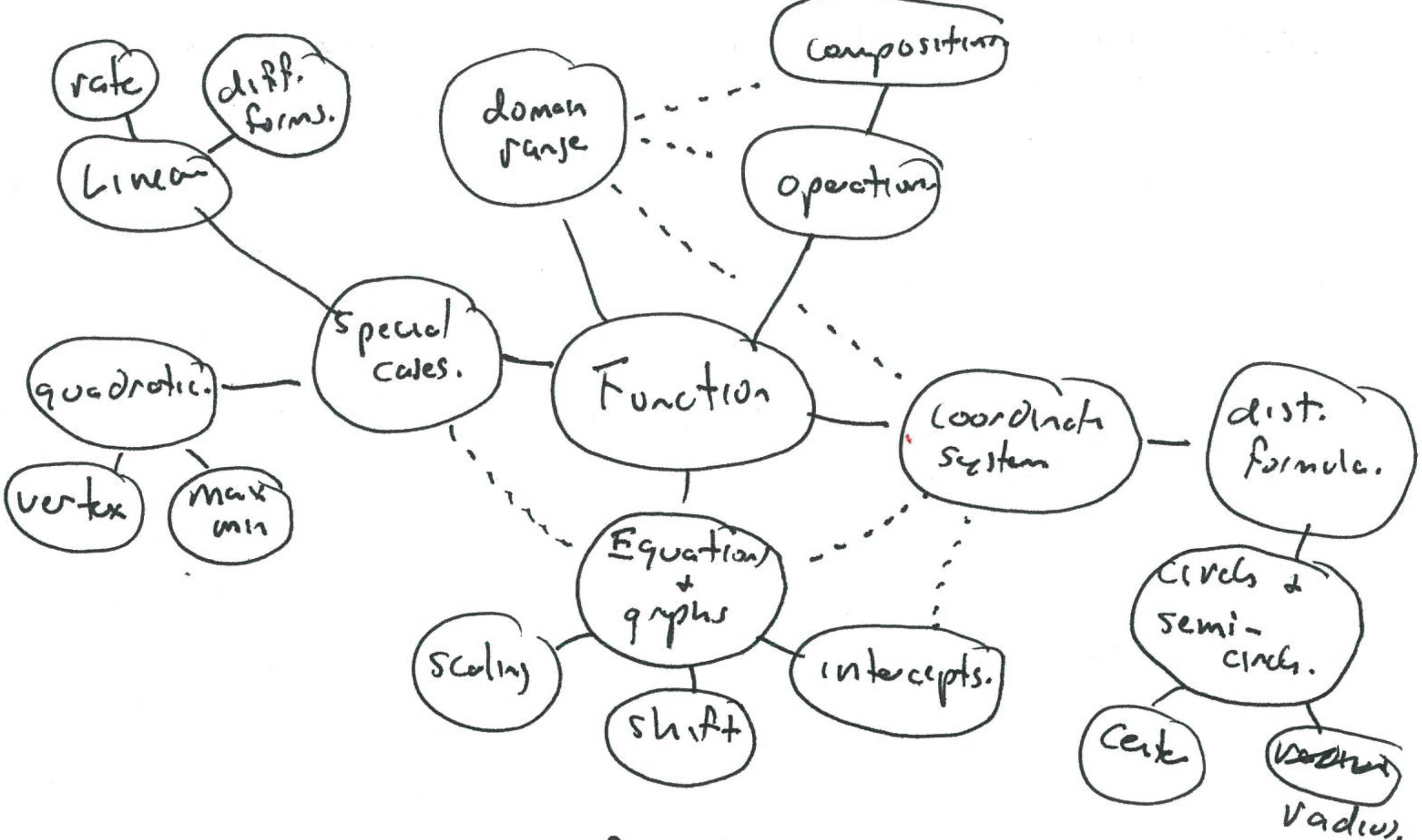
22 June Review for test.

