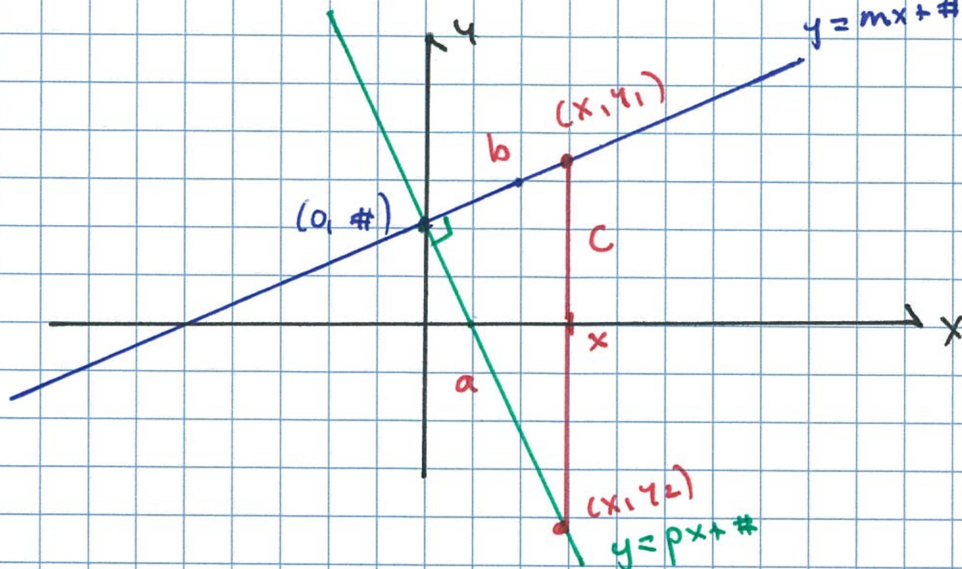
First – let's set it up nicely...

Goal: Given $y = mx+b$

determine the slope of <u>any</u> line perpendicular to the original line.



excellent - let's do it!



given $y = mx + \#$
assume $y = px + \#$
is $\perp$.
det. p?

well.... $a^2 + b^2 = c^2$ (by assuming $\perp$ + right Δ)

use dist formula!

$$\begin{cases} a^2 = (x-0)^2 + (px + \# - \#)^2 = x^2 + (px)^2 \\ b^2 = (x-0)^2 + (mx + \# - \#)^2 = x^2 + (mx)^2 \\ c^2 = (x-x)^2 + ((mx+\#) - (px+\#))^2 \\ \quad = 0^2 + (mx - px)^2 = ((m-p)x)^2 = (m-p)^2 x^2 \end{cases}$$

now put it all together.

$$\underbrace{(x^2 + p^2x^2)}_{a^2} + \underbrace{(x^2 + (m^2)x^2)}_{b^2} = \underbrace{(m-p)^2 x^2}_{c^2}$$

$$x^2 \left[1 + p^2 + 1 + m^2 \right] = (m-p)^2 x^2 \qquad \text{assume } x \neq 0$$

$$2 + p^2 + m^2 = (m-p)^2$$

$$2 + \cancel{p^2} + \cancel{m^2} = \cancel{m^2} - 2mp + \cancel{p^2}$$

$$2 = -2mp$$

$$p = -1/m$$

so, given $y = mx + b$, <u>any</u> line perpendicular to it has slope $p = -\frac{1}{m}$

ex/ Determine which line goes through $(5,-1)$ that is perpendicular to the line through $(2,1)$ and $(4,2)$.

$$m = \frac{1-2}{2-4} = \frac{-1}{-2} = 1/2$$

$$\text{so} \quad p = \frac{-1}{(1/2)} = \frac{-1}{1/2}\cdot\frac{2}{2} = -2$$

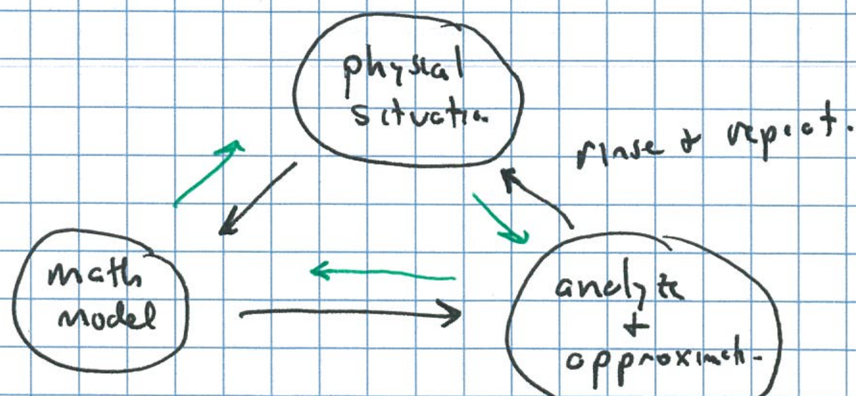
so... $y-(-1) = -2(x-5)$

or $y+1 = -2(x-5)$

<u>note</u> modeling w/ math

We use mathematical expressions to <u>approximate</u> the real world. (Hopefully it is "close enough.") Linear functions are often tried first because they are easier than other functions. We often go back and change to get a better approximation.

