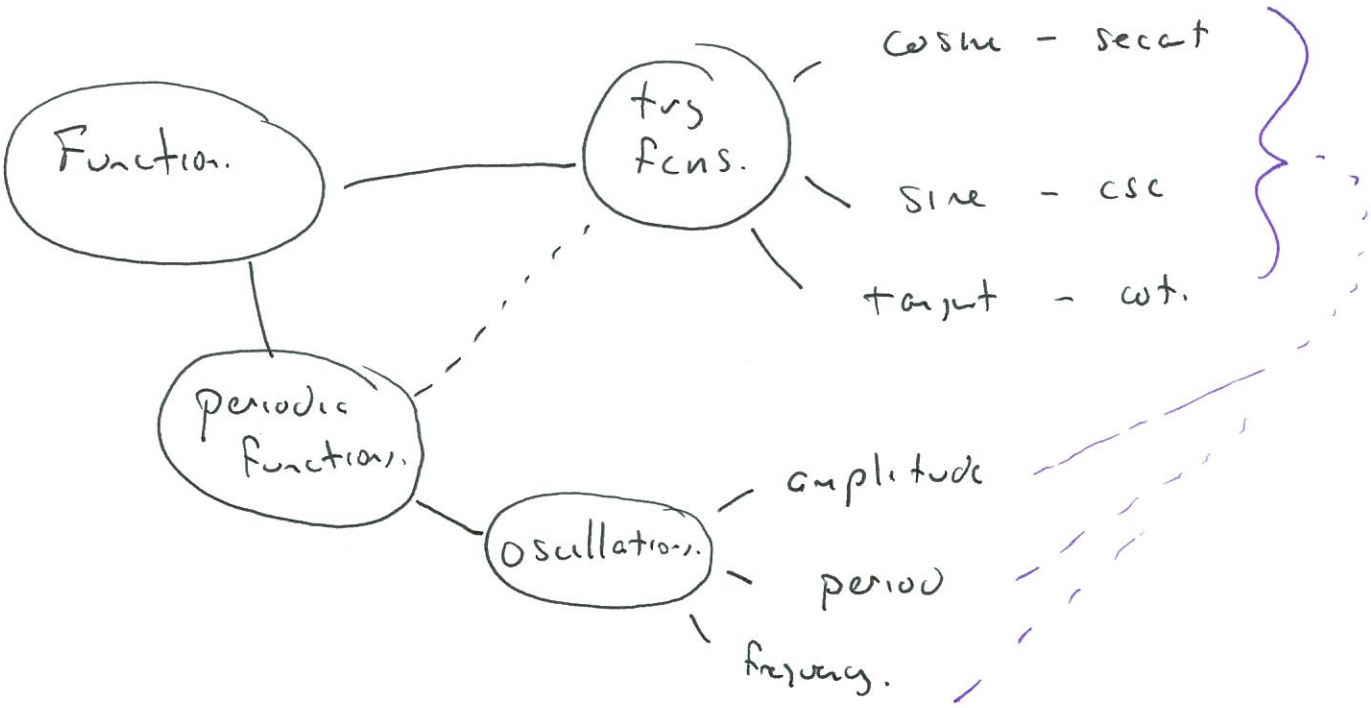
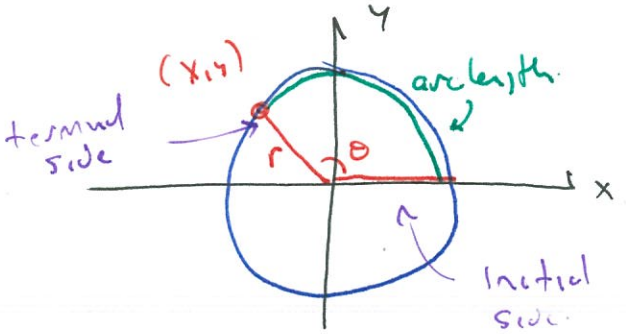
Chapter 4 - Review



1ST what is an "angle?"

