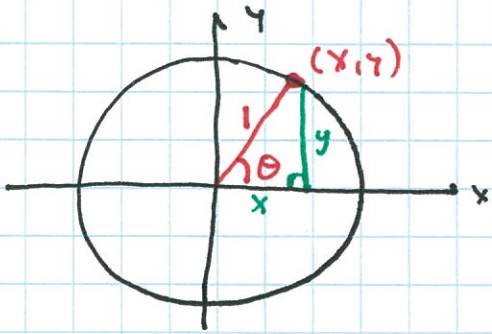
4.3 Right angle trig.

ALEKS

Right triangle and trig functions
Fundamental trig. identities
trig functions and geometry

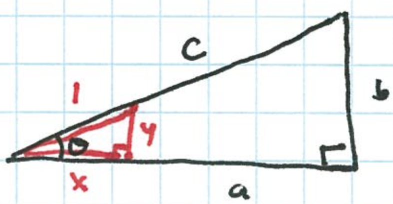
Last time



$\sin(\theta) = y$
$\cos(\theta) = x$
$\tan(\theta) = y/x$

$\sin^2\theta + \cos^2\theta = 1$

cuz of similar triangles we have that

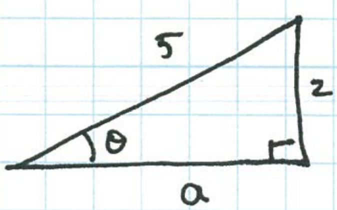


$\sin(\theta) = y/1 = b/c$
$\cos(\theta) = x/1 = a/c$
$\tan(\theta) = y/x = b/a$

so what? If we know something about sine/cosine/tangent of θ we can work w/ __any__ triangle!

(b.t.w. - you __always__ know the pythagorean identity is true!) but only for right triangles!

ex/



what is $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.

$\sin(\theta) = 2/5$ (do the easy one 1st!)

$a = ?$

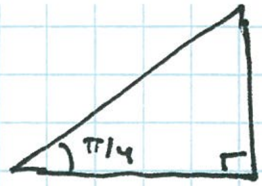
$a^2 + 2^2 = 5^2$
$a^2 = 5^2 - 2^2 = 21$

$a = \pm\sqrt{21}$
?
sign depends on context!

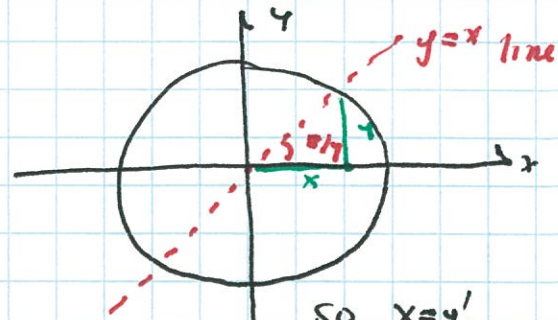
$\cos(\theta) = \frac{\sqrt{21}}{5}$

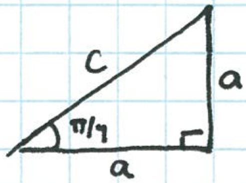
$\tan(\theta) = 2/\sqrt{21}.$

ex/



what's up w/ this?





$a^2 + a^2 = c^2$

$2a^2 = c^2$

~~[illegible]~~

$\Rightarrow c = \sqrt{2} \cdot a$ hyp. pos!

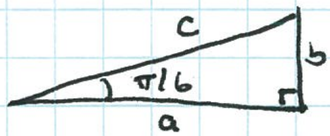
so $x = y$!

