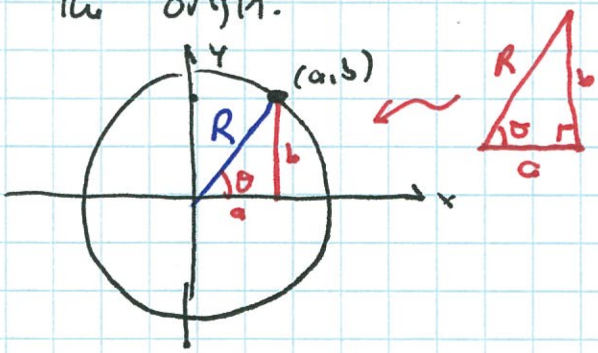
Section 4.2

ALEKS

The unit circle
Def. of trig functions
fundamental trig identities.

Suppose we have a circle of radius R centered at the origin.

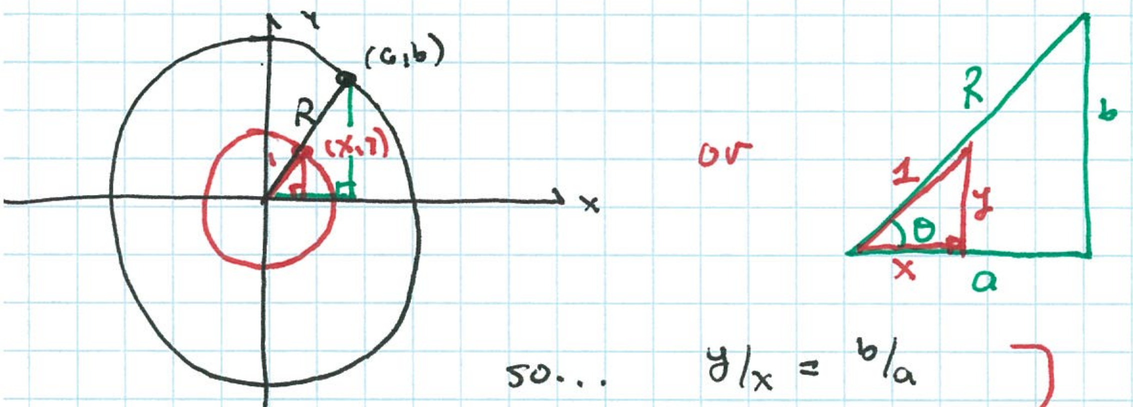


well... right triangle!

$$a^2 + b^2 = R^2$$

what are the relationships that tie θ, a, b, and R together?

That's a lot of variables! Wait, Does R really matter?



similar triangle!

so... $y/x = b/a$

and $x/1 = a/R$

$y/1 = b/R$

These ratios are the same for all radii!

we can just focus on the special case of $R=1$.

So what? – we just need 1 circle, w/ Radius 1. we shall call it the Unit circle.

– The ratios b/a, a/R, b/R are linked to the angle θ somehow. (same for all circles.)

– The angle is ~~x~~ linked to the arclength. $s = R\theta$

– This is a lot to think about...

Special case … $R=1$, the unit circle.

$s = R\theta$ & $s = \theta$.

gives dist. around sector we have the angle.

— we assume θ is in radians!

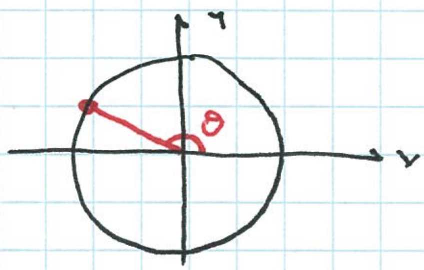
Definitions: if (x,y) is a point on the unit circle we define

$$\sin(\theta) = y/1 \qquad \csc(\theta) = 1/y$$
$$\cos(\theta) = x/1 \quad \text{Like with} \quad \sec(\theta) = 1/x$$
$$\tan(\theta) = y/x \qquad \cot(\theta) = x/y.$$

also $x^2+y^2=1$ so $(\cos(\theta))^2 + (\sin(\theta))^2 = 1$.