Intuitive - but hard to define.

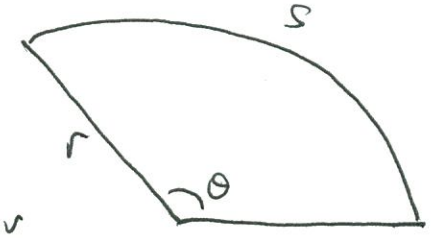
So, we do it in terms of a circle.



If the initial side is on the pos. x-axis the angle is in standard position.

Taken together it gives a sector.

or



We have $S = r\theta$ ← def'n of radian measure

$A = \frac{1}{2} r^2 \theta$

ex/ A bicycle wheel has a diameter of .70m, and a rider travels a dist. of 150km. What angle did the wheel turn through?

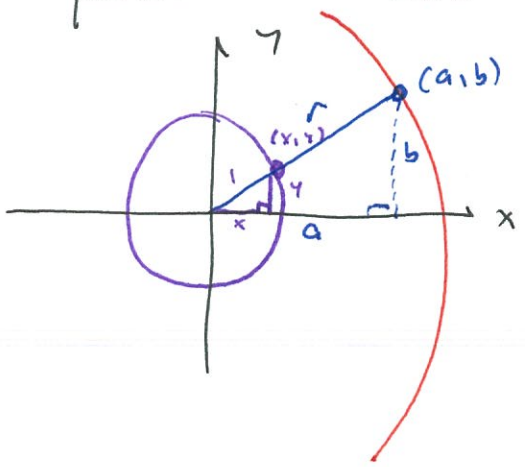
so $r = .35$

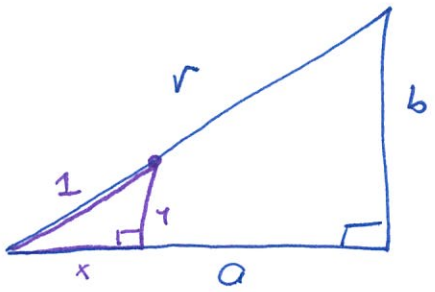
$S = 150{,}000$

$\theta = S/r = \frac{150{,}000}{.35} \approx$ 428,571.43

or about 68,209.26 rev.

Note The idea of using a circle is a powerful idea!





$\frac{y}{x} = \frac{b}{a}$ cuz similar Δs.

$\Rightarrow$ defn

$\tan(\theta) = y/x$

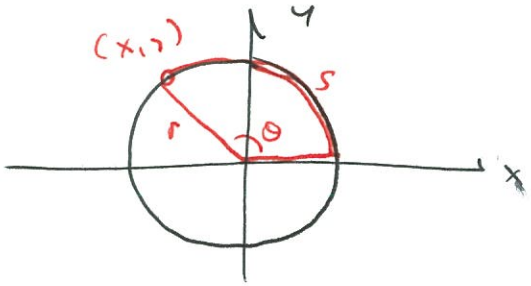
similarly $\sin(\theta) = y/1$

$\cos(\theta) = x/1.$

⇒ If we can figure out how to deal w/ a circle of radius 1 ...

- we can deal w/ any circle!
- we can deal w/ any triangle.
- we can rule Westeros.

