Section 3.5 ALEKS.

Solve exponential equation
Solve log equation
develop equation from written description

Note: Algebra still matters:

- one step at a time
- as you do onto the left do onto the right
- multiply by 1
- add zero.
- If confused - do a substitution ($u = \#$)
- If $f(x)$ is a fcn ~~any of~~ and $g(x)$ is its inverse then
$$f(g(x)) = x$$
$$g(f(x)) = x$$
- $\exp(x) = e^x$ and its inv. is $\ln(x)$
- Given an equation circle what you want and try to get it one side 1st. Then figure how to isolate it.

ex/ Solve for x ← what I want

$$3 \cdot 15^{x} = 10$$

$$15^x = 10/3 \qquad \text{div. by 3}$$

$$\ln(15^x) = \ln(10/3) \qquad \text{to get to exponents take the log.}$$

$$x \ln(15) = \ln(10/3) \qquad \text{property of logs.}$$

$$x = \frac{\ln(10/3)}{\ln(15)} \qquad \text{divide by } \ln(15).$$

Know when to stop!

go back and check work!

everything is in exponents! logs will be used!

ex/ $3 \cdot 15^{x} = 4^{x}$

$15^x / 4^x = \frac{1}{3}$ — div. by 4^x and 3

$\ln(15^x/4^x) = \ln(1/3)$ — get things out of exp. by logging it all

$\ln(15^x) - \ln(4^x) = \ln(1/3)$ — prop. of logs.

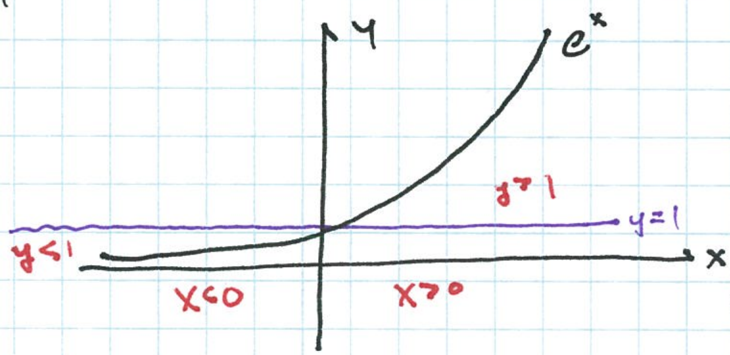
$x\ln(15) - x\ln(4) = \ln(1/3)$ — prop. of logs.

$x[\ln(15) - \ln(4)] = \ln(1/3)$ — fact the x

$x = \dfrac{\ln(1/3)}{\ln(15) - \ln(4)}$ — div. by $\ln(15) - \ln(4)$

go back and check work.

ex/



Things to keep in mind:

if $x < 0$ $\quad 0 < e^x < 1$

if $x > 0$ $\quad \cancel{1 < e^x < \infty}$ $\quad e^x > 1$

$\Rightarrow$

$\ln(y) < 0$ if $0 < y < 1$

$\ln(y) > 0$ if $y > 1$

$\ln(1) = 0$.

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so dom e^x is $(-\infty, \infty)$

range e^x is $(0, \infty)$

dom $\ln(x)$ is $(0, \infty)$

range $\ln(x)$ is $(-\infty, \infty)$

ex/ $e^{2x} + e^{x} - 6 = 0$ um... what the ~~heck~~ heck ??!?

let $u = e^x$ then $e^{2x} = (e^x)^2 = u^2$
solve for x later.

$$u^2 + u - 6 = 0$$

$$u = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2} = 2, -3$$

so $\underbrace{e^x = -3}$ or $e^x = 2$ possible!

Nope! not possible.

so $\ln e^x = \ln(2)$

$$\underline{x = \ln(2)}$$

ex/ $\log_3(2x) = 5$ inside logs. exponents will be used.

what if base was diff?

$$3^{\log_3(2x)} = 3^5$$

$$2x = 3^5 \Rightarrow x = \frac{1}{2}(3^5)$$

ex/ $\log_3(2x) = \log_5(x)$ see comment above. Convert to ln & then start over.

