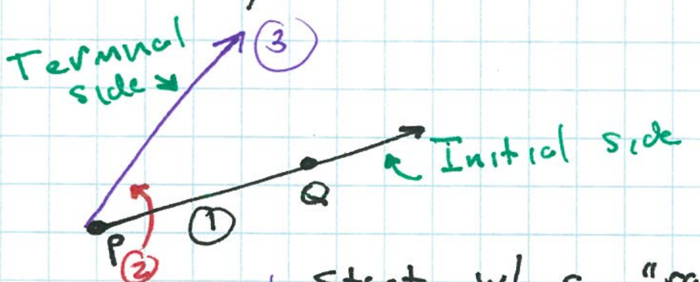
Section 4.1 Angles ALEKS

angle measure
radians
arclength
areas of a sector

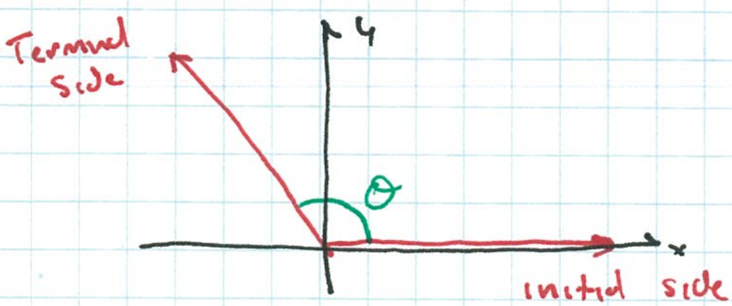
Def: An angle is formed by rotating a ray about an endpoint. The angle is the amount the ray is rotated.

wot?



1 Start w/ a "ray" - the endpoint is P.
2 rotate the ray around P.
3 It ends up at the terminal side.
The angle is a measure of the space between them.

Note: The angle is in the standard position if the vertex of the ray is at the origin, and the initial side is along the x-axis.



The measure of the angle is a quantity that indicates the magnitude of the rotation.

Yule! How?

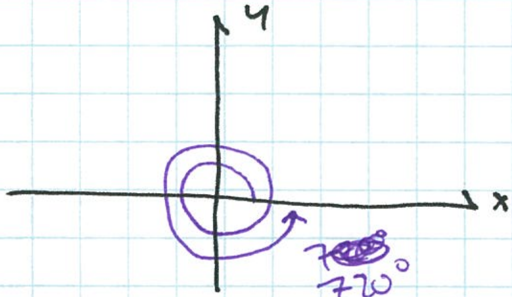
We are talking in circles here. ← That's it. That's how we get out of this double talk!

Suppose one complete rotation is 100% of a 1. wat?

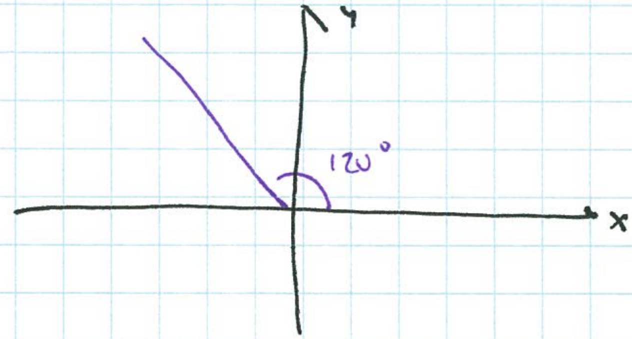
⇒ If 1 rotation = 360° deg.

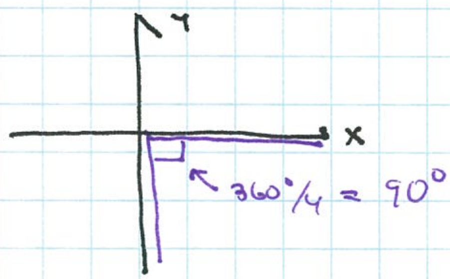
then 2 rotations is 720°.

1/3 of a rot. is $\frac{360°}{3} = 120°$.



(Initial & Terminal side overlap.)



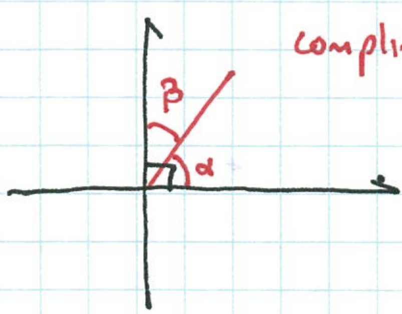


note: I assume that counter clockwise is the pos. direction for a rotation.