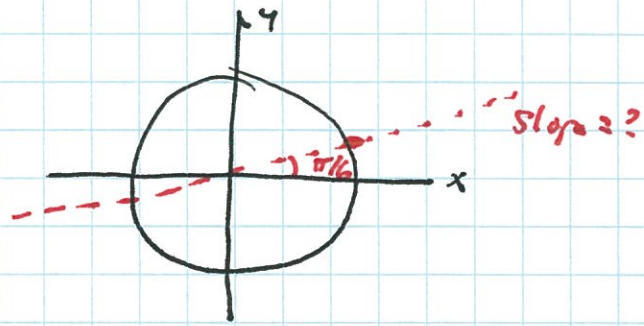
$\sin(\frac{\pi}{4}) = a/a\sqrt{2} = 1/\sqrt{2}$

$\cos(\pi/4) = a/a\sqrt{2} = 1/\sqrt{2}$

$\tan(\theta) = a/a = 1.$

ex/



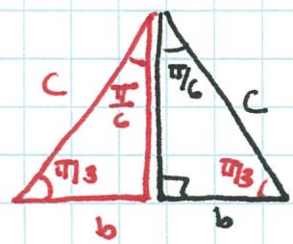


Ugly trick:

turn it on its side.

$\pi/2 - \pi/6 = \pi/3.$

$\uparrow \frac{1}{3}(\pi)$!



An equilateral triangle!

$2b = c \Rightarrow b = 1/2\, c$

$a^2 + c^2/4 = c^2$

$a^2 = 3/4\, c^2$

$a = \sqrt{3}/2\, c$

$\sin(\pi/6) = \frac{1/2\, c}{c} = 1/2$

$\cos(\pi/6) = \frac{\sqrt{3}/2\, c}{c} = \sqrt{3}/2$

$\tan(\pi/6) = \frac{1/2}{\sqrt{3}/2} = 1/\sqrt{3}.$

Also... $\sin(\pi/3) = \frac{\sqrt{3}/2\, c}{c} = \sqrt{3}/2$

$\cos(\pi/3) = \frac{1/2\, c}{c} = 1/2$

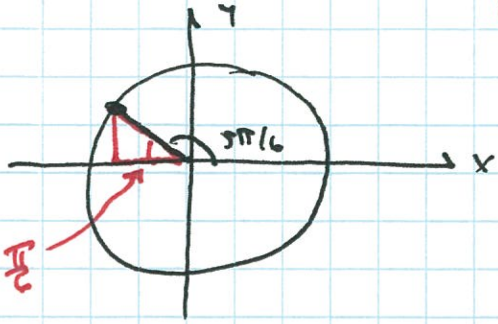
$\tan(\pi/3) = \frac{\sqrt{3}/2\, c}{1/2\, c} = \sqrt{3}.$

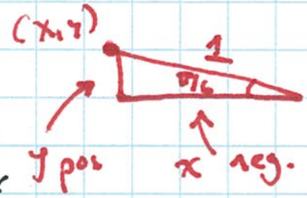
This is a <u>very</u> special case! so what?

why bother w/ acute angles and right triangles?

well - what is $\cos(5\pi/6)$? I do not know.

A - reduce to a previously solved problem!

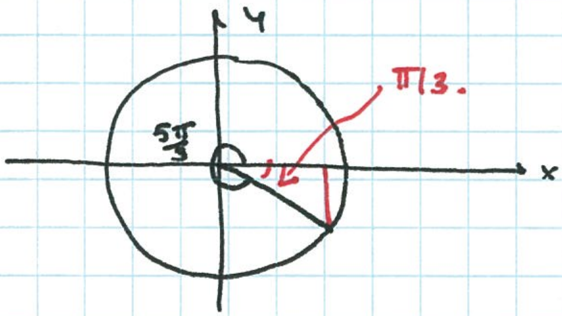


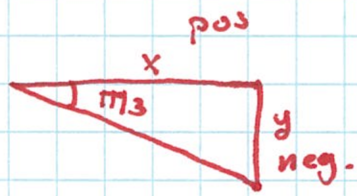


$x = \cos(\frac{5\pi}{6}) = -\cos(\frac{\pi}{6}) = \frac{-\sqrt{3}}{2}$

$y = \sin(\frac{5\pi}{6}) = \sin(\pi/6) = 1/2$

Q/ what is $\sin(5\pi/3)$?