So, now we have



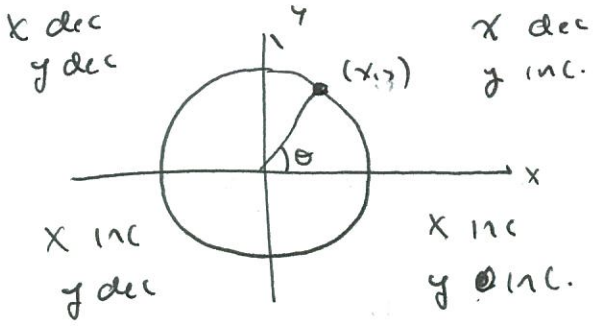
$\theta = s/r$

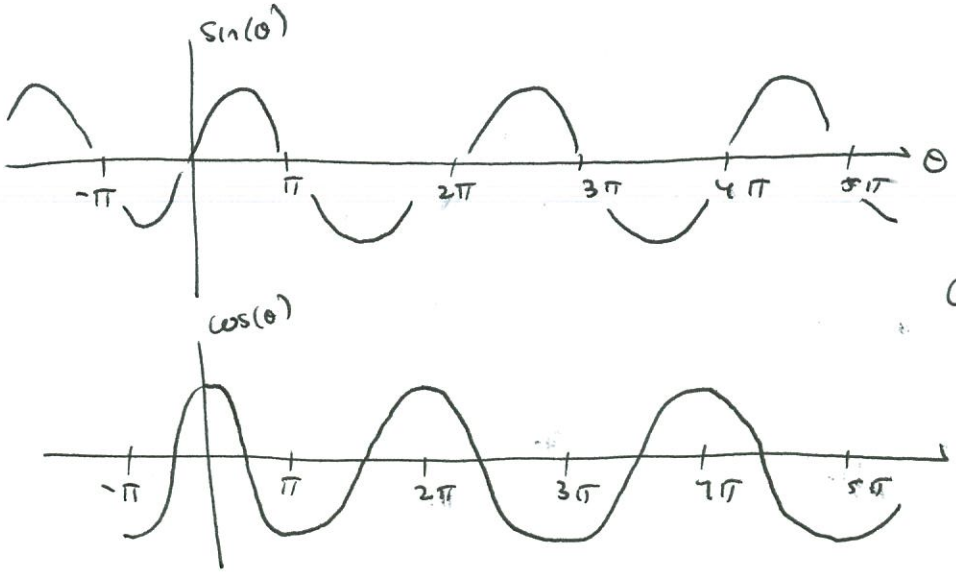
if $r=1$ then $\theta = s$.

$\cos(\theta) = x$

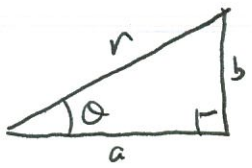
$\sin(\theta) = y$

$\tan(\theta) = y/x$





and

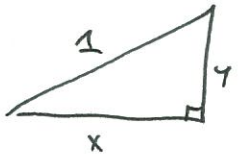


$\sin(\theta) = b/r$

$\cos(\theta) = a/r$

$\tan(\theta) = b/a$.

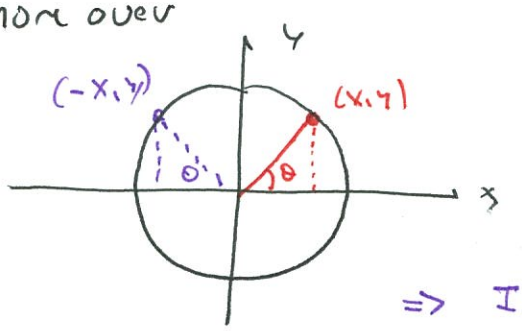
These imply the Pythagorean identities



$x^2 + y^2 = 1^2$ so $\cos^2(\theta) + \sin^2(\theta) = 1$.

div. by $\cos^2(\theta)$ so $1 + \tan^2(\theta) = \sec^2(\theta)$.