ex/ The point $(-\sqrt{3}/2, 1/2)$ is on the unit circle

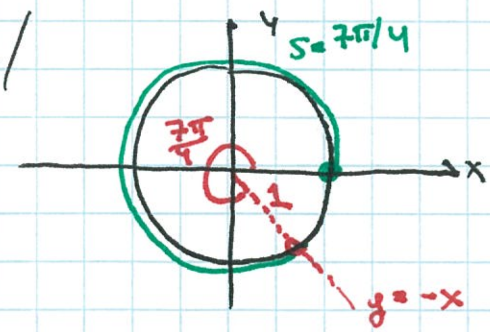
$$(-\sqrt{3}/2)^2 + (1/2)^2 = 3/4 + 1/4 = 1 \checkmark$$



Then
$$\cos(\theta) = -\sqrt{3}/2$$
$$\sin(\theta) = 1/2$$
$$\tan(\theta) = \frac{1/2}{-\sqrt{3}/2} = -1/\sqrt{3}.$$

} neg. #'s depending on the quadrant :)

ex/



suppose $s = \frac{7}{8} \cdot 2\pi = \frac{7}{4}\pi$

so $y=-x$ is the ray (quad IV)

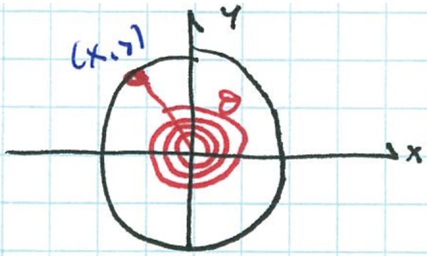
then $\cos(\theta) = -\sin(\theta)$

$$(\cos(\theta))^2 + (\sin(\theta))^2 = 1$$
$$(-\sin(\theta))^2 + (\sin(\theta))^2 = 1$$
$$(\sin(\theta))^2 + (\sin(\theta))^2 = 1$$
$$2\sin(\theta)^2 = 1$$
$$(\sin(\theta))^2 = 1/2$$
$$\sin(\theta) = \pm 1/\sqrt{2}$$

neg coz quad IV

so
$$\sin(7\pi/4) = -1/\sqrt{2}$$
$$\cos(7\pi/4) = 1/\sqrt{2}.$$

wait… $\sin(\cdot)$ is a fcn. of the angle. what is the domain?

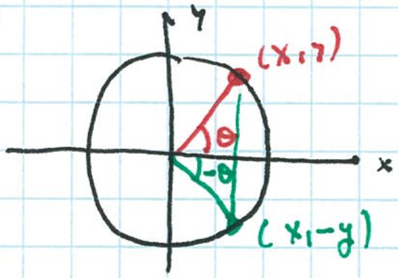


I can take any # of rev. and they can be negative.

Domain of $\sin(\theta)$ is $(-\infty, \infty)$
Domain of $\cos(\theta)$ is $(-\infty, \infty)$

Range of $\sin(\theta)$ is $[-1, 1]$
Range of $\cos(\theta)$ is $[-1, 1]$ $\Big\}$ wrt (x,y) on the unit circle.

__Wait__



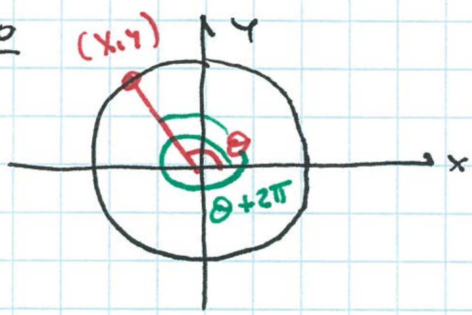
$\cos(\theta) = x$
and $\cos(-\theta) = x$

Cosine is an even function

$\sin(\theta) = y$
$\sin(-\theta) = -y$

sine is an odd function.

