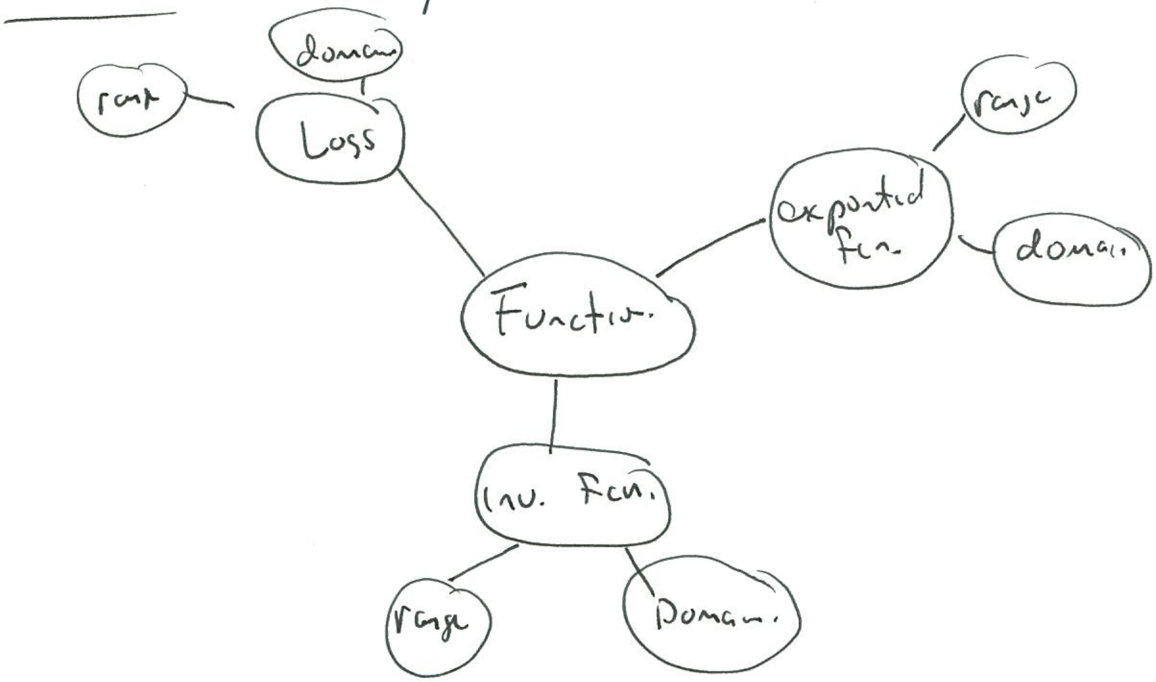
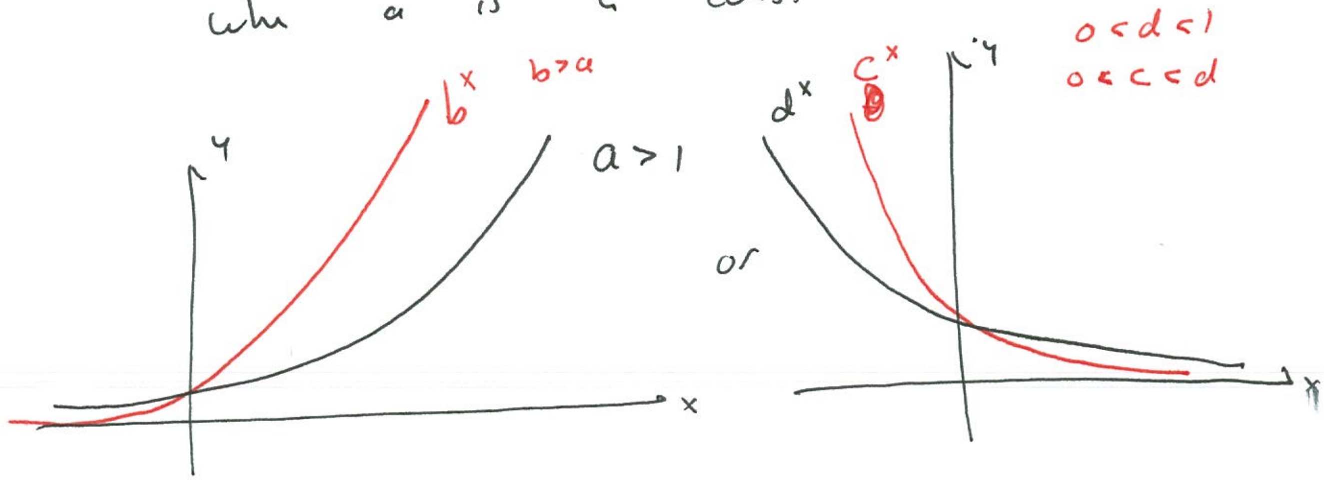
Review Chapter 3