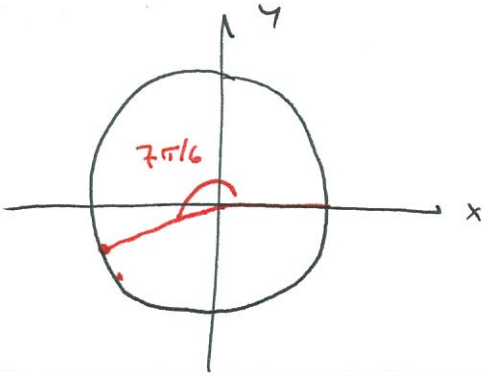
Moreover

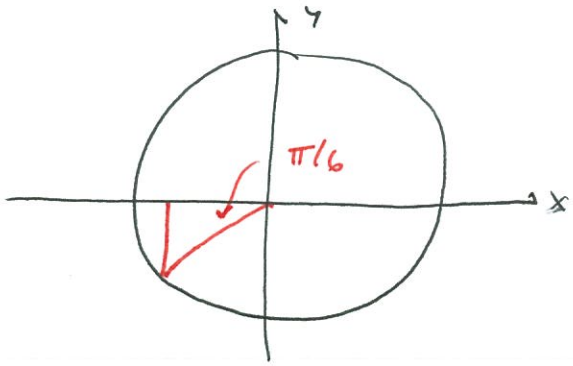


The ref. angle is the angle between the terminal side and the horizontal axis.

$\Rightarrow$ If we sort out the 1st quadrant we can get the other quadrants.

ex/ Given that $\sin(\pi/6) = 1/2$ what is $\cos(7\pi/6)$?





So $\cos(7\pi/6) = -\cos(\pi/6)$

and $\cos^2(\pi/6) + \sin^2(\pi/6) = 1$

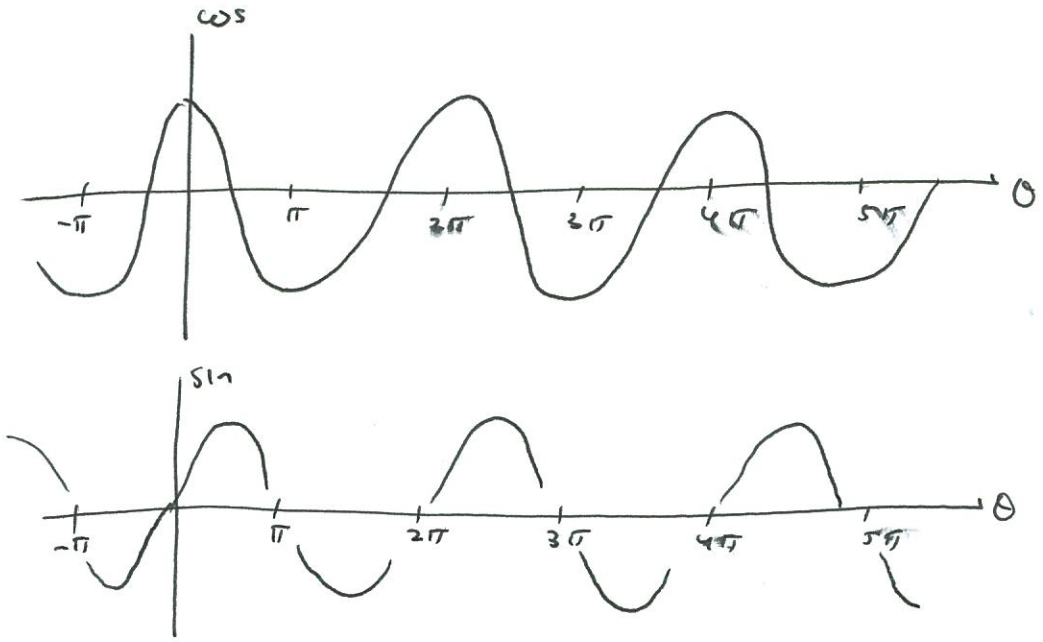
$\cos^2(\pi/6) + (1/2)^2 = 1$

$\cos^2(\pi/6) = 1 - 1/4 = 3/4$

$\cos(\pi/6) = \sqrt{3}/2$

and $\cos(7\pi/6) = -\sqrt{3}/2$.

Back to sine and cosine as a function... we can think of how this function changes



We can translate the bijections out of this! Also it is a periodic function w/ period 2.