__also__



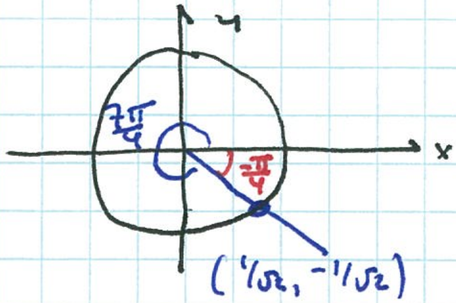
$\sin(\theta) = \sin(\theta + 2\pi)$
$\cos(\theta) = \cos(\theta + 2\pi)$

They repeat themselves! (Like me!)

__Def:__ A function is __periodic__ if $f(t+p) = f(t)$ for every t in its domain and p is a constant. p is called the __period__ of the function.

$\Rightarrow$ The period of $\sin(\cdot)$ is 2π
The period of $\cos(\cdot)$ is 2π

ex/ we now know that $\sin(7\pi/4) = -1/\sqrt{2}$.



since $\sin(\cdot)$ is periodic w/ period 2π

$$\sin(7\pi/4) = \sin(7\pi/4 - 2\pi) = \sin(-\pi/4)$$

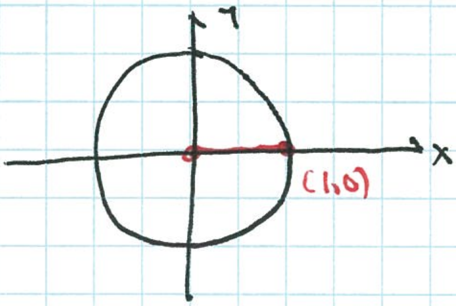
$$\sin(-\pi/4) = -1/\sqrt{2}$$

but sine is odd so

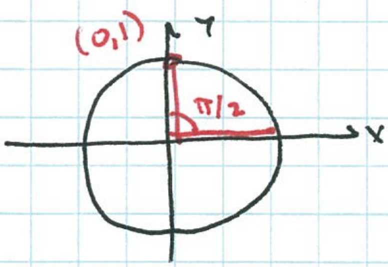
$$\sin(\pi/4) = -\sin(-\pi/4) = -(-1/\sqrt{2}) = 1/\sqrt{2}.$$

also since cosine is even and periodic $\cos(7\pi/4) = 1/\sqrt{2}$, $\cos(-\pi/4) = 1/\sqrt{2}$

Have you noticed that I have not said anything about ~~tangent~~ tangent? That is because the tangent is a bitter, bitter fcn.



$\cos(0)=1$
$\sin(0)=0$ $\Rightarrow$ $\tan(0) = 0/1 = 0$. Life is good.



$\cos(\pi/2)=0$
$\sin(\pi/2)=1$ $\Rightarrow$ $\tan(\pi/2) = \overline{}$ uh oh! no can do!

This only happens at $(0,1)$ and $(0,-1)$ so no big deal! right? <u>nope</u>!

$(0,1)$ corresponds to $\pi/2$ and $\pi/2 + 2\pi$, $\pi/2 + 4\pi, \ldots$

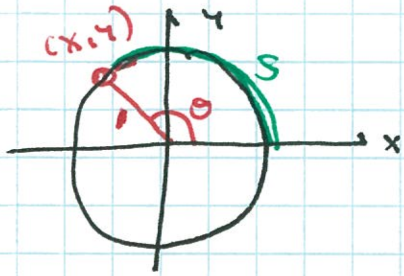
$(0,-1)$ corresponds to $3\pi/2$ and $3\pi/2 + 2\pi$, $\frac{3\pi}{2} + 4\pi, \ldots$

so the Domain of $\tan(\cdot)$ is

$$\ldots (-3\pi/2, -\pi/2) \cup (-\pi/2, \pi/2) \cup (\pi/2, 3\pi/2) \cup (3\pi/2, 5\pi/2) \cup \ldots$$

okay then.
that sucks.

What just happened?



we have the unit circle ($r=1$)

$x^2+y^2=1$

For <u>any</u> point on the circle we can get there by walking around the edge of the circle. call the dist we walk s.

