Section 2.1

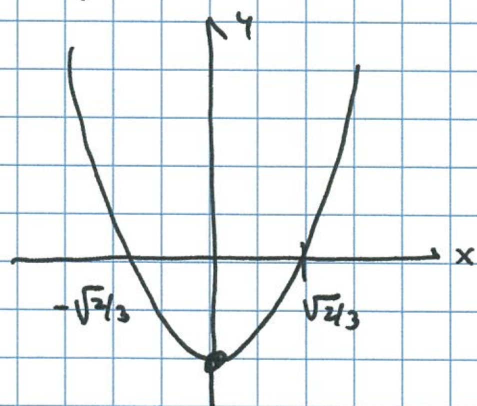
ALEKS HW

Graph of quadratic
Vertex of the quadratic
optimization w/ quadratics
Express formula for a quadratic given an explanation.

Def: A quadratic function is a function in the form $f(x) = ax^2 + bx + c$.

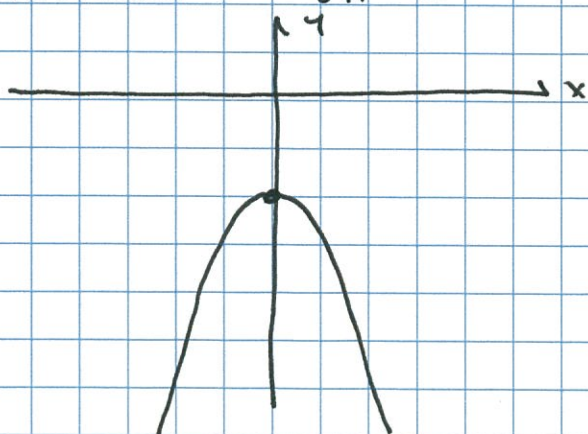
(a, b, and c are constants.)

ex/ $\text{Megan}(x) = 3x^2 - 2$



$a = 3$
$b = 0$
$c = -2$.

ex/ $\text{Mark}(x) = -3x^2 - 2$



$a = -3$
$b = 0$
$c = -2$

x-intercept?

$0 = -3x^2 - 2$

$3x^2 = -2$

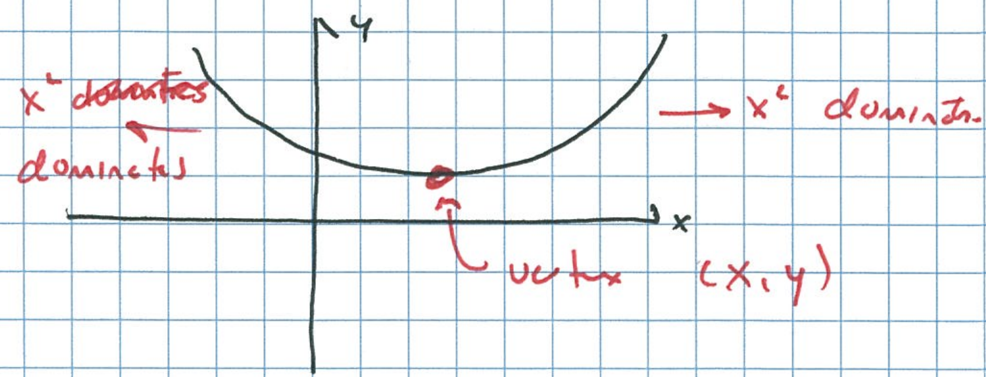
$x^2 = -2/3$

Nope!

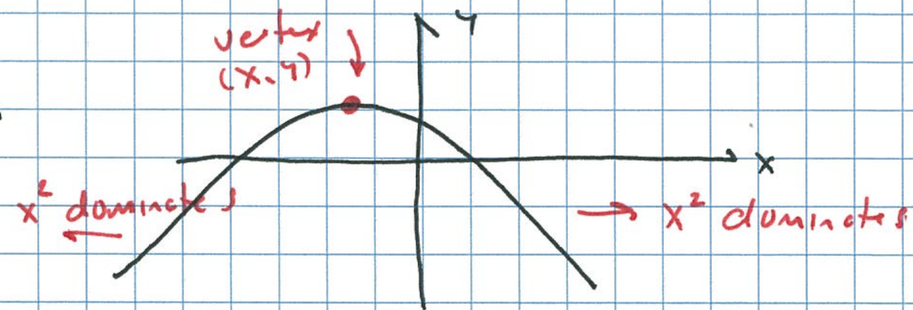
1

In general

$a>0$
opens up.



