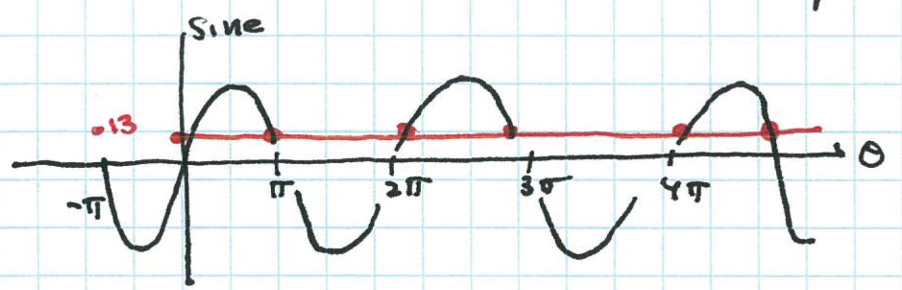
Inverse Sine / cosine / tangent
Composition of inv. trig and trig functions.

Suppose that we know that $\sin(\theta) = 1/\sqrt{2}$ and θ is in the 2^{nd} quadrant, what is θ?

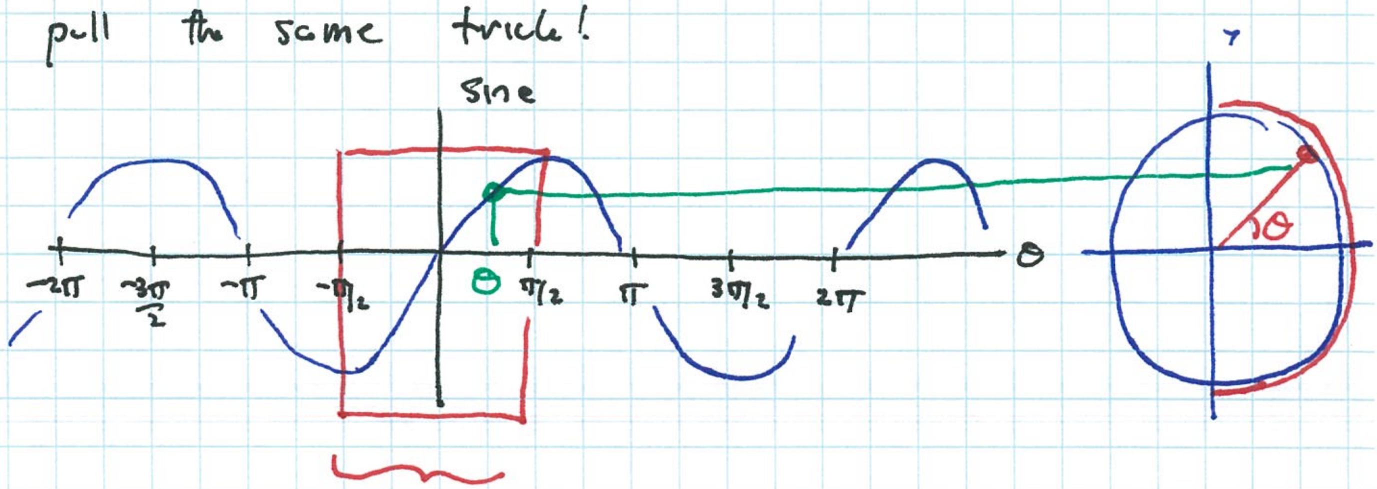
$\theta = 3\pi/4$. what if $\sin(\theta) = .13$?

First, the question is problematic



There is no inverse. sin is not 1-1.

You know, this did not stop us w/ $f(x) = x^2$.
Let's pull the same trick!

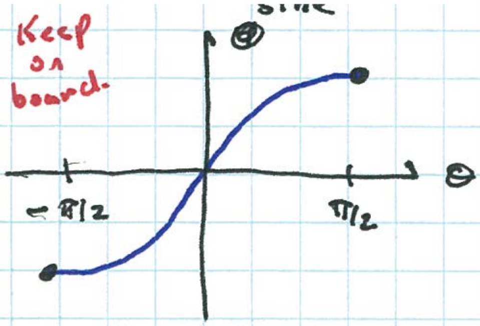


if $-\pi/2 \le \theta \le \pi/2$ then $\sin(\theta)$ is 1-1 on this restricted domain.

we can define an inverse on the restricted domain $-\pi/2 \le \theta \le \pi/2$.