$\Rightarrow$ This ~~def~~ defines a sector of radius 1 and arclength s.

$\Rightarrow$ $s = r\theta = \theta$

<u>we</u> <u>define</u> the following:

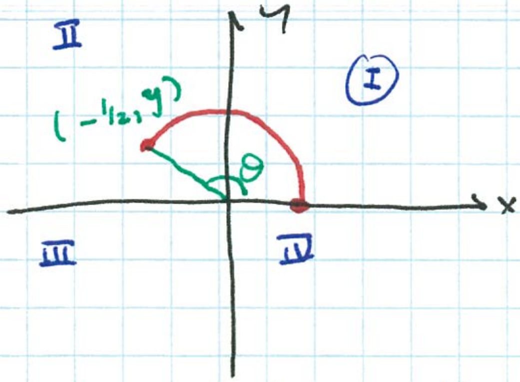
$y = \sin(\theta)$ | $1/y = \csc(\theta)$
$x = \cos(\theta)$ and $1/x = \sec(\theta)$
$y/x = \tan(\theta)$ | $x/y = \cot(\theta)$

$\Rightarrow (\cos(\theta))^2 + (\sin(\theta))^2 = 1.$

So given s we get θ. With θ there is a relationship with (x,y) the point on the circle.
and there are <u>infinite</u> ways to get to that point because I keep returning to the point each time I walk around the circle. :o

ex/ I walk around the unit circle some dist. and my x-coord. is $-1/2$. If I am in the 2^{nd} quadrant what is the cosine / sine / tangent?

Assume pos. orientation & start @ $(1,0)$



counter-clockwise is defined to be pos. dir.

well $\cos(\theta) = -1/2$ (that was easy!)

note $\sin(\theta) > 0$ cuz quad II.

$x^2+y^2 = 1$

$(-1/2)^2 + y^2 = 1$

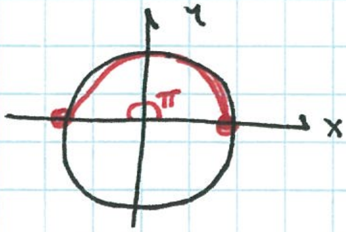
$1/4 + y^2 = 1$

$y^2 = 3/4 \Rightarrow y = \pm\sqrt{3}/2.$

so $y = +\sqrt{3}/2.$

so $\sin(\theta) = \sqrt{3}/2$ and $\tan(\theta) = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}.$

ex/ I walk around the unit circle a dist of π m. What's the dill pickle?



so $\theta = \pi$ 1/2 way around the circle.

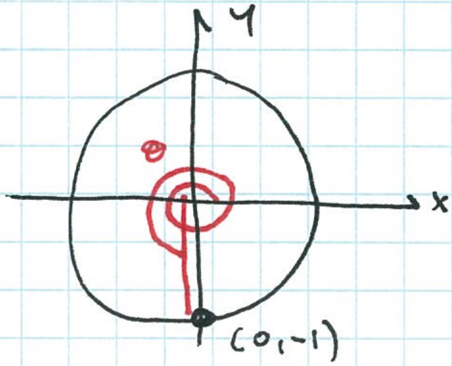
$\Rightarrow (-1, 0)$

$\cos(\theta) = \cos(\pi) = -1$

$\sin(\pi) = 0$

$\tan(\pi) = 0/1 = 0.$

ex/ I ~~walk arou~~ walk around the unit circle a dist. of $7\pi/2$ units. What's up pussy cat?