$a<0$
opens down



If $a<0$ the vertex is at the "top" of the parabola.
(maximum y)

If $a>0$ the vertex is at the "bottom" of the parabola
(minimum y)

Q/ How do you find the vertex?

complete the square!

ex/ $y = \boxed{x^2}\boxed{-3x}\boxed{+2}$

assume that $y = a(x-h)^2 + k$ why? Just wait.

$$\Rightarrow \quad y = a(x^2 - 2hx + h^2) + k$$
$$y = \boxed{ax^2}\boxed{-2ahx} + \boxed{ah^2 + k}$$

so $ax^2 = x^2$ or $a = 1$

$-2ahx = -3x$ or $h = 3/2$

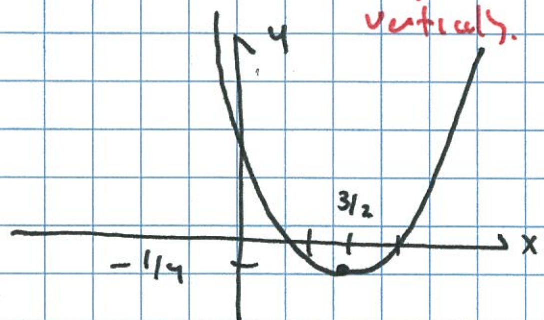
$ah^2 + k = 2$ or $k = 2 - ah^2$

$= 2 - 9/4 = -\frac{1}{4}$

so $y = x^2 - 3x + 2 = 1(x - 3/2)^2 - \frac{1}{4}$ So what?

$$y = 1 \cdot \underbrace{(x - 3/2)^2}_{\text{parabola shifted right } 3/2} \underbrace{- \tfrac{1}{4}}_{\text{shift parabola down } -1/4 \text{ vertically.}}$$

scale parabola by 1 vertically.



vertex @ $x = 3/2$, $y = -1/4$
$P(3/2, -1/4)$

Def: The standard Equation for a parabola is

$$y = a(x-h)^2 + k.$$

(also calld the vertex form.)

- ~~vertex~~ vertex @ (h, k)
- min/max @ (h, k)
- if $a < 0$ opend downward.
  if $a > 0$ opens upward.

ex/ Det min or max of

$$y = -2x^2 + 6x + 1$$

must be a max.
~~opens~~ opens downward.

ex/ Det max/min of

$$y = -2x^2 + 6x + 1$$

(max – $a = -2 < 0$ opens down)

$$y = a(x-h)^2 + k$$
$$= a(x^2 - 2hx + h^2) + k$$
$$= \boxed{a x^2} \boxed{- 2ahx} + ah^2 + k$$

$$ax^2 = -2x^2 \Rightarrow a = -2$$
$$-2ahx = 6x \Rightarrow h = 3/2$$
$$ah^2 + k = 1 \Rightarrow k = 11/2$$

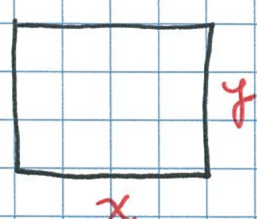
$$y = -2(x - 3/2)^2 + 11/2$$

max value is $11/2$ occurs @ $x = 3/2$.

ex/ A ~~rec~~ rectangular plot of land has a height and base to be determined. The perimeter must be 100m. What is the maximum possible area?

(0) Read!

(1) picture



(2) x = base, y = height

(3) explain

$P = 2x + 2y$ but $P = 100$

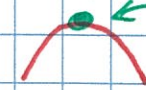
$A = x \cdot y$ $A = ?$

(4) solve perimeter for y & subst. into Area. make A as big as possible.