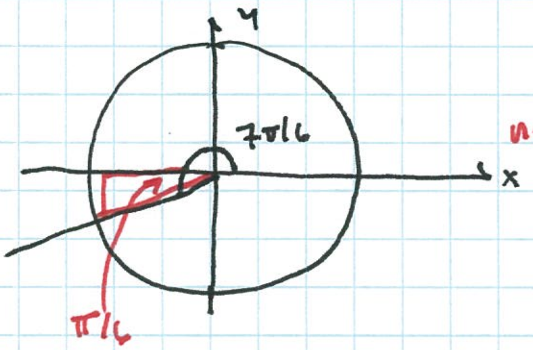


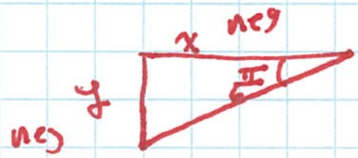


$$y = \sin(5\pi/3) = -\sin(\pi/3) = \sqrt{3}/2$$

$$x = \cos(5\pi/3) = \cos(\pi/3) = 1/2.$$

Q) what is $\tan(7\pi/6)$?





$$x = \cos(7\pi/6) = -\cos(\pi/6) = -\sqrt{3}/2$$

$$y = \sin(7\pi/6) = -\sin(\pi/6) = -1/2$$

$$\Rightarrow \tan(7\pi/6) = \frac{-1/2}{-\sqrt{3}/2} = 1/\sqrt{3}.$$

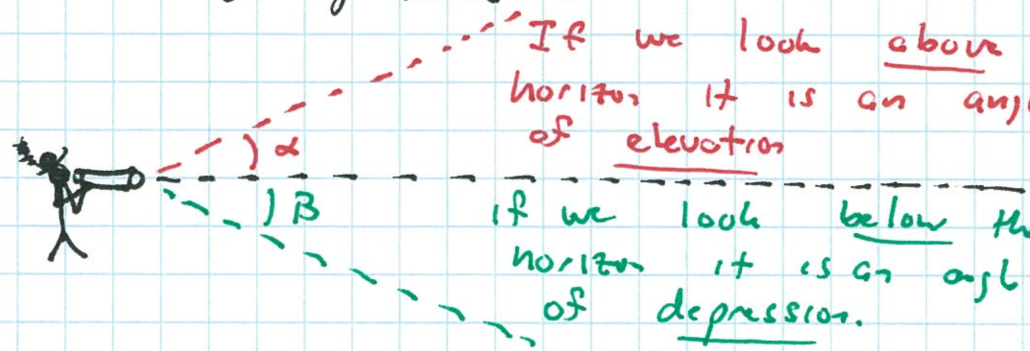
(pos!)

what nonsense is this? – we gotta be careful!

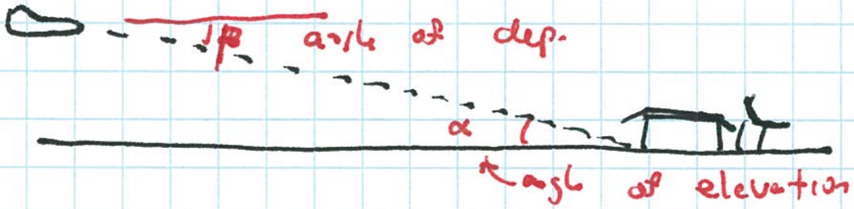
- we can take diff. geometries and condense them in to "nice" triangles.
- we can then translate them back into what we need/want.
- WE <u>HAVE</u> TO <u>DRAW PICTURES</u> AND BE <u>CAREFUL</u> ABOUT THE COORDINATE SYSTEM!