(not unique! but this is the standard, and it is what your calculator uses.)

Keep on board.



sine is 1-1 on this domain.
sine is invertible on this domain.

Def: we define $\arcsin(x)$ to be the inverse of $\sin(x)$. The $\arcsin(x)$ has domain $[-1,1]$ and range $[-\pi/2, \pi/2]$.

Recall: $\sqrt{x^2} = |x|$ not nec. x if $x<0$!
so restricting the domain does weird things!

Because of the restriction above

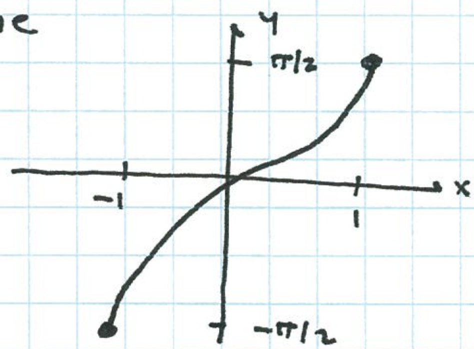
$\sin(\arcsin(x)) = x$ — no problem!

$\arcsin(\sin(x))$? — depends on which quadrant x is in!

ex/ $\arcsin(\sin(\pi/4)) = \arcsin(1/\sqrt{2}) = \pi/4$ ✓

$\arcsin(\sin(3\pi/4)) = \arcsin(1/\sqrt{2}) = \pi/4$

graph of arcsine



ex/ $\sin^2(x) - 2\sin(x) + 1/4 = 0$. what is x?

let $u = \sin(x)$

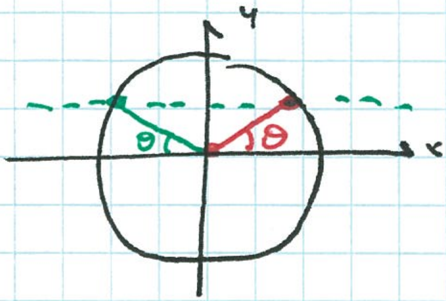
$\Rightarrow u^2 - 2u + 1/4 = 0$

$$u = \frac{2 \pm \sqrt{(-2)^2 - 4(1/4)}}{2} = \frac{2 \pm \sqrt{3}}{2} = 1 \pm \sqrt{3}/2$$

either $\sin(x) = 1 + \sqrt{3}/2$ or $\sin(x) = 1 - \sqrt{3}/2 \approx .134$ so possible.

Nope! cannot be bigger than one.

so $\theta = x = \arcsin(1 - \sqrt{3}/2)$



The angle could be from the <u>definition</u> of $\arcsin(\cdot)$ in quad 1. It could also be the angle in quad 2 whose reference angle is θ. :(

so $x = \arcsin(1 - \sqrt{3}/2)$

or $x = \pi - \arcsin(1 - \sqrt{3}/2)$

} what if it went around <u>many</u> revolutions?

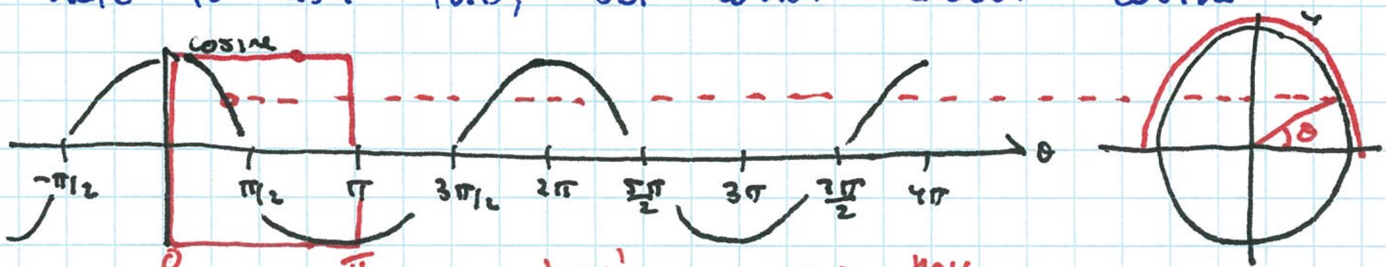
It could be off by <u>any</u> factor of 2π.

so $x = \arcsin(1 - \sqrt{3}/2) + n \cdot 2\pi$

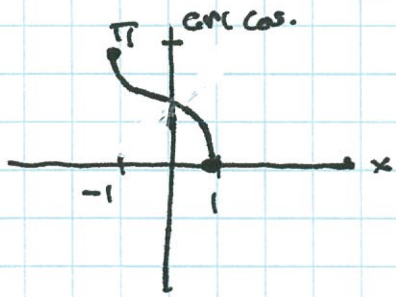
or $x = \pi - \arcsin(1 - \sqrt{3}/2) + n \cdot 2\pi$

where n is <u>any</u> integer and could be negative.

