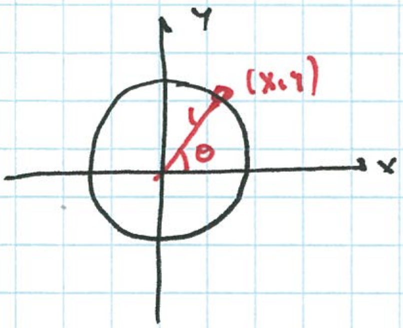
Sect 4.5 ALEKS

sin(θ) w/ cos(θ) as functions.
shifting and scaling functions.

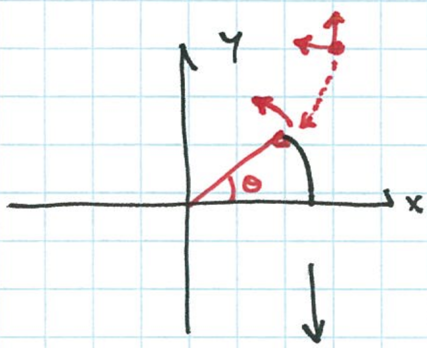


what happens to the value of cos(θ) and sin(θ) as θ changes?

Recall: if $r=1$ then

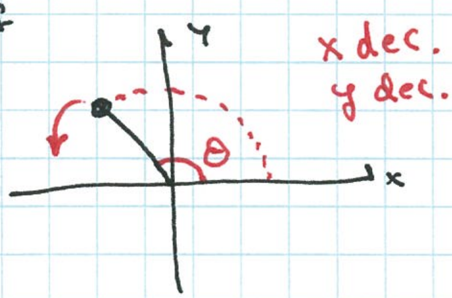
$$\left.\begin{aligned} \cos(\theta) &= x \\ \sin(\theta) &= y \end{aligned}\right\} \quad \begin{aligned} \cos(0) &= 1 \\ \sin(0) &= 0. \end{aligned}$$

We start @ (1,0). As θ gets bigger
x decrease w/ y increase.

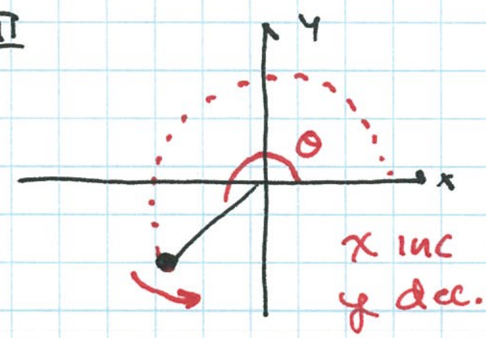


(Quad I)