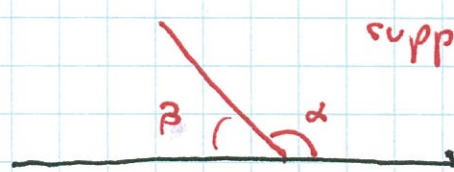
(This makes it consistent w/ right hand rule in physics.)

<u>Def:</u> If the sum of two angles is 90° the two angles are <u>complimentary</u>

<u>Def:</u> If the sum of two angles is 180° the two angles are <u>supplimentary</u>.



complimentary.

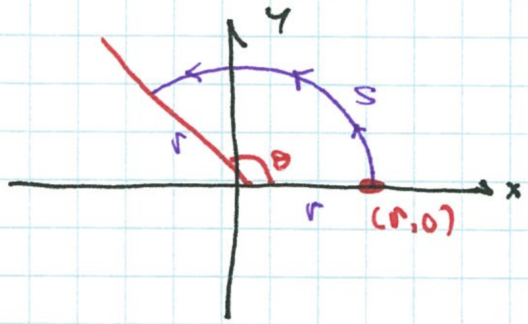


supplementary.

Wait, why 360°? That makes no sense. Is this some kind of idolatry? yes. yes it is.

Actually, they used 360° back in the good old days cuz it was easily divisible by a lot of numbers. We now have calculators. We can do better!

– The idea – Geometry is good! – let's keep things physical.

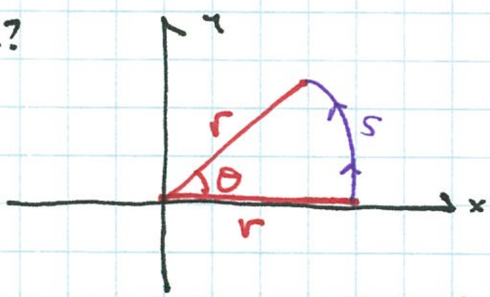


suppose an ant starts at $(r,0)$ and walks around the edge of a circle of radius r centered at the origin.

- If you double the ant's dist. the angle doubles.
- If you halve the ant's dist the angle is cut in half.

⇒ The arclength of the sector of radius r and angle θ is proportional to the angle θ.

wot?



a sector is a slice of a circle w/ radius r and angle θ.

s is the arclength + " the length of the outside edge.

So, s is prop. to θ.

$\Rightarrow \quad s = k\theta$ where k is a constant.

wait a second, a bunch of dead greek guys say that the circumference of a circle is $2\pi r$.

arclength of a full circle.

Oy! so if angle $\gamma = 100\%$ of a circle

the circ. $= k \cdot \gamma$

If $r=1$ and $\gamma = 360°$ then

$$2\pi = k \cdot 360 \implies k = \frac{2\pi}{360} = \frac{\pi}{180}.$$

yuk! Let's do like the metric people do, and redefine our units. We will figure out a set of units that will make $k=1$. I like one.

Do it backwards. If we want $k=1$ then

$$2\pi = 1 \cdot \gamma \quad \text{or} \quad \gamma = 2\pi.$$

not just Greeks did this. Persians and Chinese were in on the fun.

Def: Radian angle measure is the units where

$$s = \theta \qquad \text{if} \quad r = 1.$$

In general, the circ. of a circle is

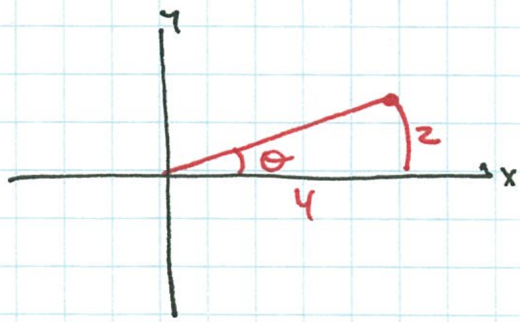
$$2\pi r = k\,\gamma \qquad \text{then} \quad k = r.$$

and...

$$s = r\theta \quad \text{or} \quad \boxed{\theta = s/r}$$

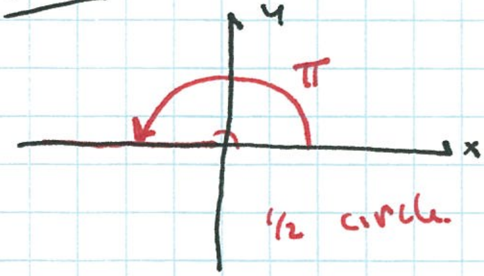
Better def: The radian measure of a sector is the arclength of the sector divided by the radius.