I hate to ask this, but what about cosine?



1-1 in here! Again — not unique, but this is what your calc. uses.



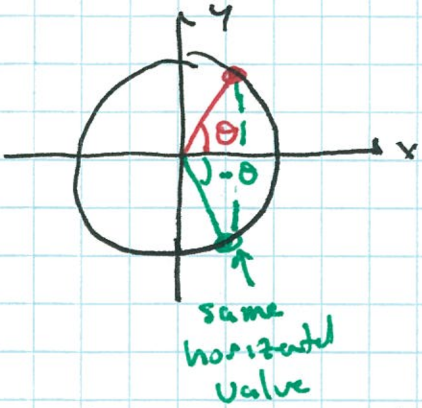
ex/ $e^{\cos(x)} = 2$ what is x? (remember how natural logs seemed?)

$\ln(e^{\cos(x)}) = \ln(2)$ $\approx .693$ so ok.

$\cos(x) = \ln(2)$

$x = \arccos(\ln(2)) = \theta$

yeah, no.

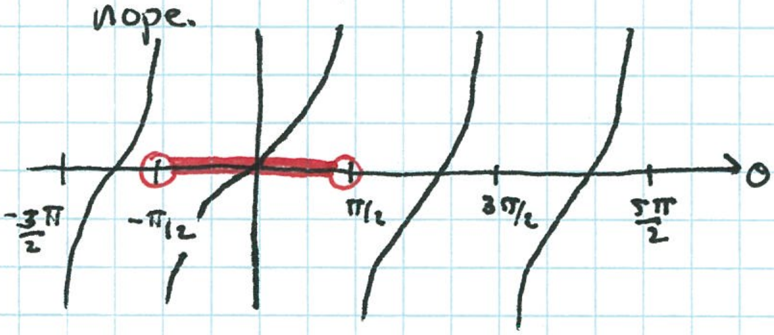


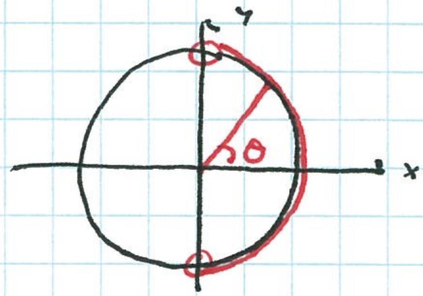
so $x = \arccos(\ln(2)) + n \cdot 2\pi$

or $x = -\arccos(\ln(2)) + n \cdot 2\pi$

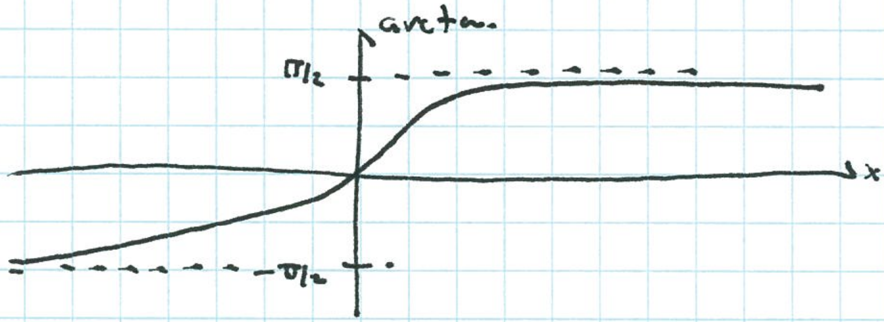
when n is <u>any</u> integer, could be neg.

well, surely tangent must be better? right? please?
nope.





