Section 1.2

See ALEKS for HW

- Equation for circles
- std. eqn for a circle
- general forms for circles.

Recall: Given two points, $P(x_1, y_1)$ and $Q(x_2, y_2)$, the distance between them is

$$d(P,Q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

so what?

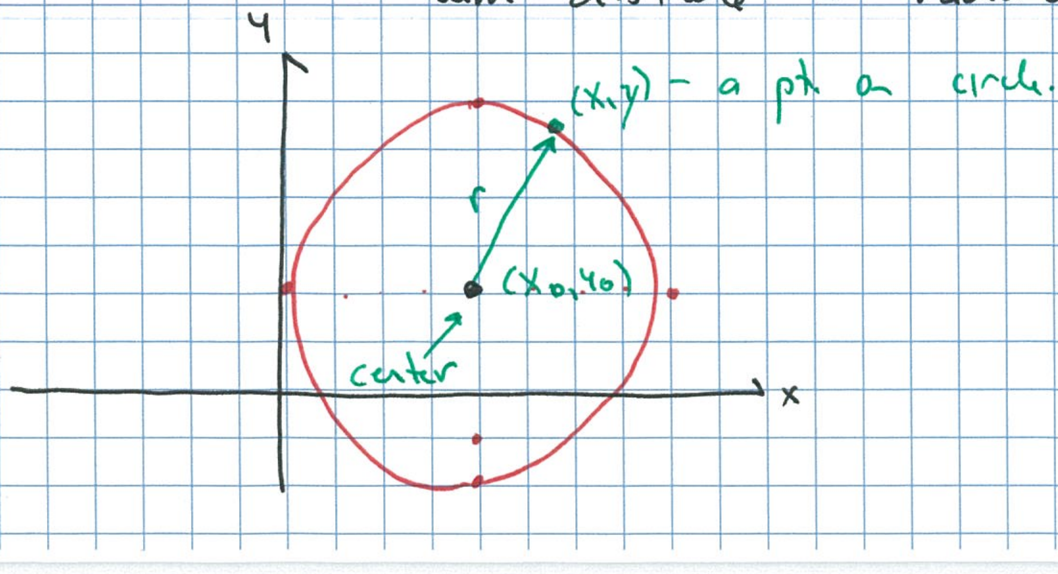
This is a building block to get to other ideas!

Like what?

Well, like circles!

ex/ A circle is the set of __all__ __points__ that are the __same__ __distance__ from a central point.

- "central point" = center of circle.
- "same distance" = radius of circle.



dist. from (x_0, y_0) to <u>any</u> (x, y) on circle is equal to the radius.

or:

$$R = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

$$\text{or } R^2 = (x - x_0)^2 + (y - y_0)^2$$

(x_0, y_0) are fixed #'s.

ex/ $(x-3)^2 + (y+2)^2 = 16$

a circle of radius = 4 centered at $(3, -2)$

ex/ circle centered at $(-1, 5)$ w/ radius 3 is

$$(x - (-1))^2 + (y-5)^2 = 3^2$$

$$\text{or } (x+1)^2 + (y-5)^2 = 9.$$

<u>Note</u> $(x+1)^2 + (y-5)^2 = 9$

$$\Rightarrow (x^2 + 2x + 1) + (y^2 - 10y + 25) = 9$$

$$\underbrace{x^2 + 2x + y^2 - 10y}_{\substack{\text{given this can you get}\\ \text{the center and radius?}}} + 17 = 0$$

we have to complete the square!

How? we start w/ what we want and work backwards.

so... $(x+a)^2 = x^2 + \boxed{2a}x + \underline{\underline{a^2}}$

given this # we figure out what a is and add/subtract in a clever way.

so... suppose

$x^2 + \boxed{2}x = 8$

Then have to be equal.

$(x+a)^2 = x^2 + \boxed{2a}x + a^2$

$2 = 2a$ so $a = 1$

$\Rightarrow a^2 = 1$

$x^2 + 2x + 1 - 1 = 8$

$(x+1)^2 - 1 = 8$

$(x+1)^2 = 9 \Rightarrow x+1 = 3$ or $x+1 = -3$

$\underline{x = 2}$ or $\underline{x = -4}$

but we had

$x^2 + 2x + y^2 - 10y + 17 = 0$ is it a circle? you betcha!

divide and conquer!