Recall $A f(mx+c)+b$

horizontal change.

vertical change.

so $A\cos(mx+c)+b$

or

$A\sin(mx+c)+b$.

ex/ If we have

$$y = 3\sin(4x + \tfrac{\pi}{2}) - 4$$

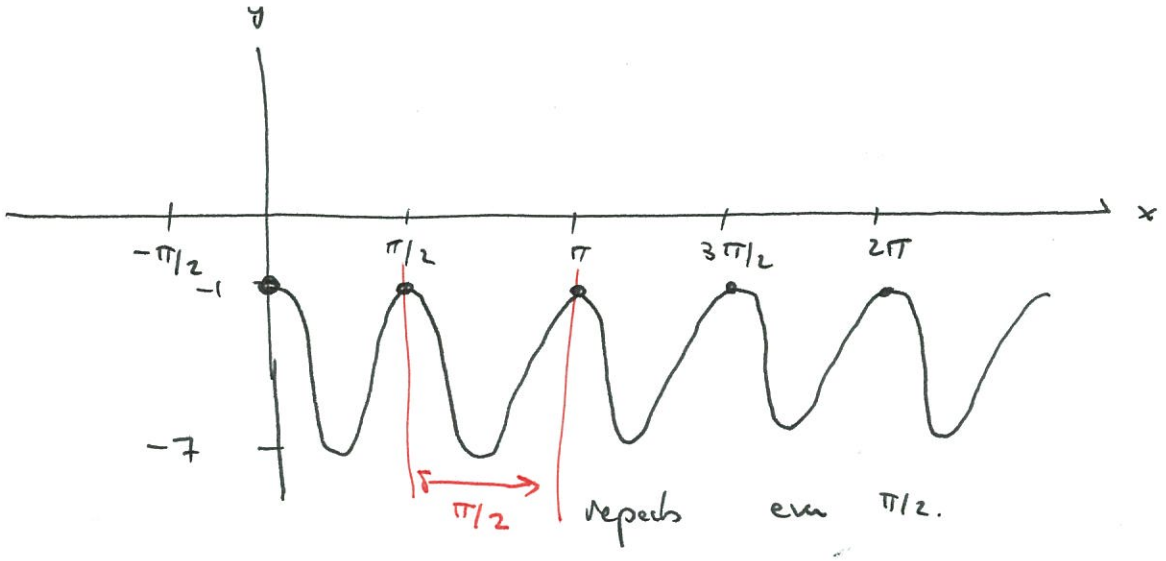
oscillates between $3-4=-1$ and $-3-4=-7$.

period: $4x = 2\pi \Rightarrow$ Period $= \frac{2\pi}{4} = \pi/2$.

max @ $4x + \frac{\pi}{2} = \pi/2$

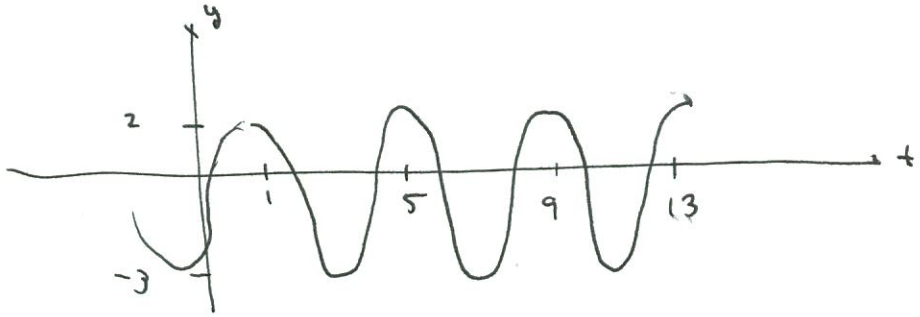
$4x = \pi/2 - \pi/2$

$x = 0$



Or... suppose that we want a cosine wave that oscillates between -3 and 2, and it has a period of 4 and a max @ $t=1$, what is the eqn?