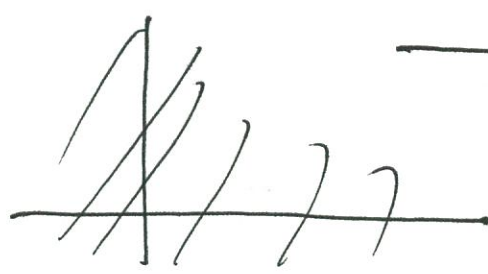


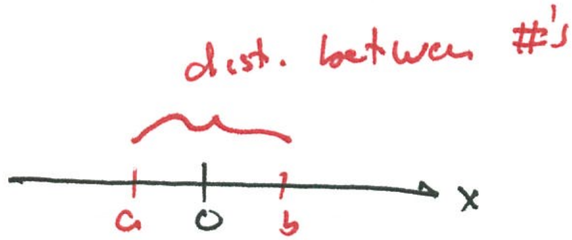
Central idea is function.

idea – given one input
– you get one output.

In calculus we want to quantify "change"
⇒ what is change?

on the number line





dist = $|b-a|$

For a functn

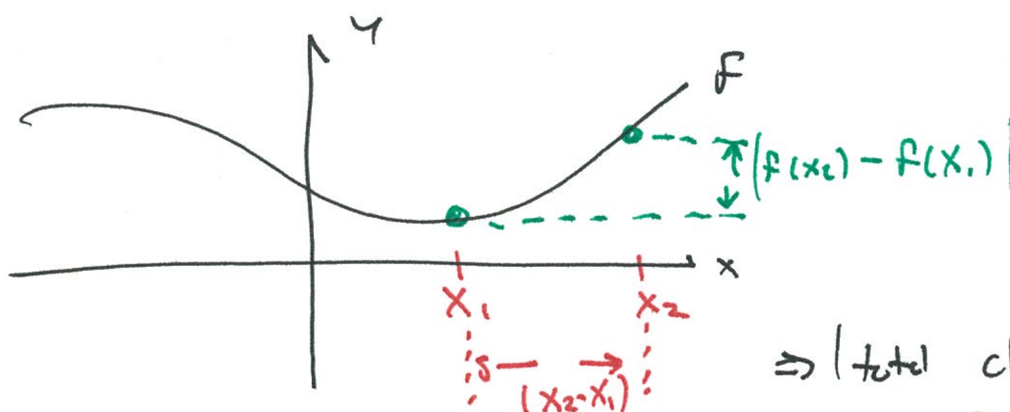
we have input $\underbrace{X_1, X_2}$

change betwn is $(X_2 - X_1)$

output $\underbrace{f(X_1), f(X_2)}$

change betwn is $|f(X_2) - f(X_1)|$

How do we represent these two related things? The coordinate plane.

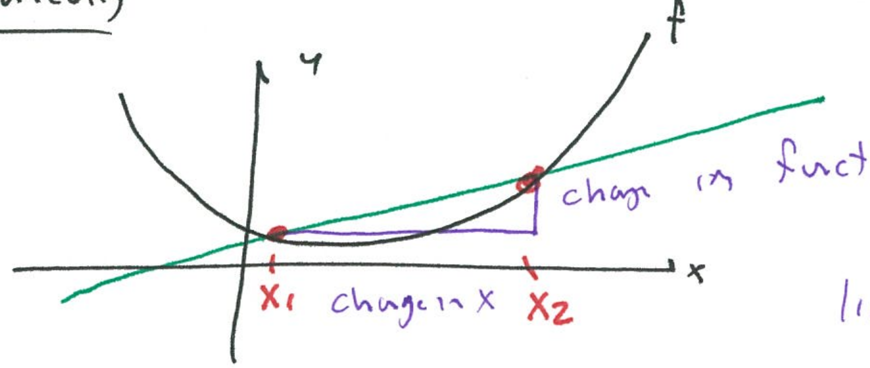


$\Rightarrow$ |total change| $= |f(X_2) - f(X_1)|$

What about "relative change"?

$$\text{average rate of change} = \frac{\text{total change in funct.}}{\text{change in inputs.}}$$ can be neg!