(5) see next page.

$$2x + 2y = 100 \quad \Rightarrow \quad 2y = 100 - 2x$$
$$y = 50 - x$$

a quadratic.



$$\text{Area} = x \cdot y = x(50 - x) = 50x - x^2$$

$$-x^2 + 50x = a(x-h)^2 + k$$
$$= a(x^2 - 2ahx + h^2) + k$$
$$= ax^2 - 2ahx + ah^2 + k$$

$$-x^2 = ax^2 \quad \Rightarrow \quad a = -1$$
$$-2ahx = 50x \quad \Rightarrow \quad h = 25$$
$$ah^2 + k = 0 \quad \Rightarrow \quad k = (25)^2$$

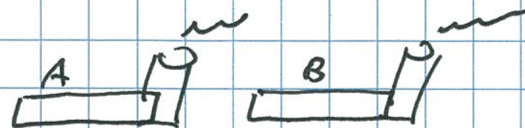
since the area is a downward opens parabola the max is @ when $x = 25$ and the area is $(25)^2$.

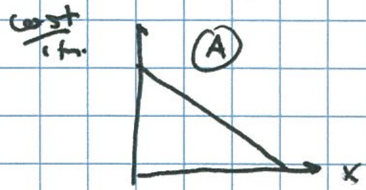
Note if we were ask for the dimensions we would need
$x = 25$
$y = 50 - 25 = 25$
for the answer.

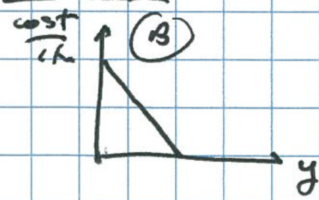
ex/ A company will set up two plants to produce an item. It is estimated that the cost per item to produce x items @ plant A is $1000 - 2x$, and the ~~cost to produce~~ cost per item to produce y items @ plant B is $1800 - 4y$. A total of 150 items must be produced. How many should be produced @ each plant?

(0) Read!
(1) next pg.

① Draw pict.







Does this make sense?

② x = # items plant A
y = # items, plant B.

③ Total = $150 = x + y$

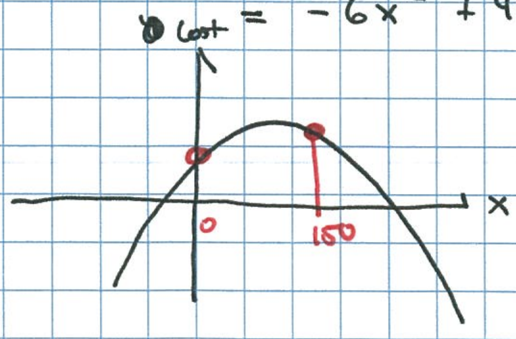
cost (A) = # items $\times$ $\frac{\text{cost}}{\text{item}}$ = $x(1000 - 2x)$

cost (B) = # items $\times$ $\frac{\text{cost}}{\text{item}}$ = $y(1800 - 4y)$

④ get ~~total~~ cost, solve constraint for y, subst. min cost.

⑤ $x + y = 150 \Rightarrow y = 150 - x$

$$\begin{aligned}
\text{cost} &= x(1000 - 2x) + y(1800 - 4y) \\
&= 1000x - 2x^2 + 1800y - 4y^2 \\
&= 1000x - 2x^2 + 1800(150 - x) - 4(150 - x)^2 \\
&= 1000x - 2x^2 + 1800 \cdot 150 - 1800x - 4(150^2 - 2x + x^2) \\
\text{cost} &= -6x^2 + 400x + 180{,}000
\end{aligned}$$



ooops! There is a max cost! we want min!

min @ either $x = 0$ or $y = 0$ $\Rightarrow x = 150$

all or nothing!

option 1 $x = 0$ $y = 150$ cost = $150(1800 - 4(150)) = 180{,}000$ \$

option 2 $x = 150$, $y = 0$ cost = $150(1000 - 2(150)) = 105{,}000$ \$

option 2 is best. Produce all in plant A and shut plant B down.

Just for kicks, what is the max?

$$\text{Cost} = -6x^2 + 400x + 180{,}000$$
$$= a(x-h)^2 + k$$
$$= ax^2 - 2ahx + ah^2 + k$$

$$a = -6$$
$$h = \frac{-400}{-2(6)} = \frac{100}{3}$$
$$k = 180{,}000 - (-6)\left(\frac{100}{3}\right)^2 = \frac{560{,}000}{3}$$

So max cost when $x = \frac{100}{3}$ and it is

$\frac{560{,}000}{3}$ \$.