Assm $y = A\cos(nt + c) + b$



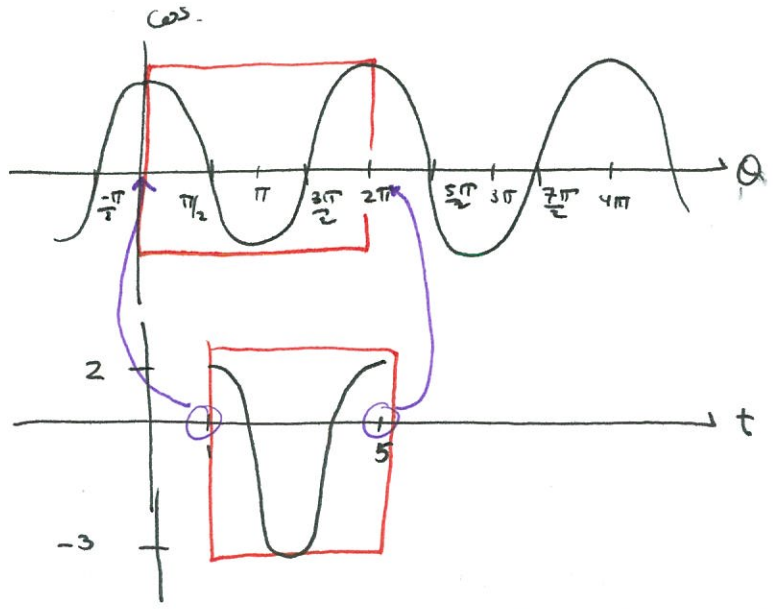
(A) do vertical stuff 1st

$$\begin{aligned} 2 &= A + b \\ +\quad -3 &= -A + b \end{aligned}$$

$$-1 = 2b \qquad b = -1/2$$

$$A = 2 - b = 2 + 1/2 = 5/2.$$

(B) do horizontal stuff 2nd.