The language we use is precise & difficult to parse. we need some definitions.

angles are w.r.t. some initial line!
If we use an imaginary horizontal line ("horizon")
Then

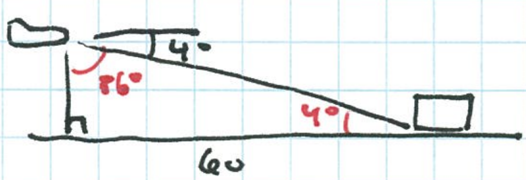


So if an airplane is 60 miles from the Athens Airport (KAHN!!!)



note $\alpha = \beta$ in this case.

ex/ An airplane is 60 miles west of the Athens Airport. The pilot's angle of depression is 4°. what is the plane's elevation?



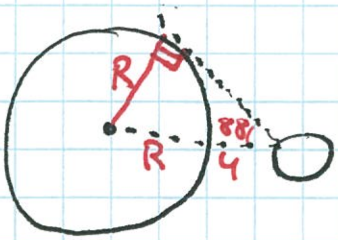
$\tan(4°) = h/60$

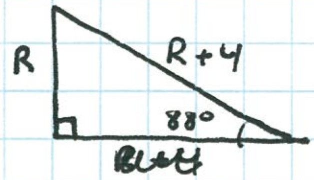
$h = 60\tan(4°) \approx 4.20$ mi

units!

hmmm

ex/ In Rogue 1 the Death star shoots at a planet at the end of the movie and kills everybody. Suppose instead they ~~wat~~ want to fire a playful warning shot that just grazes the planet. If the Death star is 4 km above the surface of the planet and turns 88° away from directly below it what is the radius of the planet? (Darth Vader is curious.)





$$\sin(88^\circ) = \frac{R}{R+4}$$

$R = ?$

$$(R+4)\sin(88^\circ) = R$$

$$R\sin(88^\circ) + 4\sin(88^\circ) = R$$

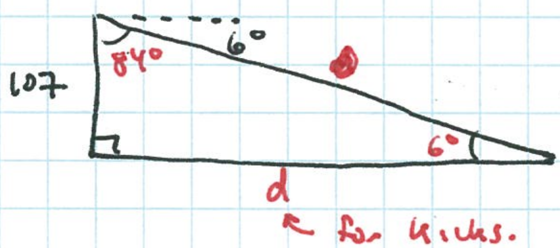
$$R\sin(88^\circ) - R = -4\sin(88^\circ)$$

$$R[\sin(88^\circ) - 1] = -4\sin(88^\circ)$$

$$R = \frac{-4\sin(88^\circ)}{\sin(88^\circ) - 1} \approx 6562 \text{ km.}$$

← units!

ex/ ~~prince stands~~ Prince William stands on the white cliffs of Dover. He waves to his true love, Kate, who is on a fishing boat. The cliff is 107m tall, ~~and~~ and the prince's angle of dep. is 6°. How far away is Kate?



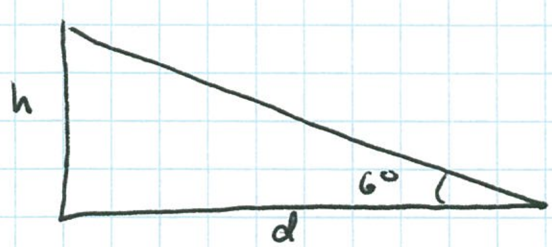
$$\tan(84^\circ) = d/107$$

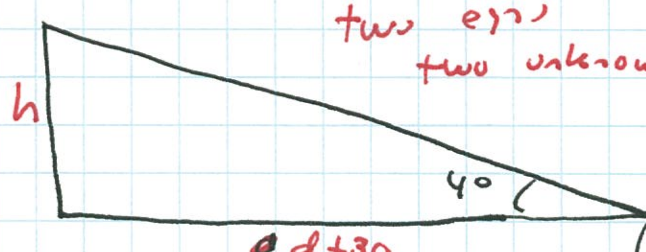
$$d = 107\tan(84^\circ) \approx 1018 \text{ m.}$$

If William wants to blow a kiss, how far will it travel?

$$\cancel{\cos(6^\circ)} \quad \cos(84^\circ) = \frac{107}{r} \Rightarrow r = \frac{107}{\cos(84^\circ)} \approx 1023.6 \text{ m.}$$

ex/ A surveyor points a transit at a bluff and measures an angle of elevation of 6°. The surveyor backs up 30m and measures an angle of elevation of 4°. How tall is the bluff?





two eqn's
two unknowns!