graphically



slope of secant line = $\dfrac{\text{change in func.}}{\text{chge in x.}}$

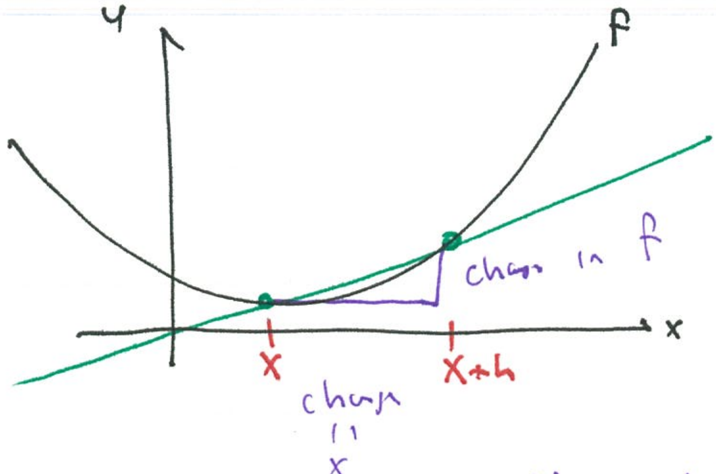
So Avg. rate of change from x_1 to x_2 is slope of the secant line ~~between~~ through two points on the graph of a function.

$\Rightarrow$ Avg. rate of change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

(if neg the fcn. dec. overall ~~...~~)

if pos. the fcn. inc. overall)

This leads to the def. of the diff. quotient – a general form of the average rate of change.



Diff Quotient $= \dfrac{f(x+h) - f(x)}{(x+h) - x}$

This is a fcn. of x!

ex/ det the diff. quotient for $f(x) = 4x^2 - x$.

$$\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{\left(4(x+h)^2 - (x+h)\right) - (4x^2 - x)}{h}$$

$$= \frac{4(x^2 + 2hx + h^2) - x - h - 4x^2 + x}{h}$$

$$= \frac{\cancel{4x^2} + 8hx + 4h^2 - \cancel{x} - h - \cancel{4x^2} + \cancel{x}}{h}$$

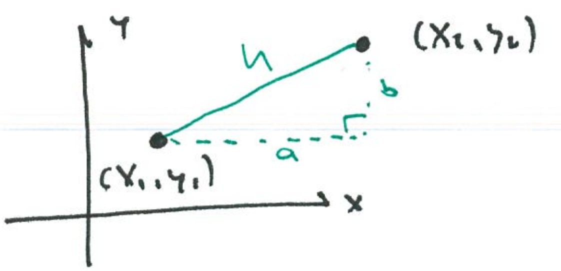
$$= \frac{8hx + 4h^2 - h}{h} = \cancel{\frac{h(8x + 4h + 1)}{h}}$$

$$= \frac{h(8x + 4h - 1)}{h} = \underline{\underline{8x + 4h - 1}}.$$

Whoa we did all that?! yes. it is a lot.

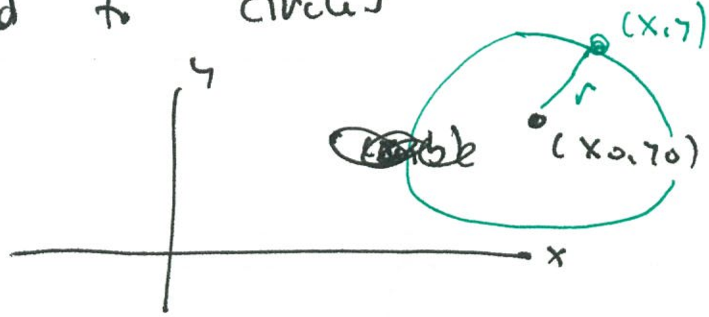
we needed some intermediate stuff. :(

like distance between points



$$h^2 = a^2 + b^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

which led to circles



$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

and we focused on "interesting points"

ex - x-intercept - points that intersect the x-axis. (i.e. $y=0$)

y-intercept - points that intersect the y-axis. (i.e. $x=0$).