$x^2 + 2x$

$(x+a)^2 = x^2 + 2ax + a^2$

$2 = 2a$ so

$a = 1$

$a^2 = 1$

$y^2 \boxed{-10}y$

$(x+b)^2 = x^2 + \cancel{2b}\boxed{2b}y + b^2$

$-10y = 2by$

$-10 = 2b \Rightarrow b = -5$

$b^2 = 25$

(next page)

$$x^2+2x \quad + \quad y^2-10y \quad + 17 = 0$$

$$x^2+2x+1-1 \quad + \quad y^2-10y+25-25 \quad + 17 = 0$$

$$(x+1)^2-1 \quad + \quad (y-5)^2-25 + 17 = 0$$

$$(x+1)^2+(y-5)^2 = 1+25-17 = 9$$

so $(x+1)^2+(y-5)^2=9$

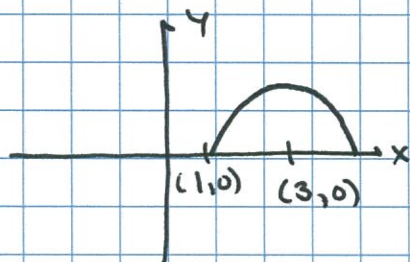
Radius $= \sqrt{9} = 3$

center @ $(-1,5)$

Extra problems (Time Permitting)

if no time state summary on p. 6.

what about semi-circles? (good question!)



so radius = 2
center @ (3,0)
but only top half.

well a whole circle is

$$(x-3)^2+\underbrace{(y-0)^2}_{\text{we want to limit } y} = 2^2 = 4$$

we want to limit y take
solve for y a ~~take~~
the part we want

$$(y-0)^2 = 4-(x-3)^2$$

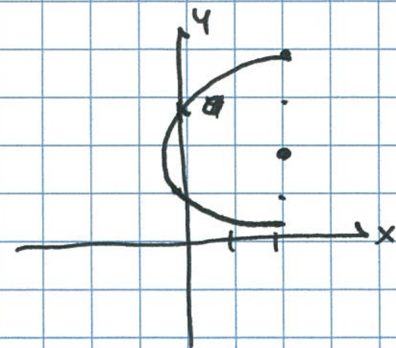
$$y-0 = \pm\sqrt{4-(x+3)^2}$$

which is it?
we want y>0 so it's pos.

so $y = +\sqrt{4-(x+3)^2}$.

ex/ <u>Left</u> half of a circle of radius 3
center at $(2,4)$

step 1 draw a picture!



we want to limit x values.

step 2 start with <u>whole</u> circle.

$$(x-2)^2 + (y-4)^2 = 3^2 = 9$$

step 3 limit x-values.

$$(x-2)^2 = 9-(y-4)^2$$

$$x-2 = \pm\sqrt{9-(y-4)^2}$$

we want $x<2$ so $x-2<0$
use neg. part.

$$x-2 = -\sqrt{9-(y-4)^2}$$

or $x = 2 - \sqrt{9-(y-4)^2}$

ex/ sketch

$$y = -2 - \sqrt{\underbrace{-x^2+6x+16}_{}}$$

<u>not</u> in a nice form.