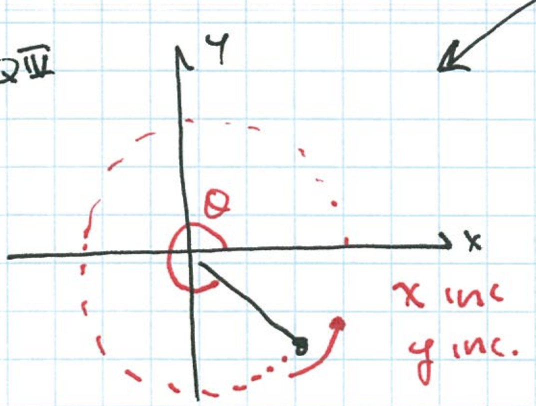
QII



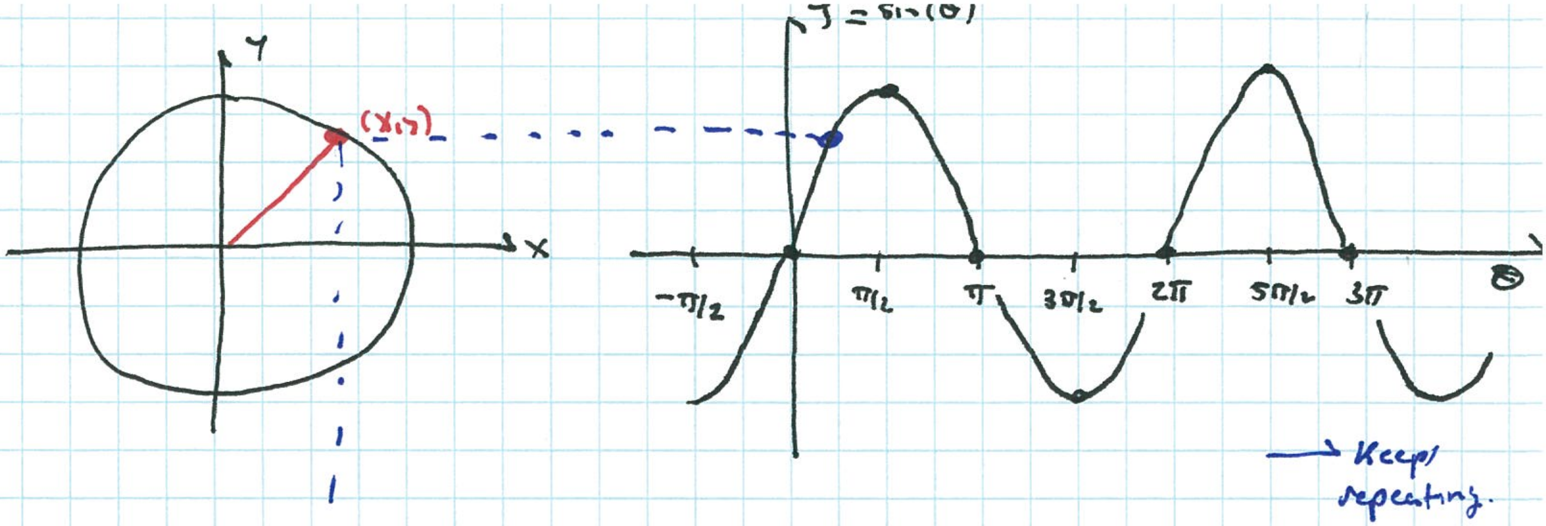
QIII

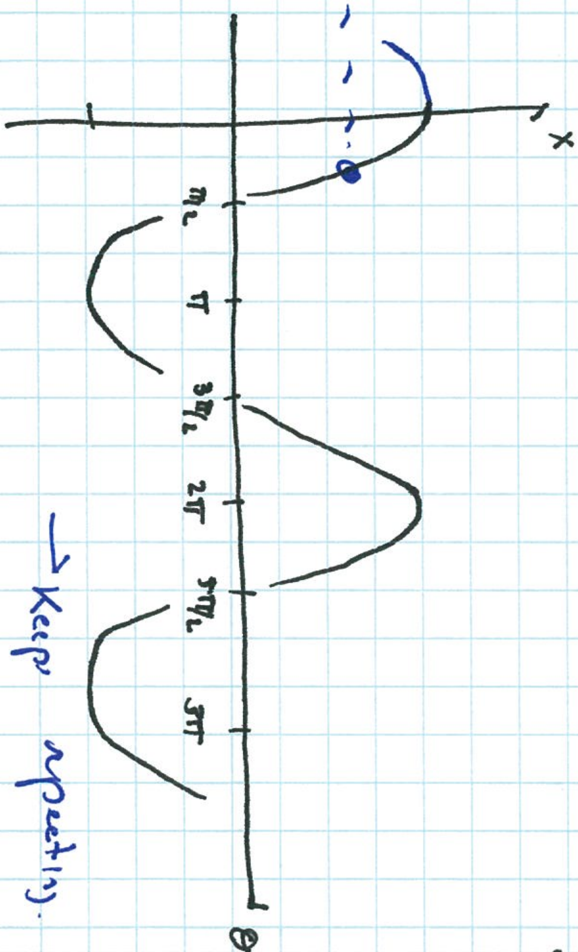


QIV



if $\theta > 2\pi$ then whole thing <u>repeats</u>



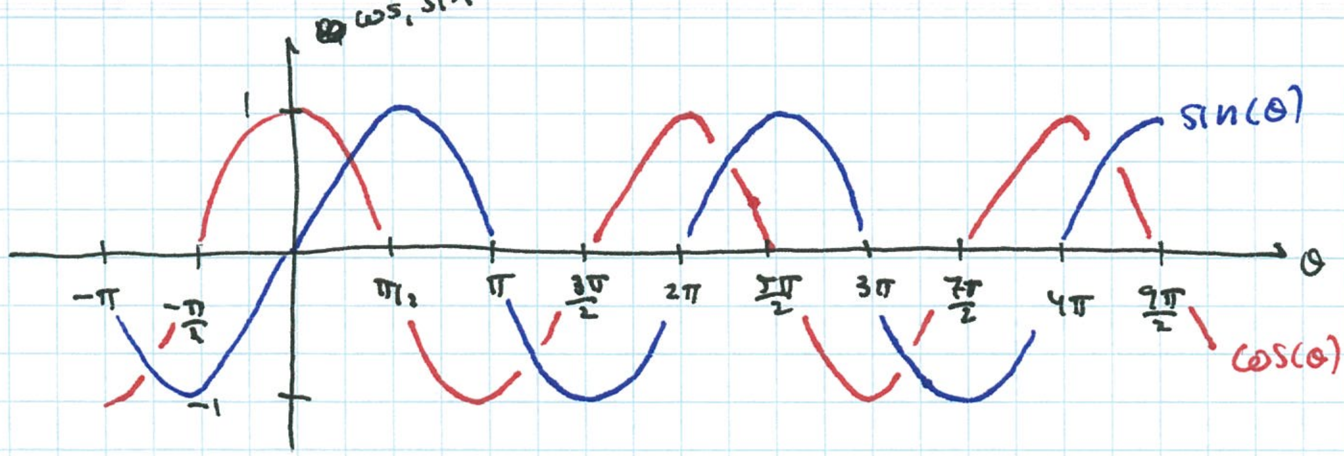


- what if θ neg?
  repeats in neg. direction

- both functions repeat every 2π radians.

- <u>Both</u> graphs are important! you need to know them!

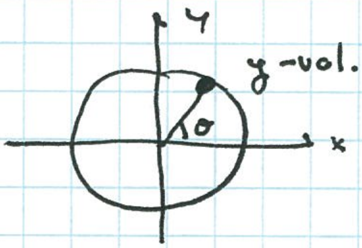
$\sin(\theta)$	$\cos(\theta)$
domain: $(-\infty,\infty)$	domain: $(-\infty,\infty)$
range: $[-1,1]$	range: $[-1,1]$



Also note that

$$\cos(\theta) = \sin(\theta + \pi/2) = -\sin(\theta - \pi/2)$$