$7\pi/2 = 6\pi/2 + \pi/2 = 3\pi + \pi/2$

$= 2\pi + \pi + \pi/2$

$\cos(7\pi/2) = 0$

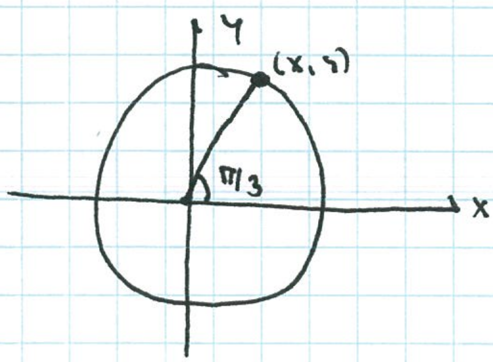
$\sin(7\pi/2) = -1$

$\tan(7\pi/2) = \frac{-1}{\text{uh oh!}}$ not defined! :o

~~sec(θ)~~ $\sec(7\pi/2)$ not defn but $\cot(\frac{7\pi}{2}) = 0.$

Time Permitting

ex/ I walk around the unit circle a dist. of $\pi/3$ m. What's cookin?



$x = \cos(\pi/3) = ?$ (on calc.)

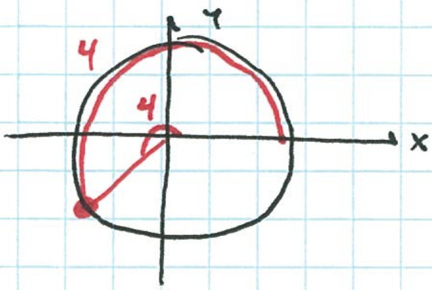
$y = \sin(\pi/3) = ?$ (on calc.)

$\tan(\pi/3) = ?$ (on calc.)

Quad 1 so $\cos > 0$, $\sin > 0$, $\tan > 0$. ✓

Time permitting.

ex/ I walk around the unit circle a dist of 4m. ~~sup~~ sup?



$\theta = 4$

$x = \cos(4) \approx ?$ $\quad x < 0$ Quad III

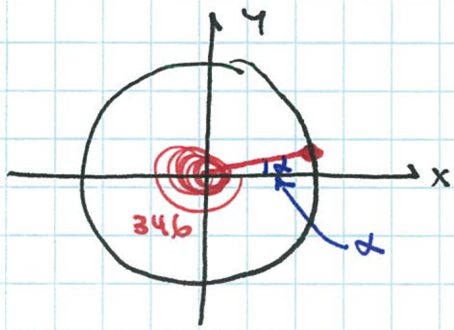
$y = \sin(4) \approx ?$ $\quad y < 0$

$\tan(4) \approx ?$ $\quad > 0$!

Time Permitting

ex/ I walk around the unit circle 346m. (wot?)

Howz it shaking, bacon? ugh!



$346 - 2\pi n \geq 0$ what's the biggest n to make this so

$2\pi n \approx 346$

$n \approx \frac{346}{2\pi} \approx 55.068$ let $n = 55$

$\theta = 346 = 55(2\pi) + \alpha$

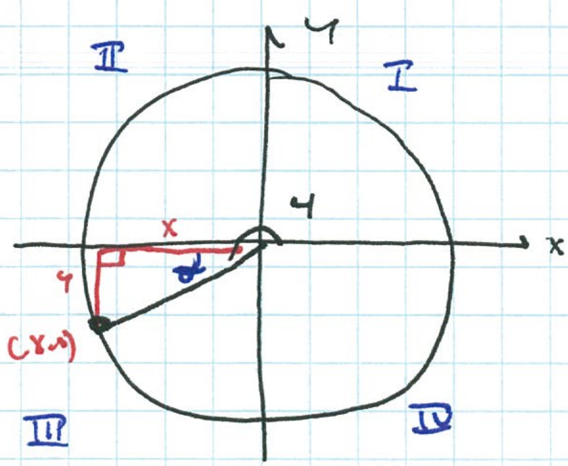
$\alpha \approx .425$

$x \approx \cos(.425) \approx .9111$

$y \approx \sin(.425) \approx .4122$

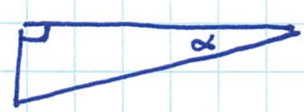
$\tan(.425) \approx .452$

ex/ lets go back to $\theta = 4$ + use same idea as prev. ex.



There is a hidden right triangle!

$4 = \alpha + \pi$



since (x,y) is quad III I know $x<0, y<0$. but if I get...

the $\cos(4-\pi) = -\cos(4)$. !