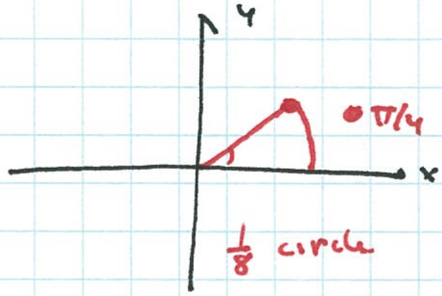
ex/ A sector has radius 4m and the arclength is 2m. what is the angle

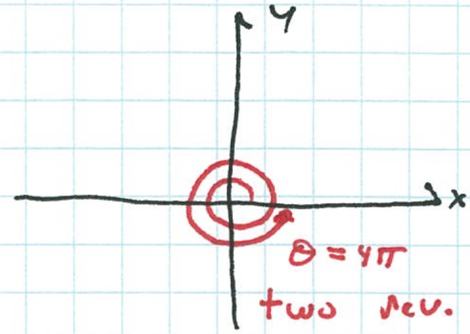


$$\theta = 2/4 = 1/2.$$

Note

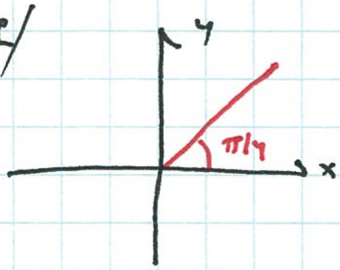




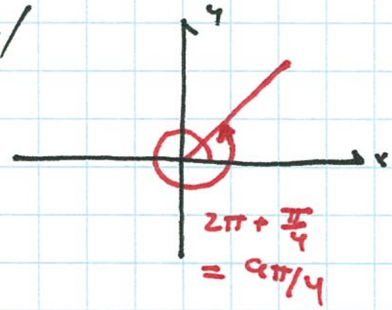


uh oh... two ways to get to the x axis - actually, there are more!

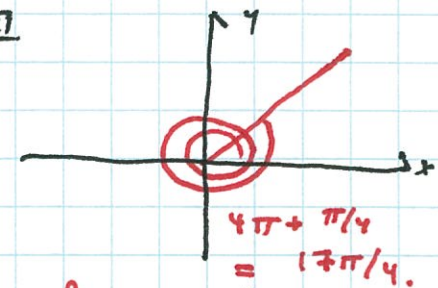
I/



II/



III



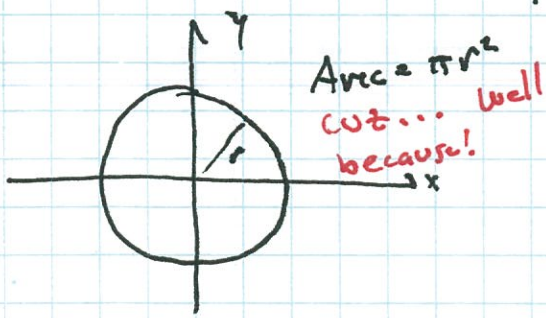
inf. ways!

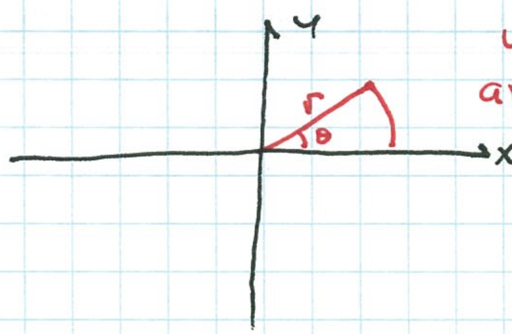
Def: If two angles have the same initial side and same terminal side we say that they are coterminal.

ex: $\frac{\pi}{4}$ and $\frac{9\pi}{4}$ above are coterminal.

Read p. 454 about angle speed. we won't spend much time in class on that.

Okay, then. We ~~are~~ are now angle experts.
what about ~~areas~~ areas of sectors?
That is where the pie is!





what is the area ~~angle~~ of this?

(assume angle is in radians)

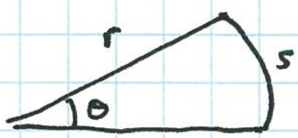
$$\text{Area} = \underbrace{(\text{\% of circle})}_{\theta/2\pi} * \pi r^2$$

assume $0 \le \theta \le 2\pi$

$$\boxed{\begin{aligned} \text{Area} &= \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{1}{2}\theta r^2 \\ \text{arclength} &= r\theta \end{aligned}}$$

if θ in radians.

ex/ A pie slice has a crust length of 8cm. The area of the slice is 4 cm². What is the angle?