m+c is the line that goes through the pts

$(1, 0)$ & $(5, 2\pi)$

$$m = \frac{2\pi - 0}{5 - 1} = \pi/2$$

$$L(t) - 0 = \pi/2 (t - 1)$$

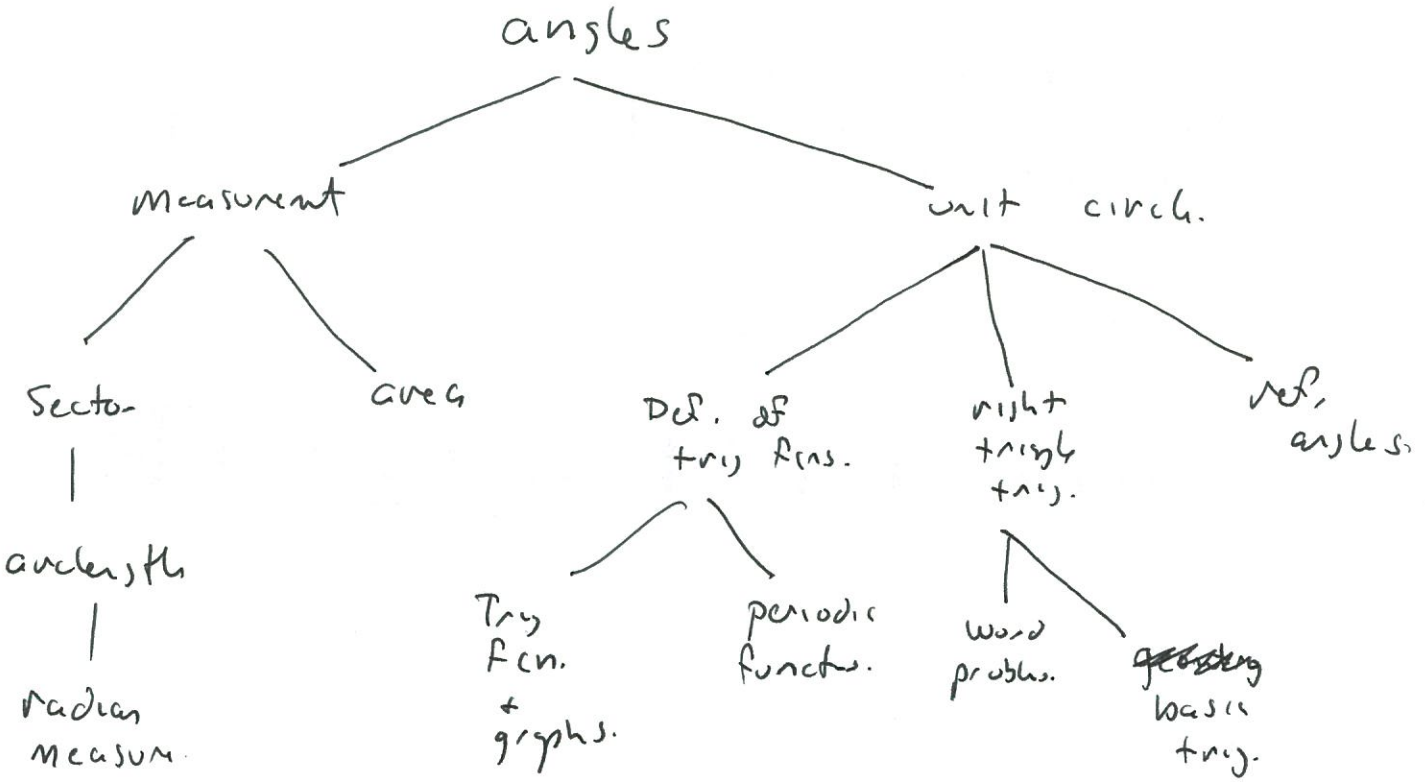
$$\text{so } L(t) = \pi/2 (t-1) + 0$$

$$= \pi/2 t - \pi/2$$

so

$$y = 5/2 \cos(\pi/2 t - \pi/2) - 1/2$$

on the test

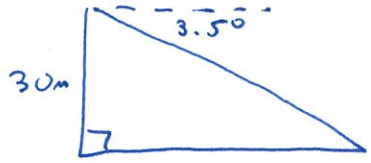


Keep in mind:

- $s = r\theta$
- everything is in radians
  unless otherwise stated!
- you <u>always</u> have
  $\sin^2(\theta) + \cos^2(\theta) = 1$
  $\tan^2(\theta) + 1 = \sec^2(\theta)$
- Draw, Draw, Draw!
  unit circle & triangles.
- Draw everything multiple times

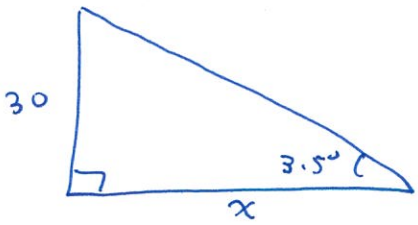
ex/ A sailor is in the mast of a ship and is 30m above the water. Another ship is spotted and the angle of depression is 3.5°. How far is the ship?

a) Read. (1)



(2) x = dist.
$\alpha = 3.5°$
height = 30.

(3)



$\tan(3.5°) = \frac{30}{x}$ (4) soln for x!

(5) $x = \frac{\tan(3.5°)}{30} \approx 490.5$m.

(6) check.

ex/ A sector has an arclength of 4m and an area of $2m^2$. What is the angle of the sector?