$$\sin(\theta) = \cos(\theta - \pi/2) = -\cos(\theta + \pi/2)$$

ex/ $\sin(\theta)$



$\sin(\theta)$		$\cos(\theta)$	
$-\pi/2 < \theta < \pi/2$	inc.	$-\pi < \theta < 0$	inc
$\pi/2 < \theta < 3\pi/2$	dec	$0 < \theta < \pi$	dec
$\frac{3\pi}{2} < \theta < \frac{5\pi}{2}$	inc.	$\pi < \theta < 2\pi$	inc.
⋮		⋮	

wait a second... This just keeps doing the same things over and over (Like my life)

note/ spring mass / tides / my mood swings — things that "cycle"

These are good functions to approximate situations that have some periodic aspect.

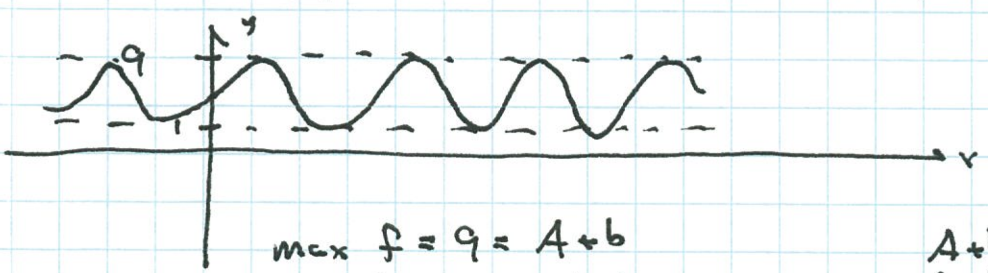
Problem: – Time to repeat things is not always 2π.
– Range of values is not always -1 to 1.

