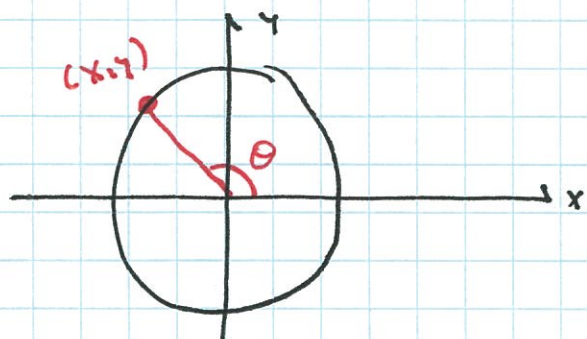Try function given the angle
Reference Angles

we have done this!

$$\cos(\theta) = x/1$$
$$\sin(\theta) = y/1$$
$$\tan(\theta) = y/x$$

$$\sec(\theta) = 1/x$$
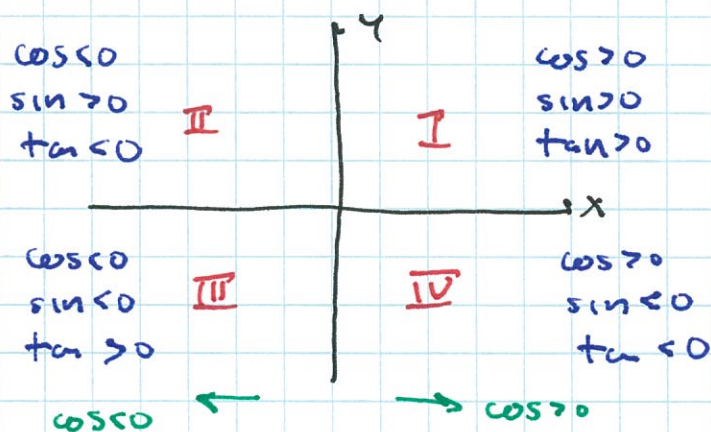$$\csc(\theta) = 1/y$$
$$\tan(\theta) = x/y$$

what do we do in practice?   Use a calculator!
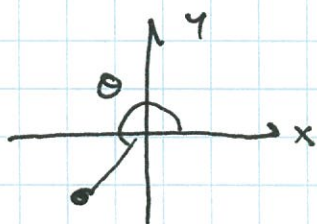
The calculator is a tool with limitations.
We still have to think and understand!

Quadrant II:
$\cos < 0$
$\sin > 0$
$\tan < 0$

Quadrant I:
$\cos > 0$
$\sin > 0$
$\tan > 0$

Quadrant III:
$\cos < 0$
$\sin < 0$
$\tan > 0$

Quadrant IV:
$\cos > 0$
$\sin < 0$
$\tan < 0$

$\sin > 0$
$\sin < 0$

$\cos < 0$ $\cos > 0$

units matter!
rad vs. deg.

quadrants matter
~ keep track
by context

ex/ θ is in the 4th quadrant + $\cos(\theta) = 1/4$.
what is $\sin(\theta)$?

$$\sin(\theta) < 0 \qquad \text{and} \qquad \sin^2(\theta) + \cos^2(\theta) = 1$$
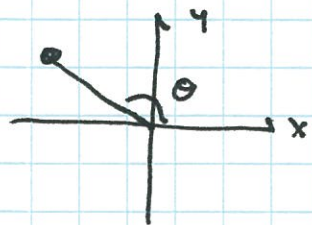$$\sin^2(\theta) + (1/4)^2 = 1$$
$$\sin^2(\theta) = 1 - (1/4)^2 = 1 - \tfrac{1}{16} = 15/16$$
$$\sin(\theta) = -\sqrt{15}/4.$$

also $\tan(\theta) = \dfrac{-\sqrt{15}/4}{1/4} = -\sqrt{15}.$

ex/ θ is in the 2nd quadrant and $\sin(\theta) = 1/3$, what is $\tan(\theta)$?

$\tan(\theta) < 0$, $\cos(\theta) < 0$, $\quad \sin^2(\theta) + \cos^2(\theta) = 1$

$\Rightarrow (1/3)^2 + \cos^2(\theta) = 1$

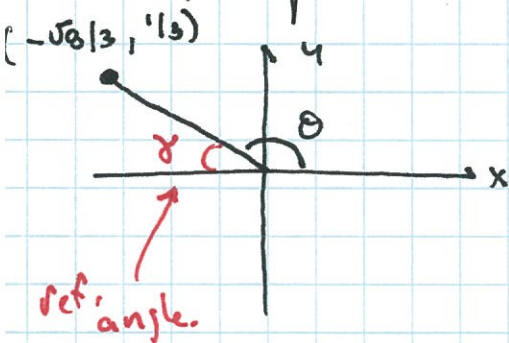$\cos^2(\theta) = 1 - (1/3)^2 = 1 - 1/9 = 8/9$