$A = \frac{1}{2}r^2\theta$
$s = r\theta$

$\Rightarrow$ $2 = \frac{1}{2}r^2\theta$
$4 = r\theta$

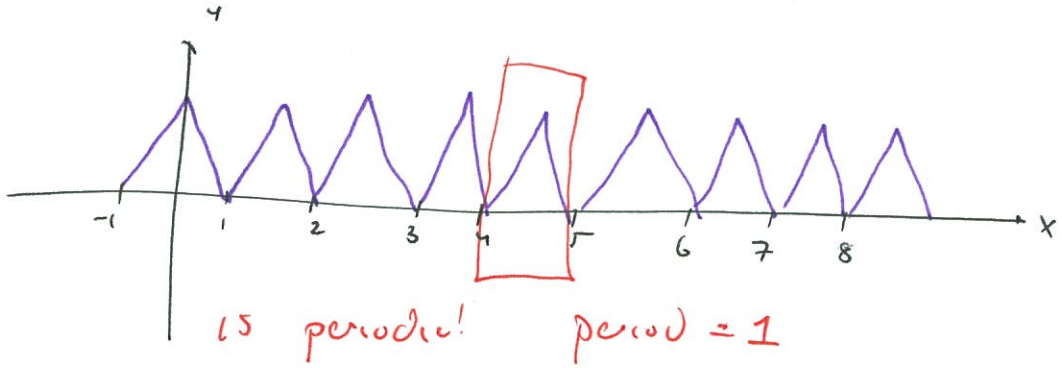
} 2 eqn's 2 unknowns.

$r = \frac{4}{\theta}$

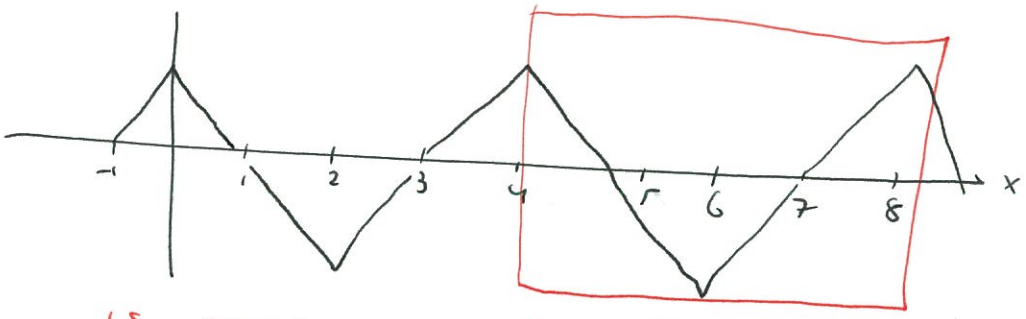
$2 = \frac{1}{2}\left(\frac{4}{\theta}\right)^2\theta = \frac{1}{2}\cdot\frac{16}{\theta^2}\theta = \frac{8}{\theta}$

so $\theta = 4$ rad.

ex/ Is the following fcn. periodic? If so what is the period?



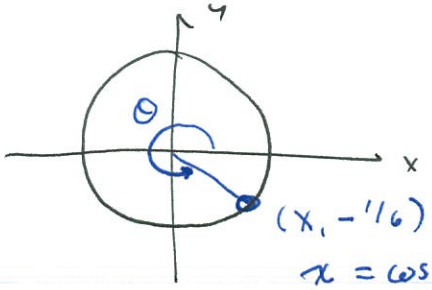
is periodic! period = 1



is periodic. — period = 4.

ex/ θ is in Quad IV & $\sin(\theta) = -1/6$.

what is $\cos(\theta)$?



$x = \cos(\theta)$ is pos.

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\left(-\frac{1}{6}\right)^2 + \cos^2(\theta) = 1$

$\frac{1}{36} + \cos^2(\theta) = 1$

$\cos^2(\theta) = 1 - 1/36 = \frac{35}{36}$

so $\cos(\theta) = \pm\sqrt{\frac{35}{36}}$

need pos. root.

$\cos(\theta) = \sqrt{\frac{35}{36}}$.