ugh.... let $u = \log_3(2x)$

$$\Rightarrow 3^u = 2x$$

$$\ln(3^u) = \ln(2x)$$

$$u \ln(3) = \ln(2x) \Rightarrow u = \frac{\ln(2x)}{3}$$

also let $v = \log_5(x)$

$$5^v = x$$

$$\ln(5^v) = \ln(x)$$

$$v \ln(5) = \ln(x) \Rightarrow v = \frac{\ln(x)}{\ln(5)}$$

so plug back in to get

$$\frac{\ln(2x)}{\ln(2)} = \frac{\ln(x)}{\ln(5)}$$

$$\ln(2x) = \frac{\ln(2)}{\ln(5)} \cdot \ln(x)$$

$$e^{\ln(2x)} = e^{\ln(2)/\ln(5) \cdot \ln(x)}$$

(cont.)

$$2x = \left(e^{\ln(x)}\right)^{\ln(2)/\ln(5)}$$

$$2x = (x)^{\ln(2)/\ln(5)}$$

$$2 = x^{\ln(2)/\ln(5) - 1}$$

$$\ln(2) = \ln\left\{x^{\ln(2)/\ln(3) - 1}\right\}$$

$$\ln(2) = \left(\ln(2)/\ln(5) - 1\right)\ln(x)$$

$$\ln(x) = \frac{\ln(2)}{\ln(2)/\ln(5) - 1}$$

$$x = e^{(\ln(2))/(\ln(2)/\ln(5) - 1)}$$

<u>Yuk!</u> :o

ex/ $14 \cdot 7^{x} = 33 \cdot 17^{x}$ all in exp. we need loss!

we need a palate cleanser.

you just log it all.

$$\ln(14 \cdot 7^x) = \ln(33 \cdot 17^x)$$

$$\ln(14) + \ln(7^x) = \ln(33) + \ln(17^x)$$

$$\ln(14) + x\ln(7) = \ln(33) + x\ln(17)$$

$$x\ln(7) - x\ln(17) = \ln(33) - \ln(14)$$

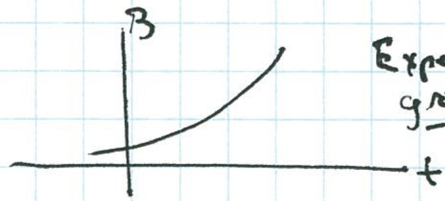
$$x\left[\ln(7) - \ln(17)\right] = \ln(33) - \ln(14)$$

$$x = \frac{\ln(33) - \ln(\cancel{17}\,14)}{\ln(7) - \ln(17)}$$

no pos.
~~neg. #?~~
~~ok.~~

ex/ I open an account that has 1.2% annual int. compounded monthly. How long till the balance doubles?

0) Read.

① 

Expect growth

② P = principle.

$n = 12$

$r = .012$

$t = ?$

$P_0 = ?$

③ $$P = P_0\left(1 + \frac{.012}{12}\right)^{12t}$$

want P wn $P(t) = 2P(0)$

$P(0) = P_0$

④ set $P = 2P_0$ and solve for t. (Hope P_0 does not matter.)

⑤ $$2P_0 = P_0\left(1 + .012/12\right)^{12t}$$

woot!

$$2 = \left(1 + .012/12\right)^{12t}$$

$$\ln(2) = \ln\left(\left(1 + \frac{.012}{12}\right)^{12t}\right) = 12t\ln(1 + .012/12)$$

$$t = \frac{\ln(2)}{12\ln(1 + .012/12)} \approx 57.8 \text{ years.}$$

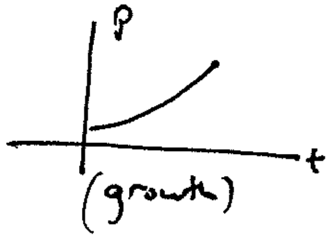
That kinda sucks :(

⑥ check work.

ex/ I want to open an account and have the principle triple in 50 years. The int. will be compounded weekly. What should the int. rate be?

0) Read

1)



(2) B = balance

B_0 = initial balance

$r = ?$

$n = 52$

$t = 50$

(3) $B = B_0 (1 + r/52)^{52 \cdot 50}$

$B(0) = B_0$

$B(50) = 2B_0$.

(4) solve $B(50) = \cancel{2}\,3 B_0$ for r.