We have a new function: The exponential

$$f(x) = a^x$$

where a is a const.



Why is this helpful?

cuz in nature there are situations where the rate of change is not constant.

ex/

- Every min 30 gallons of water is added to a pool.

$\Rightarrow$ const. rate of $+30$ gallon/min.

each min gives $+30$ gall.

$\Rightarrow \text{Vol} = 30 \cdot t + (\ \ )$ const.

- 10g of chemical is put in a vat.

Every 3 days half the remaining chemical decays.

$\Rightarrow$ day 0 — 10g
day 3 — 5g
day 6 — 5/2 g
day 9 — 5/4 g

avg rate of chg $= \frac{5-10}{3} = \frac{-5}{3}$

avg rate of chg $= \frac{5/2-5}{3} = \frac{-5}{6}$

avg rate of chg $= \frac{5/4-5/2}{3} = \frac{-5}{12}$

$\vdots$

changes in time!
NOT linear!

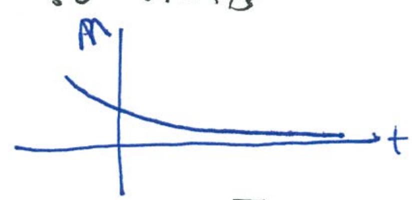
here $M(n) = 10 \cdot (1/2)^n$

$n = \#$ time periods

$t = n \cdot 3$ so $n = t/3$

$\Rightarrow M(t) = 10\,(1/2)^{t/3}$

not lin.



- Like with growth.

Every 10 days a pop. of rodents inc. by ~~10~~ 7%.

$P(0) = P_0$

$P(10) = 1.07\,P_0$

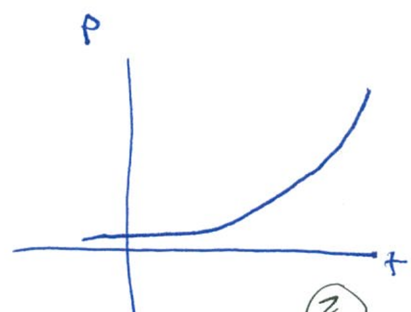
$P(20) = 1.07\,P(10) = 1.07^2\,P_0$

$P(30) = 1.07\,P(20) = (1.07)^3\,P_0$

$\vdots$

$P(t) = P_0\,(1.07)^{t/10}$.

not linear!



Ah, but, we do not want to keep track of 1/2's or 7%'s or whatever base the kids use today. We can just use one base!

why? First - <u>which</u> base!

2.7183 of course — we will tell you the real reason in calc, but for now we need <u>some</u> mystery in the relationship.