If only there was some way to shift / scale a function.

wait a second!

If $f(\theta) = A\sin(\theta) + b$ ← vert. shift.
↑ vert scale

ex/ I want a cosine fcn. to oscillate between 9 and 1. What is the function?



$f(\theta) = A\cos(\theta) + b$
max when $\cos(\theta) = 1$
min when $\cos(\theta) = -1$.

max $f = 9 = A + b$
min $f = 1 = -A + b$

$$\begin{aligned} A + b &= 9 \\ -A + b &= 1 \quad \text{(add)} \\ \hline 2b &= 10 \end{aligned} \qquad b = 5$$

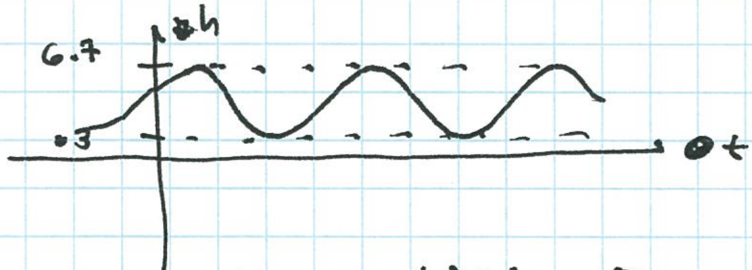
$\Rightarrow A = 9 - b = 4.$

$f(\theta) = 4\cos(\theta) + 5$ (check)

D8: A is called the "amplitude"

warning: it is 1/2 of what you think it should be.

ex/ The tides at St. Simon's Island oscillate between 0.3 ft and 6.7 ft from the mean lower water limit. Express the tide as a sine wave.



$$\text{Tide}(t) = A\sin(\overset{?}{\cdot}) + b$$

max when $\sin(\cdot) = 1 \Rightarrow$ max Tide $= 6.7 = A + b$,

min when $\sin(\cdot) = -1 \Rightarrow$ min Tide $= .3 = -A + b$.

(add)
$$\begin{array}{r} A + b = 6.7 \\ -A + b = .3 \\ \hline 2b = 7.0 \end{array} \Rightarrow b = 3.5 \text{ ft}$$

$$A = 6.7 - 3.5 = 3.2 \text{ ft.}$$

$$\text{Tide}(t) = \underline{3.2}\sin(\underline{?}) + 3.5$$

amplitude — horizontal scale / shift!

So... what happens on the inside? — horizontal change!

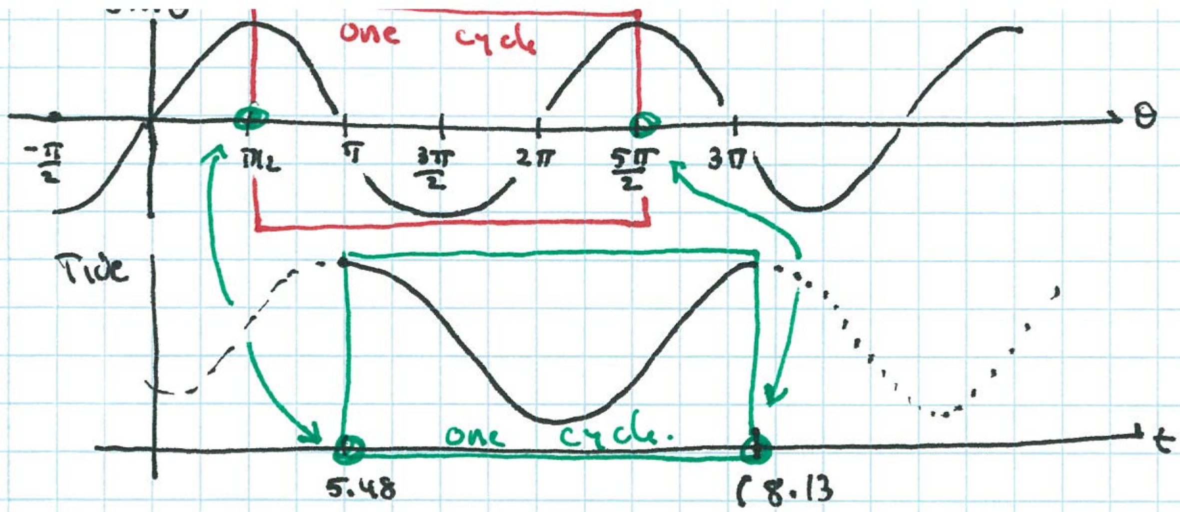
According to www.tides4fishing.com on 4 July high tide was @ 5:29am and ~~low tide was @~~ the next high tide was @ 6:08 pm.