range of arctan $(-\pi/2, \pi/2)$
domain of arctan $(-\infty, \infty)$:(



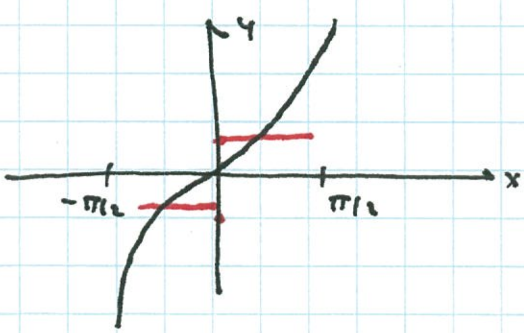
ex/ $\sec^2(x) - 9/8 = 0$ det. x

well... $\sec^2(x) = \tan^2(x) + 1$

so $\tan^2(x) + 1 - 9/8 = 0$

$\tan^2(x) = 9/8 - 1 = 1/8.$

so $\tan(x) = \pm 1/\sqrt{8}.$



so $x = \arctan(1/\sqrt{8}) + n\pi$

or $x = \arctan(-1/\sqrt{8}) + n\pi$

where n is any integer.

Okay. then, that was fun. Can we make this more confusing? why yes. yes we can!

ex/ what is $\sin(\arccos(2/3)) = ?$ are you mad?

well, it is the sine of the angle ...
whose cosine is 2/3.

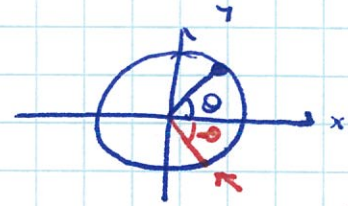
work inside out. (This is backwards from other stuff!)

① whose angle has a cosine of 2/3?

② det. the sine.

$\sin(\theta) = \sqrt{5}/3.$

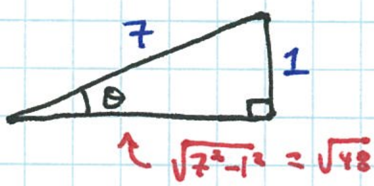
wait!



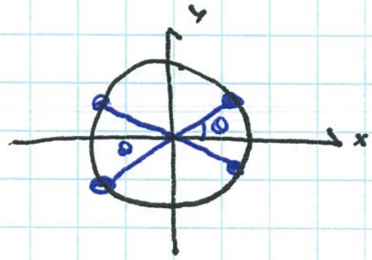
how do you know it is not $-\sqrt{5}/3$?
— depends on context! —

ex/ $\tan(\arcsin(1/7)) = ?$

① let $\theta = \arcsin(1/7)$ so $\sin(\theta) = 1/7$

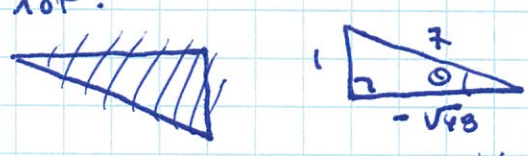


② $\tan(\theta) = 1/\sqrt{48}$



how do we know it is not $\frac{-1}{\sqrt{48}}$.

we do not!

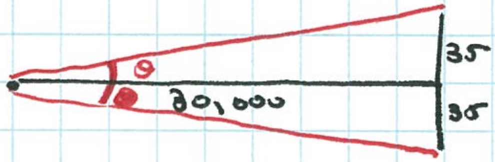


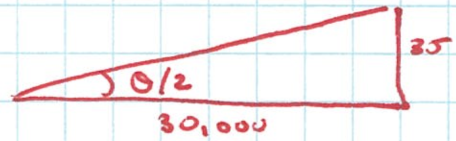
is possible!

depends on context.

Time permitting.

ex/ A weather radar can discern clouds whose angle of resolution is θ radians or more.
The smallest cloud size resolvable @ 30,000 m is 70 m wide. What is the angle of resolution?





$$\tan(\theta/2) = 35/30{,}000$$

$$\theta/2 = \arctan(35/30{,}000)$$

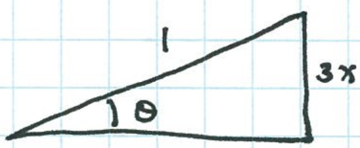
$$\theta = 2\arctan(35/30{,}000) \approx .0011 \text{ rad.}$$

Time permitting

ex/ What is the value of $\tan(\arcsin(3x))$?

note we need $-1 \le 3x \le 1$ or $-\frac{1}{3} \le x \le 1/3$.

① $\theta = \arcsin(3x)$ or $\sin(\theta) = 3x = \frac{3x}{1}$



$\sqrt{1^2-(3x)^2} = \sqrt{1-9x^2}$

② $\tan(\theta) = \dfrac{3x}{\sqrt{1-9x^2}}$ (+ or − ?)