So we just had...

$$(1.07)^{t/10} = \left[(1.07)^{1/10}\right]^t$$

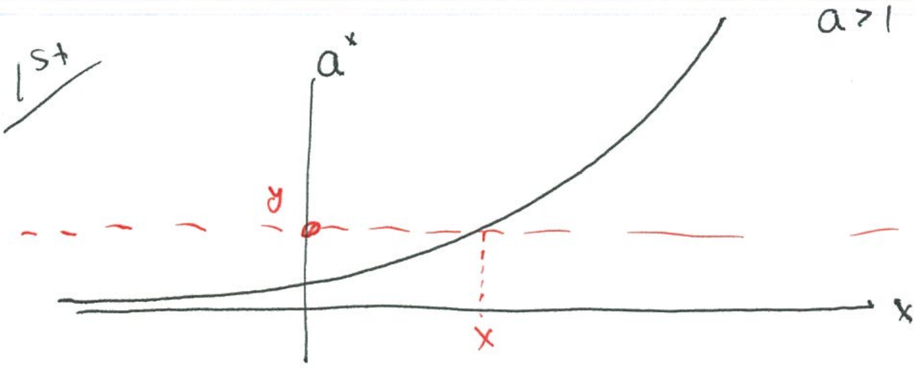
note if $e^r = (1.07)^{1/10}$ for <u>some</u> r then

$$(1.07)^{t/10} = (e^r)^t = e^{rt}.$$

we can do this for any pos #!

How do we know that?
How can we get r?
We need more definitions & tools!

1st



<u>if $y>0$</u> then there is <u>exactly</u> <u>one</u> x where $y = a^x$.

y in domain.

⇒ This fcn. is 1-1.

⇒ given y the <u>process</u> of getting x is a function.

so... A funct is 1-1 if

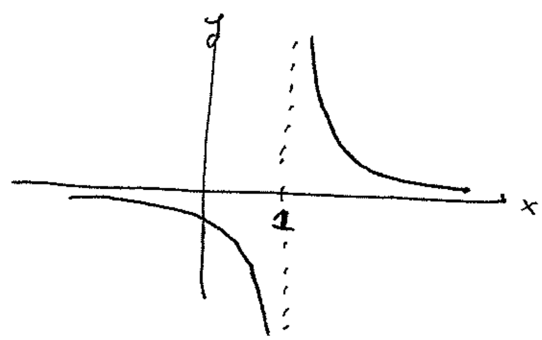
(B) $f(a) = f(b)$ implies that $a = b$

(A) if $a \neq b$ then $f(a) \neq f(b)$

— for any a, b in dom. of f. —

And <u>if</u> a fcn. is <u>1-1</u> the fcn. can be inverted. (i.e. if $y = f(x)$ we can get x given y).

ex/ $f(x) = \dfrac{1}{x-1}$



1-1 coz if

$a \neq 1$ $\quad f(a) = f(b)$

$$\frac{1}{a-1} = \frac{1}{b-1}$$

$$\Rightarrow \quad b-1 = a-1$$

$$b = a \ \checkmark$$

so an inv. exists

$y = 1/x-1 \Rightarrow$ $(x-1)y = 1$

$xy - y = 1$

$xy = 1 + y$

$x = \dfrac{1+y}{y}$ = inv. of f.

notation $f^{-1}(y)$

and $f(f^{-1}(y)) = f\left(\dfrac{1+y}{y}\right) = \dfrac{1}{\frac{1+y}{y} - 1} \cdot \dfrac{y}{y} = \dfrac{y}{1+y-y} = y. \checkmark$

likewise $f^{-1}(f(x)) = x$

We do not have a nice way to figure out the inv. of $f(x) = a^x$ but it can be approx. (what your calc. does). So we have to be adept at using the notation to help us use the tool correctly.

so $f(x) = a^x$ define $\log_a(x)$ to be the inv. of $f(x)$.

$\Rightarrow$ $a^{\log_a(x)} = x$ and $\log_a(a^x) = x$.

note - exponential functions have some weird properties.

$$a^u \cdot a^v = a^{u+v}$$
$$(a^u)^r = a^{ru}$$

This results in a couple weird properties of logs.