Let's call midnight to be 0 hours.

$$\Rightarrow \quad 5{:}29\text{ am} = 5 + \frac{29}{60} \approx 5.48 \text{ hours.}$$

$$6{:}08\text{ pm} = 6 + 12 + 8/60 \approx 18.13 \text{ hours.}$$

Now – how to shift the sine wave to make this work!



so $\text{Tide}(t) = 3.2\sin(\underbrace{mt+c}_{\text{a line - call it } L(t)}) + 3.5$

The line goes through the points $(5.48, \pi/2)$ and $(18.13, 5\pi/2)$.

$$m = \frac{5\pi/2 - \pi/2}{18.13 - 5.48} = 2\pi/12.65$$

$$L(t) - \pi/2 = \frac{2\pi}{12.65}(t - 5.48)$$

so $\text{Tide}(t) = 3.2\sin\left(\frac{2\pi}{12.65}(t-5.48) + \pi/2\right) + 3.5$ phew.

ex/ Elvis is a cosine ~~wave~~. wave. It repeats every 6 seconds, and it oscillates between 3 and -1. what is the function? also it has a min @ 3 seconds.

$$\text{Elvis}(t) = A\sin(mt+c) + b$$

Do outside first:

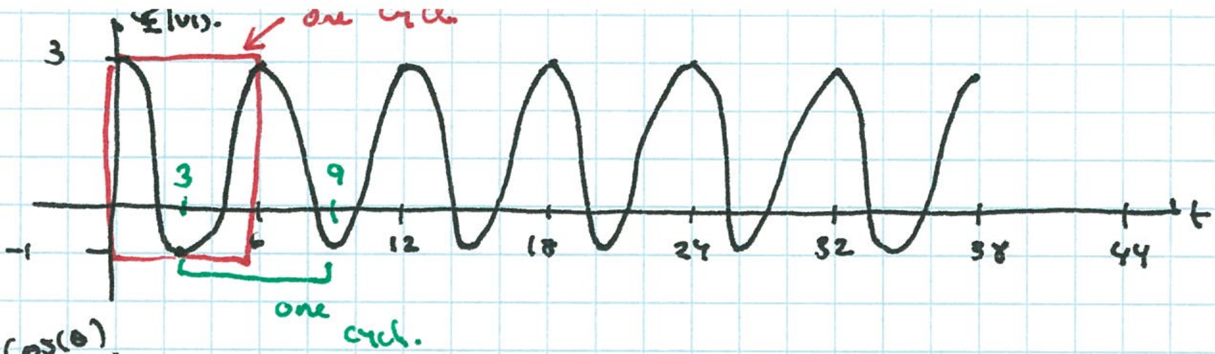
max: $3 = A + b$

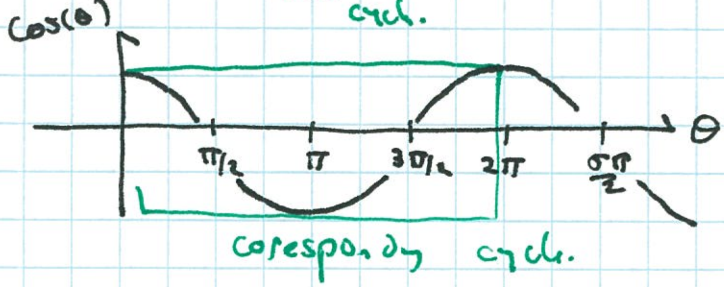
min: $-1 = -A + b$

add $+2 = 2b$ $\qquad b = +1$

$A = 3 - b = 2$

Amplitude $= 2$.