(5) $\cancel{2}\,3 B_0 = B_0 (1 + r/52)^{52 \cdot 50}$

$3\,\cancel{2} = (1 + r/52)^{52 \cdot 50}$

$\ln \cancel{2}\,3 = \ln\left((1 + r/52)^{52 \cdot 50}\right) = 52 \cdot 50 \ln(1 + r/52)$

$\frac{1}{50 \cdot 52} \cdot \ln(3) = \ln(1 + r/52)$

$e^{\frac{\ln(3)}{50 \cdot 52}} \cancel{2} = 1 + r/52$

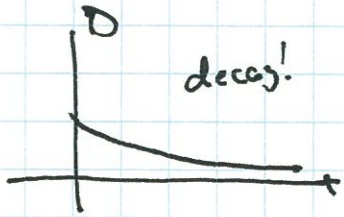
$r = 52\left[\cancel{2}\, e^{\ln(3)/50 \cdot 52} - 1\right] \approx .0220$

or 2.20 %.

Time Permitting

ex/ 30 mg of a drug is administered to a patient. The amount in the bloodstream follows an exponential function. After 4 hours there is 10mg in the patient's blood stream. How much will be in the PT's blood stream in 7 hours?

(0) Read (1)



(2) $B(t)$ = amount in blood str
t = time (hours)

$B(0) = 30$
$B(\cancel{8}\,4) = 10$ $\qquad B(7) = ?$

(3) $B(t) = Ae^{-rt}$ $\qquad r > 0.$

$$\left.\begin{aligned} 30 &= Ae^{-0} \\ 10 &= Ae^{-4r} \end{aligned}\right\} \begin{array}{l} 2 \text{ eqn's} \\ 2 \text{ unknowns.} \end{array}$$

(4) Solve for A, r
plug in to $B(7)$.

(5) $30 = Ae^{0} = A \Rightarrow A = 30$

$$10 = 30e^{-4r}$$

$$\frac{1}{3} = e^{-4r}$$

$$\ln(1/3) = \ln(e^{-4r}) = -4r$$

$$r = -\frac{1}{4}\ln(1/3) \quad \underline{>0!}$$

$$B(t) = 30e^{+t/4\,\ln(1/3)} \qquad -\left(-\tfrac{1}{4}\ln(1/3)\right)$$

$$B(7) = 30e^{+7/4\,\ln(1/3)} \approx 4.39 \text{ hours.}$$

Time Permitting

ex/ #84 in book p.419

A company spends x hundred dollars for advertising.

Sales $= S(x) = 5 + 7\ln(x+1)$ in thousands of \$

if sales = 19,100\$ what was the adv. budget?

$$\cancel{19{,}000}\ 19 = 5 + 7\ln(x+1)$$

$$14\ \cancel{18{,}995} = 7\ln(x+1)$$

$$2\ \cancel{18{,}995/7} = \ln(x+1) \Rightarrow e^{\cancel{18{,}995/7}\ 2} = x+1$$

$$x = e^{\cancel{18{,}995/7}} - 1 \approx e^{2} - 1 \approx 6.39\$$$

ex/ Time permitting

$Sam(x) = \ln(7x+1) - 5$ det. the inv. of sam.

$y = \ln(7x+1) - 5$

$y+5 = \ln(7x+1)$

$e^{y+5} = 7x+1$

$x = \dfrac{e^{y+5}-1}{7} = Diane(y)$

Domain of $sam(x)$ $(-\frac{1}{7}, \infty)$ Dom of Diane: $(-\infty, \infty)$

range of $sam(x)$ $= (-\infty, \infty)$ range of Diane $(-\frac{1}{7}, \infty)$

ex/ Time Permitting

$Tammy(x) = e^{2x+1} + 3$ det. the inv.

$y = e^{2x+1} + 3$

$y - 3 = e^{2x+1}$

$\ln(y-3) = 2x+1 \Rightarrow x = \dfrac{\ln(y-3) - 1}{2}$

dom. of Tammy : $(-\infty, \infty)$ | dom. inv: $(3, \infty)$

range of Tammy : $(3, \infty)$ | range of inv: $(-\infty, \infty)$

ex/ Time Permitting

solve $3 = \dfrac{10}{1 + 4e^{-2x}}$ for x.

$3(1 + 4e^{-2x}) = 10$

$1 + 4e^{-2x} = 10/3$

$4e^{-2x} = 10/3 - 1 = 7/3$

$e^{-2x} = 7/12$

$\ln(e^{-2x}) = \ln(7/12)$

$-2x = \ln(7/12)$

$x = -\frac{1}{2}\ln(7/12) \approx .270$