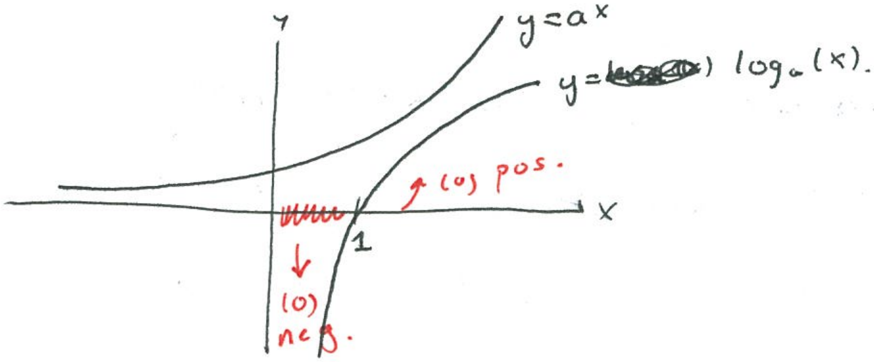
$$\log_a(u \cdot v) = \log_a(u) + \log_a(v)$$

and $$\log_a(u^r) = r \log_a(u).$$

Special notations:

$$\log(x) = \log_{10}(x)$$
$$\ln(x) = \log_e(x)$$

These are button so your calc. & have nice properties.



so.... If an isotop has an half-life of 1,000 years what is its one third-life?

assum exp. fcn. (decay)

$$P(t) = C e^{-rt}$$

- $P(1000) = \frac{1}{2} P(0)$

$$P(?) = 1/3 \, P(0)$$

$$P(0) = C e^{0} = C$$

$$\Rightarrow \quad P(1000) = \tfrac{1}{2} C$$

$$C e^{-r \cdot 1000} = \tfrac{1}{2} C$$

$$e^{-1000r} = \tfrac{1}{2}$$

$$-1000 r = \ln(1/2)$$

$$r = \frac{-1}{1000} \ln(1/2)$$

$$P(t) = C e^{+t/1000 \, \ln(1/2)}$$

$$\tfrac{1}{3} C = C e^{t/1000 \, \ln(1/2)}$$

$$\tfrac{1}{3} = e^{t/1000 \cdot \ln(1/2)}$$

$$\ln(1/3) = \frac{t}{1000} \ln(1/2) \quad \Rightarrow \quad t = 1000 \frac{\ln(1/3)}{\ln(1/2)}$$

note...

$$e^{t/1000 \, \ln(\frac{1}{2})} = e^{\ln((1/2)^{t/1000})} = \left(\tfrac{1}{2}\right)^{t/1000} \qquad !$$

So – on the test –

Look @ the question.

<u>Read</u>

- algebra (solve)
  - exp.
    - inv. is logs.
      - alg. math
  - logs.
    - inv. is exp.
      - alg math.
- function question
  - exp.
    - 1-1
      - def. of log.
  - log.
    - 1-1
      - def. of exp.
  - inverse
    - 1-1
    - gen. inv.
    - find inverse.
- word problem.
  - growth
    - $y = Ce^{rt}$
      - 6 step program (TM)
  - decay.
    - $y = Ce^{-rt}$
      - 6 step prog. (TM)
  - logistic.
    - $y = \dfrac{c}{1 + ae^{-rt}}$
      - 6 step program (TM)

– Do not stare at a problem – it will not help!

– explain and <u>write</u> stuff out.

– I care only about your reasoning / communication skills.

– Read question & parse it.
  Draw picture / ask basic questions! (growth / decay / fcn?)
  identify / define the variables.
  explore.

plan

solve

check work

This is why it is important to be organized and neat.

If you do not know where to start the issue is not about understanding the math. It is about not knowing how to solve problems! (something different and not particular to math.)

Note Algebra matters.

- parenthesis must be respected.
- work from outside to inside.
- deal w/ + & - first.
- deal w/ $\times$ and $\div$ 2nd

keep parens in mind.

- know when to factor vs. distribute
- understand functions & their notations.
  - our function have multi-letter names now!
- ask yourself what operations are on the page.