Solve for one & subst. into eqn. (we want h!)

$\tan(6^\circ) = \frac{h}{d}$ or $d\tan(6^\circ) = h$

so $d = \frac{h}{\tan(6^\circ)}$

$\tan(4^\circ) = \frac{h}{d+30}$

$$\tan(4^\circ) = \frac{h}{h/\tan(6^\circ) + 30}$$

$$\tan(4^\circ)\left[h\frac{1}{\tan(6^\circ)} + 30\right] = h$$

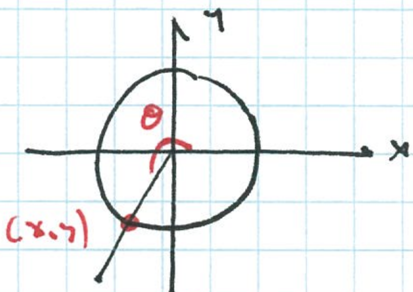
$$h\frac{\tan(4^\circ)}{\tan(6^\circ)} + 30\tan(4^\circ) = h$$

$$h\frac{\tan(4^\circ)}{\tan(6^\circ)} - h = -30\tan(4^\circ)$$

$$h\left[\frac{\tan(4^\circ)}{\tan(6^\circ)} - 1\right] = -30\tan(4^\circ) \Rightarrow h = \frac{-30\tan(4^\circ)}{\frac{\tan(4^\circ)}{\tan(6^\circ)} - 1}$$

≈ 6.26 m.

ex/ if $\tan(\theta) = \sqrt{8}$ and θ is in the 3rd quad. what is $\sec(\theta)$?



$\sec(\theta) = 1/x$ is neg.

$\tan(\theta) = \sqrt{8} = y/x$

$\Rightarrow y = x\sqrt{8}$ $\quad y^2 = x^2 \cdot 8$

$x^2 + y^2 = 1$

$x^2 + 8x^2 = 1$

$9x^2 = 1$

$x^2 = 1/9$ so $x = -1/3$

$\sec(\theta) = \frac{1}{(-1/3)} = -3.$

<u>note</u> $\tan(\theta) = y/x$ $\quad y = \sin(\theta)$ $\quad x = \cos(\theta) \Rightarrow \tan(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

and $\sec(\theta) = \frac{1}{\cos(\theta)}$

so... Algebra!

$$(\tan(\theta))^2 \cdot (\cos(\theta))^2 = \left(\frac{\sin(\theta)}{\cos(\theta)}\right)^2 \cos^2(\theta)$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta = \sin^2\theta.$$

what is $\tan^2\theta + 1$?

$$\frac{\sin^2(\theta)}{\cos^2(\theta)} + 1 = \frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \frac{\sin^2(\theta) + \cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)} = \sec^2(\theta)$$

wait - how did I know to look to this?

we know that $\sin^2(\theta) + \cos^2(\theta) = 1$

$$\Rightarrow \frac{1}{\cos^2(\theta)} \cdot [\sin^2(\theta) + \cos^2(\theta)] = \frac{1}{\cos^2(\theta)} \cdot 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \sec^2(\theta)$$

$$\underline{\tan^2(\theta) + 1 = \sec^2(\theta)} \checkmark$$

← a new identity!

ex/ $(2\sec^2(\theta) - 2)\cos(\theta) = ?$

$$2(\sec^2(\theta) - 1)\cos(\theta) = 2\tan^2(\theta)\cos\theta$$

$$= 2\,\frac{\sin^2(\theta)}{\cos^2(\theta)}\cos(\theta)$$

$$= 2\sin^2(\theta)/\cos(\theta)$$