<u>process</u> of modeling.

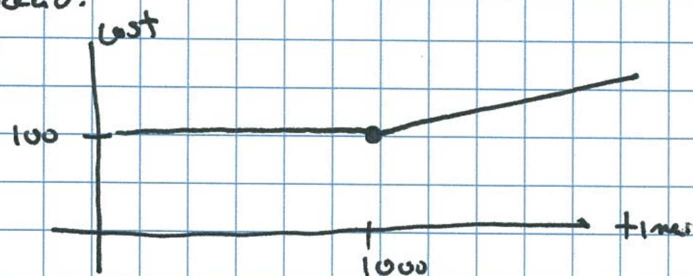


we go the other way to gain insight into the physical situation.

Fine. How do we do it though?

0. Read the problem twice!
1. draw a picture of the physical situation.
2. identify variables.
3. explore the relationships.
4. make a plan
5. execute!
6. check your work. (does it make sense?)
(I skip in class cuz I already did it! and to avoid wasting time)

ex/ The cost to ~~cho~~ use a phone is 100\$ for the 1st 1000 minutes and 10¢ each following minute. $\boxed{\text{What are the costs for any time?}}$

0. Read!
1. 

2. c = cost \$, t = time. (minutes)
3. if $t < 1000$ min $c = 100$ is constant.
   if $t > 1000$ min. $c = 100 + (?)$ ← 10¢ per min.
   units matter
4. Figure out constant for slope.
   Figure out time after 1000 min:
   put it together
5. if $t < 1000$ then $c =$ 100.

(next page)

if $t \geq 1000$ then slope $= \cancel{*}10¢ * \frac{1\,\$}{100\,¢}$ per min.
$= .10\ \$/\text{min.}$

time to change $= t - 1000$

(1000, 100) is on the line.

so cost $= 100 + .10(t-1000)$

so cost $=$ · 100\$ if time ≤ 1000
$100 + .10(t-1000)$ if $t > 1000$.

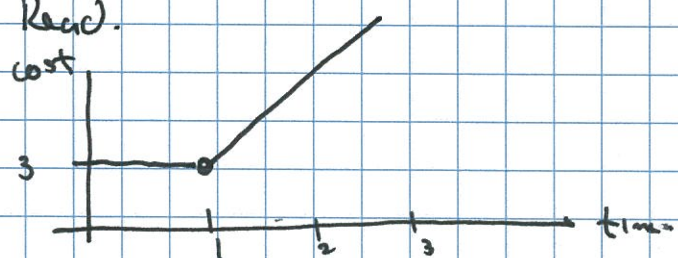
(6) check!

— Time Permitting —

ex/ If you park your car in a lot it is 3\$ for the 1st hour plus 1\$ each additional hour.
What is the cost for any time?

(0) Read.

(1)



(2) t = time in hours.
c = cost in \$

(3) cost $= 3$ if $t \leq 1$ / after 1 hour it is lin. w/ slope $= 1$.
(1,3) is on the line.

(4) get formula in slope int. form.

(5) if $t \geq 1$ ~~cost =~~ cost $- 3 = 1 \cdot (t-1)$
cost $= 3 + (t-1)$
$= t+2$

so cost $=$ 3\$ if time < 1, otherwise it is
$t+2$ \$.

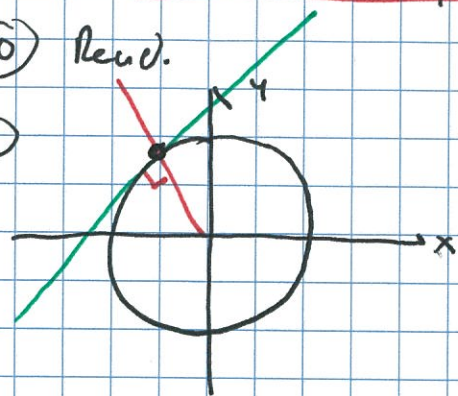
See discussion on cost/revenue/profit in book on page 171 of book.

— Tim Permitting. —

ex/ Determine the line tangent to a circle centered at the origin w/ radius 5 through the point $(-3, 4)$.

(0) Read.

(1)



(2) m = slope of line.
P is a pt. on the circle and the line.

(3) red line is $\perp$ to green line above.

$$\text{slope of one} = \frac{-1}{\text{slope of other}}$$

(4) I can get slope of red line then use $\frac{-1}{\bullet}$ to get slope of green, use pt. slope form after that.

(5) slope of red line: $\frac{4-0}{-3-0} = -4/3$

slope of green line $= \frac{-1}{(-4/3)} * \frac{3}{3} = \frac{3}{4}$

so $y - 4 = \frac{3}{4}\left(x - (-3)\right)$

(6) check!