Time permitting

ex/ Det. the value of x that satisfies

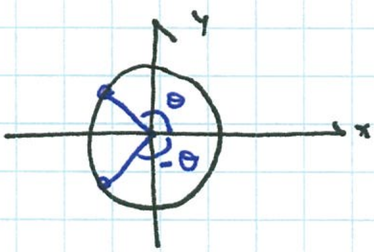
$$\cos^2(x) + 3\cos(x) + 1 = 0.$$

let $u = \cos(x) \Rightarrow u^2 + 3u + 1 = 0$ so $u = \dfrac{-3 \pm \sqrt{9-4}}{2} = \dfrac{-3 \pm \sqrt{5}}{2}$

$u = \dfrac{-3-\sqrt{5}}{2} \approx -2.62$ not possible.

$u = \dfrac{-3+\sqrt{5}}{2} \approx -.38$ possible

$\cos(x) = \dfrac{-3+\sqrt{5}}{2}$



$x = \arccos\left(\dfrac{-3+\sqrt{5}}{2}\right) + n \cdot 2\pi$

or

$x = -\arccos\left(\dfrac{-3+\sqrt{5}}{2}\right) + n \cdot 2\pi$

Time permitting

ex/ $\sec(\arccos(-3/5)) = ?$

$$\sec(\cdot) = \frac{1}{\cos(\arccos(-3/5))} = \frac{1}{-3/5} = -5/3$$

Time permitting

ex/

$\tan(\operatorname{arcsec}(4/3)) = ?$

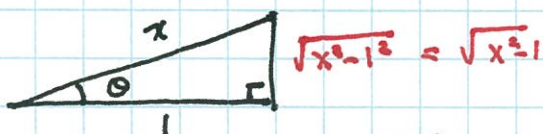
① $\theta = \operatorname{arcsec}(4/3) \Rightarrow \sec(\theta) = 4/3$

② $\tan(\theta) = \pm \sqrt{7}/3.$

Time permitting

ex/ $\tan(\operatorname{arcsec}(x)) = ?$

① $\theta = \operatorname{arcsec}(x) \Rightarrow \sec(\theta) = x = x/1$



② $\tan(\theta) = \pm \frac{\sqrt{x^2-1}}{1} = \pm\sqrt{x^2-1}.$

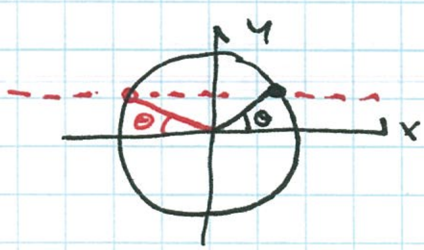
let $x = 4/3$ as above.

Time permitting

ex/ $\operatorname{arccsc}(10/3) = ?$ $\theta = \operatorname{arccsc}(10/3)$

well $\csc(\theta) = 10/3$

$\Rightarrow 1/\sin(\theta) = 10/3$ so $\sin(\theta) = 3/10$



$\theta = \arcsin(3/10) + n\cdot 2\pi$

or

$\theta = \pi - \arcsin(3/10) + n\cdot 2\pi.$

Time permitting

ex/ $\operatorname{arccot}(1/10) = ?$

$\theta = \operatorname{arccot}(1/10) \Rightarrow \cot(\theta) = 1/10$

$\cot(\theta) = \dfrac{1}{\tan(\theta)} = 1/10$ so $\tan(\theta) = 10.$

$\theta = \operatorname{arccot}(1/10) = \arctan(10) + n\pi$

n <u>any</u> integer.

Time permitting

ex/ $\operatorname{arcsec}(x) = ?$ let $\theta = \operatorname{arcsec}(x)$

$\Rightarrow \sec(\theta) = x$

$\sec(\theta) = \dfrac{1}{\cos(\theta)} = x \Rightarrow \cos(\theta) = 1/x$

$\theta = \arccos(1/x)$

Time permitting.

ex/ $3\arctan(2x+1) = 7$

$\arctan(2x+1) = 7/3$

$2x+1 = \tan(7/3) + n\pi$ n is <u>any</u> integer.

$2x = \tan(7/3) + n\pi + 1$

$x = \frac{1}{2}\left[\tan(7/3) + n\pi + 1\right]$