ex/ det. the intercepts of the function

$$\text{Bud}(x) = 3x^2 + 9x - 1.$$

y-intercept: $x=0 \Rightarrow y = \text{Bud}(0) = 3(0)^2 + 9(0) - 1 = -1$

$(0,-1)$

x-intercept: $y=0 \Rightarrow 0 = 3x^2 + 9x - 1$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(3)(-1)}}{2 \cdot 3} = \frac{-9 \pm \sqrt{81+12}}{6} = \frac{-9 \pm \sqrt{93}}{6}$$

so $\left(\frac{-9+\sqrt{93}}{6}, 0\right)$ and $\left(\frac{-9-\sqrt{93}}{6}, 0\right)$.

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we also had the idea of domain + range

domain: set of all possible input values.

range: set of all possible output values.

ex/ $f(x) = \sqrt{3x+1} + 2$

what x can I plug in? well

$\sqrt{\bullet}$ ← must be ≥ 0.

so $3x+1 \geq 0$

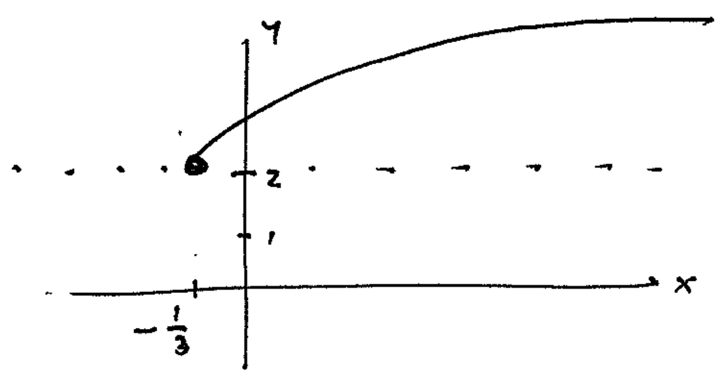
$3x \geq -1$

$x \geq -1/3$ so $[-1/3, \infty)$ = dom.

range? $\sqrt{3x+1} \geq 0$ and can be zero!

$\Rightarrow \sqrt{3x+1} + 2 \geq 2$ so $[2, \infty)$ = range

no - x - intercept!



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we also did some mathematical modeling.

i.e. given a physical situation

determine a <u>set</u> <u>of</u> <u>equations</u>

that <u>approx</u> the situation.

(A) Linear Equations. — constant rate of change

(B) quadratic Equations. — Either or

(A) if rate of change is const.

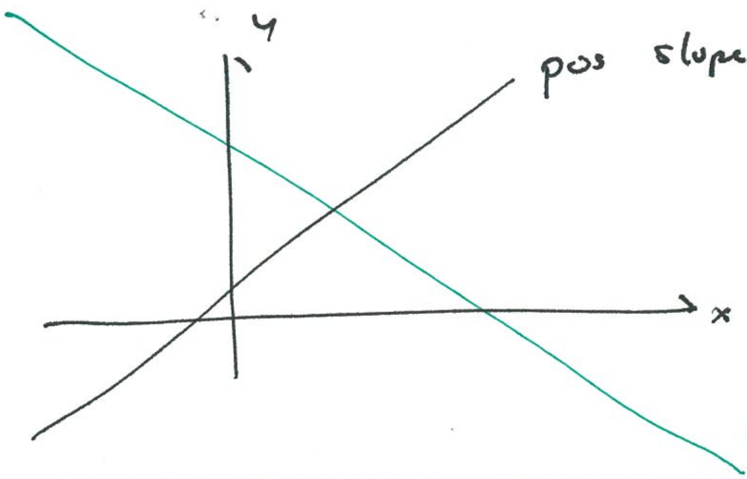
$\Rightarrow$ Lin. Equation.