$s = r\theta$
$A = \frac{1}{2}\theta r^2$ $\Rightarrow$ (A) $8 = r\theta$
(B) $4 = \frac{1}{2}\theta r^2$ } 2 eqn 2 unknown

— solve for r + subst —

(A) $r = 8/\theta$

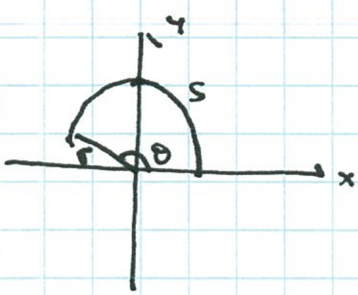
(B) $4 = \frac{1}{2}\theta\,(8/\theta)^2 = \frac{1}{2}\theta \cdot \frac{64}{\theta^2} = 32/\theta$

$\theta = 32/8 = 4 \cdot \underline{\text{rad}}$

That's a big slice of pie!

yum!

ex/ A turtle walks around a circle of radius 3m. It walks a distance of 6m. What angle did it circumscribe?



$s = r\theta$
$6 = 3\theta$
$\underline{\theta = 2 \text{ rad.}}$

(units matter!)

Okay, this is wonderful. What if my loser roomate wants everything in degrees? Then what?

(I) Get a new roomate * ⟶ * Best option.

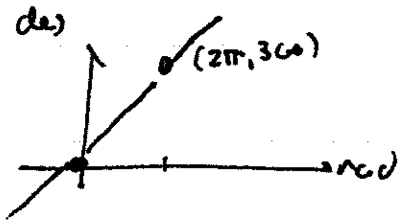
(II) Convert the units!

i) give roomate long lecture on why only losers use degrees.

ii) recognize that rad. to deg. is a linear fcn.

$0^\circ = 0 \text{ rad}$ $\quad (0,0)$

$360^\circ = 2\pi \text{ rad}$ $\quad$ ~~(360, 2π)~~ $(2\pi, 360)$



$$\text{slope} = \frac{360-0}{2\pi-0} = 180/\pi$$

$$\text{Deg} - 0 = \frac{180}{\pi}(\text{rad}-0)$$

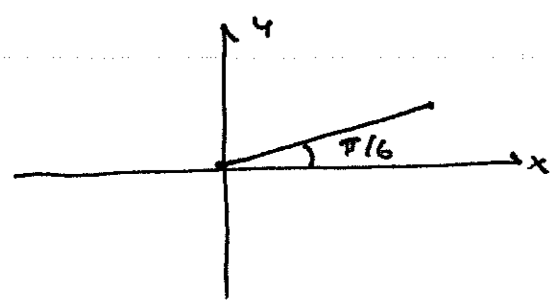
$$\text{deg} = 180/\pi \cdot \text{rad}.$$

also $\quad \text{rad} = \pi/180 \cdot \text{deg}.$

ex/ An angle is measured to be 30°. What is it in radians?

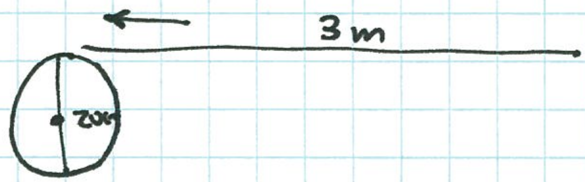
$$\text{Rad} = \frac{\pi}{180} \cdot \text{deg}.$$

$$= \frac{\pi}{180} \cdot 30 = \pi/6 \text{ rad}.$$



Time permitting

ex/ A winch on a sailboat has a diameter of 20cm. It will be used to take up 3m of slack. What angle must you turn it?



20 cm = .2 m

radius = .1 m

$s = 3$

$s = r\theta$

plug in vals for $\overset{r,s}{\cancel{s,\theta}}$ & get θ

$\Rightarrow \quad 3 = .1\,\theta$

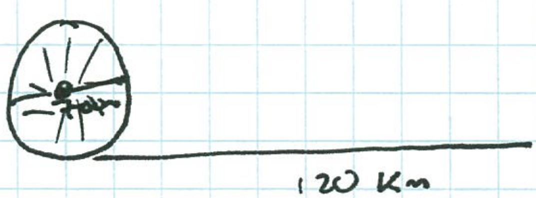
$\Rightarrow \theta = 30$ rad. wot? how?

well $\#$ revolutions $= \frac{30}{2\pi} = {}^{15}/_{\pi} \approx 4.77$ revolutions

so turn roughly 4 & 3/4 times.

Time permitting

ex/ A bike wheel has a diameter of 700mm. If you ride it 120km how many revolutions did it make?



Dia = 700mm = .7 m

Dist = 120 km = 120,000m.

$s = r\theta$

$r = .35$ m

$120{,}000 = .35 \cdot \theta$

$\theta = \frac{120{,}000}{.35}$ rad.

There are 2π rad per rev.

$\#$ rev. $= \frac{120{,}000/.35}{2\pi} \approx$ ~~[illegible]~~ 54,567 rev.