$\cos(\theta) = -\sqrt{8}/3$

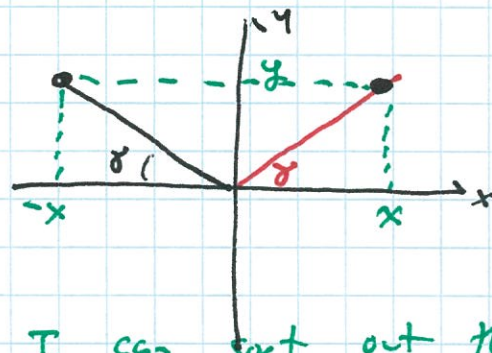$\Rightarrow \tan(\theta) = \dfrac{1/3}{-\sqrt{8}/3} = -1/\sqrt{8}$.

<u>Note:</u> we have another tool: The <u>reference angle</u>.

If an angle is in std. position the <u>reference angle</u>, γ, is the acute angle between the termind side and the horizontal axis.

In the prev. example we had

$(-\sqrt{8}/3, 1/3)$
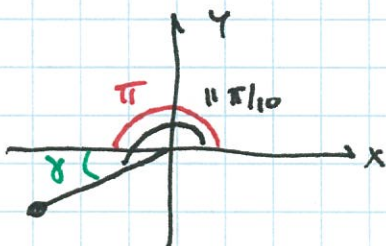
ref angle.

note it is a mirror image

If I can sort out the 1st quadrant I can handle the others!

ex/ what is the ref. angle for $\theta = \frac{11}{10}\pi$?

$\pi < \theta < \frac{3\pi}{2}$
so Q III

$\gamma + \pi = \frac{11\pi}{10}$

$\gamma = \frac{11}{10}\pi - \pi = \pi/10$.

②

ex/ Determine the cosine of ~~the~~ $87\pi/4$. ugh...

$$4\overline{\smash)87} \quad 21$$
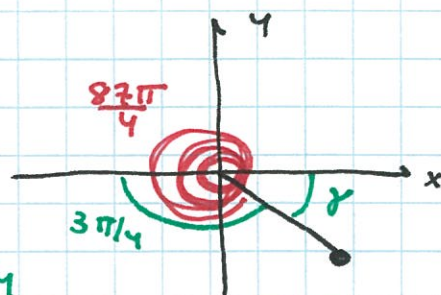$$\frac{8}{7}$$
$$\frac{4}{3}$$

$$\frac{87\pi}{4} = 21\pi + \frac{3}{4}\pi$$

$$= 20\pi + \pi + \frac{3}{4}\pi$$

$\underbrace{}_{10 \text{ rev.}}$ $\underset{\frac{1}{2}\text{ rev.}}{\uparrow}$ $\underset{3/8 \text{ rev.}}{\uparrow}$

$$\frac{3\pi}{4} + \gamma = \pi \Rightarrow \gamma = \pi - \frac{3\pi}{4} = \pi/4$$

Ref. angle $= \pi/4$.

$\cos\left(\dfrac{87\pi}{4}\right) = \cos(\pi/4) = \sqrt{2}/2$.

$\sin\left(\dfrac{87\pi}{4}\right) = -\sin(\pi/4) = -\sqrt{2}/2$.

$\cos > 0 \; / \sin < 0$

---

ex/ what is the secant of $43\pi/4$?

$$4\overline{\smash)43} \quad 10$$
$$\frac{4}{3}$$

$$\frac{43\pi}{4} = 10\pi + \frac{3}{4}\pi$$

$\underset{5 \text{ rev.}}{\uparrow}$ $\underset{3/8 \text{ rev.}}{\uparrow}$

$$\gamma + 3\pi/4 = \pi \Rightarrow \gamma = \pi/4$$

Ref. angle $= \pi/4$.

$\sec\left(\dfrac{43\pi}{4}\right) = -\dfrac{1}{\cos(\pi/4)} = -\sqrt{2}$.

what is wrong w/ me

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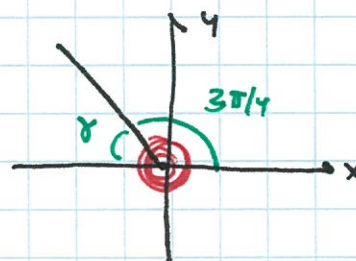
ex/ what is the cosecant of 15π?

$$15\pi = 14\pi + \pi$$

$\underset{7 \text{ rev.}}{\uparrow}$ $\underset{\frac{1}{2}\text{ rev.}}{\uparrow}$

$$\pi + \gamma = \pi \Rightarrow \gamma = 0$$

The ref. angle is 0

$$\csc(0) = \frac{1}{\sin(0)} = \cdots \quad \text{oh oh!} \\ \text{not defined!}$$

③