$y = mx + b$ slope-intercept

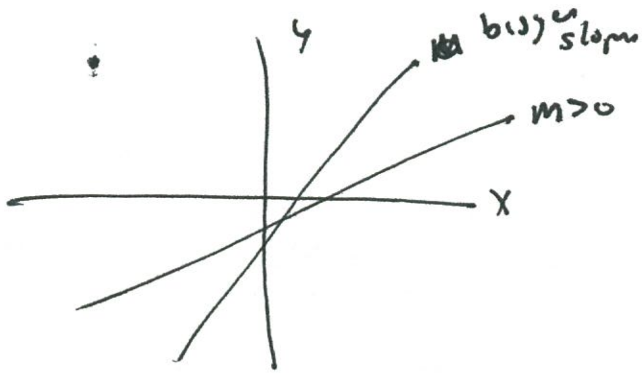
or

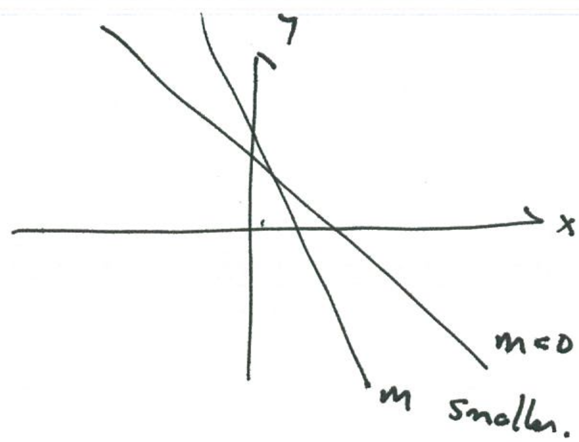
$y - y_0 = m(x - x_0)$ slope-pt.

often time ~~use it~~ preferable, Leave in this form is Ok!



you should understand what the slope "means" what it tells you!



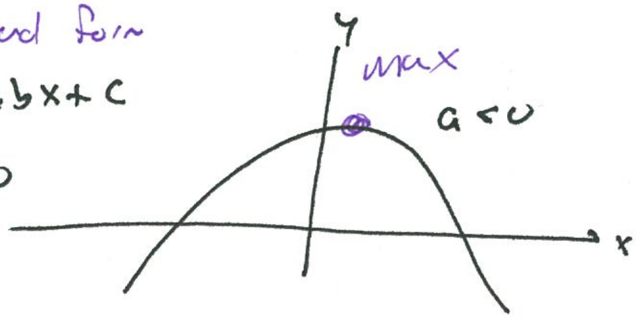


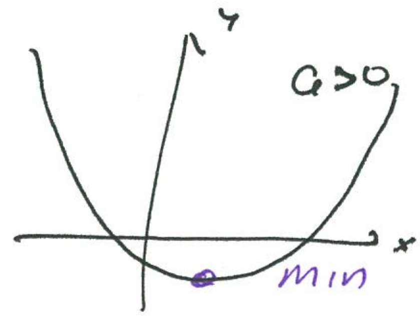
quadratics.

general form

$y = ax^2 + bx + c$

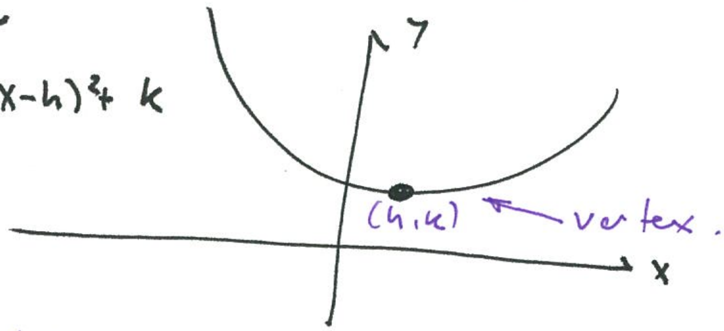
$a \neq 0$





vertex form

$y = a(x-h)^2 + k$



complete the square to go from general form to vertex form.

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misc. topics not cover here due to time constraints:

shifts $\begin{cases} \text{vertical} \\ \text{horizontal.} \end{cases}$

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piecewise defined functions.

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optimization

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even / odd

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increasing + decreasing

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composition of functions.