we <u>want</u> $R^2 - (x-a)^2$

complete the square. :(

$$-x^2 + \boxed{6x} + 16$$

$$-(x+a)^2 = -(x^2 + 2ax + a^2)$$

$$= -x^2 \boxed{-2ax} - a^2$$

$6x = -2ax \quad \Rightarrow \quad 6 = -2a$ or $\underline{a = -3}$

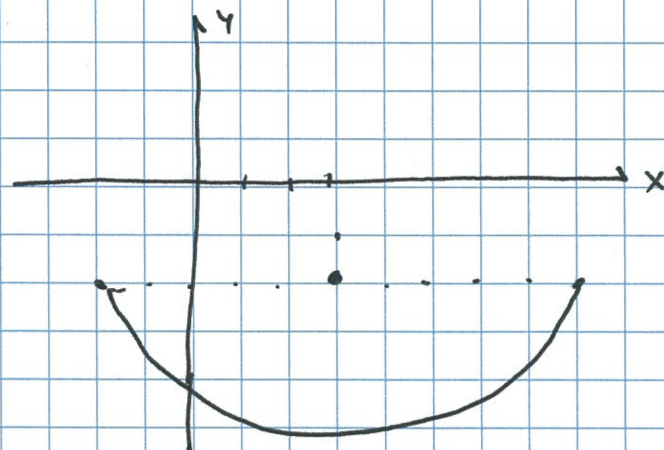
$a^2 = 9$

$$y = -2 - \sqrt{-x^2 + 6x - 9 + 9 + 16}$$

$$y = -2 - \sqrt{-(x^2 - 6x + 9) + 25}$$

$$y = -2 - \sqrt{25 - (x-3)^2}$$

$\Rightarrow$ bottom half of a circle of radius 5 center @ $(3, -2)$



summary

so we have

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

↑ standard form of a circle of radius R centered @ (x_0, y_0)

$$x^2 + y^2 + ax + bx + c = 0$$

↑ general form of a circle.

(Only if time permits)

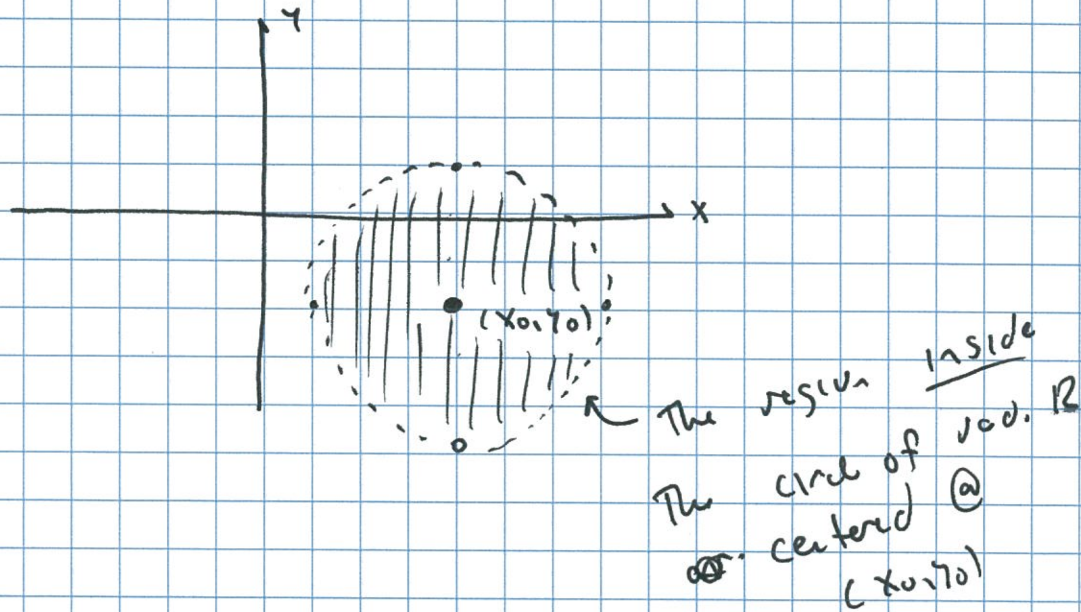
so $\cancel{x^2}$ $(x-x_0)^2 + (y-y_0)^2 = R^2$

represents <u>all</u> the points whose distance from (x_0, y_0) is <u>equal</u> to R.

What about all the points whose distance is <u>less</u> than R?

$$d^2 = (x-x_0)^2 + (y-y_0)^2$$

so $d^2 = \underline{(x-x_0)^2 + (y-y_0)^2 < R^2}$



$$(x-x_0)^2 + (y-y_0)^2 \le R^2$$

would also contain the edge of the circle.

ex/ what is up w/

$$x^2 + 4x + y^2 - 8y < 16 \quad ?$$

well...

$x^2 + \boxed{4x}$

$(x+a)^2 = x^2 + \boxed{2ax} + a^2$

$4x = 2ax$

$\Rightarrow a = 2$

$a^2 = 4$

$y^2 \boxed{-8y}$

$(y+b)^2 = y^2 \boxed{+2by} + b^2$

$-8y = 2by$

$\Rightarrow b = -4$

$b^2 = 16$

$$(x^2 + 4x + 4 - 4) + (y^2 - 8y + 16 - 16) < 16$$

$$(x+2)^2 - 4 + (y-4)^2 - 16 < 16$$

$$(x+2)^2 + (y-4)^2 < 16 + 16 + 4 = 36$$

everything inside a circle of radius 6 centered @ $(-2, 4)$

oops.