L(t) is a line through (0,0) and (6, 2π)

$$m = \frac{2\pi - 0}{6 - 0} = \pi/3$$

$$L(t) - 0 = \pi/3\,(t - 0)$$

$$E(t) = 2\cos(\pi/3\, t) + 1$$

Time permitting

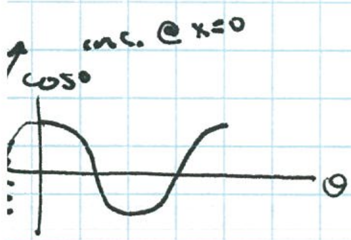
ex/ Plot the fcn. Marie (x) $= 5\cos(\pi x - \pi/2) + 2$.

max: $5 + 2 = 7$

min: $-5 + 2 = -3$

center @ $0 + 2 = 2$.

Marie (0) $= 5\cos(-\pi/2) + 2 = 0 + 2 = 2$.



x=0 arg to cos. is $-\pi/2$

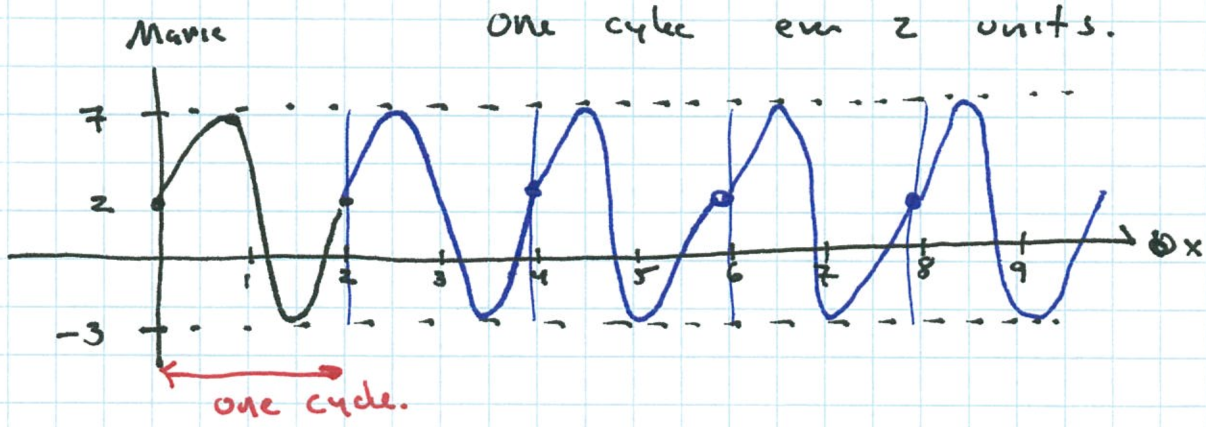
what is x after 1 cycle?

$$-\pi/2 + 2\pi = 3\pi/2$$

$$3\pi/2 = \pi x - \pi/2$$

$$2\pi = \pi x \Rightarrow x = 2.$$

One cycle every 2 units.



Note: $f(t) = A\sin(mt+c)+b$

$\sin(\cdot)$ goes through one cycle as $(mt+c)$ goes from 0 to 2π. what value of T will do that? (we call this the period.)

subtract

$$mt+C=0$$
$$m(t+T)+c=2\pi$$

$$mt+c=0$$
$$mt+mT+c=2\pi$$
$$-mT=-2\pi$$

$\Rightarrow$ $T = 2\pi/m$ = Period of oscillation.

Time permitting

ex/ A person's heart beats @ 60 beats/min. The rate of blood flow oscillates between 8 l/min and 0 l/min. At the start of monitoring the flow rate is @ a min. Det. the formula for the flow rate.

$$V(t) = A\cos(mt+c)+b$$

Period = 1/60 min.
Amplitude = 4 l/min.

$$A+b=8$$
$$-A+b=0$$
$$2b=8$$

$b=4 \quad A=8-4=4.$

min cos!

as t goes from 0 to 1/60 θ goes from π to 3π.

$(0,\pi)$ to $(1/60, 3\pi)$

$$m = \frac{3\pi-\pi}{1/60-0} = 120\pi$$

$$L(t)-\pi = 120\pi(t-0)$$

Flow rate = $4\cos(120\pi t+\pi)+4$