Math 1060: Lecture Notes

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Summer 2017*

Abstract

Lecture notes for the section of *Math 1060: Mathematics of Decision Making* I taught in Summer 2017.

1 Management Science and Graph Theory

Mathematics is a tool which allows us to take complicated, noisy, real-world problems and isolate the interesting or difficult bits. In this unit, we will be concerned with the notion of a *graph*, which abstracts the notion of *binary connections*, i.e., two objects of interest are either connected or not connected. In this unit, we will see how to turn appropriate real-world problems into problems about graphs. We will also investigate some algorithms to solve efficiency problems in this context.

1.1 Urban services

Lesson 1

- Talk about syllabus, course expectations etc.
- In this course we'll distill real-world problems into mathematical problems, and learn how to solve the mathematical problems.
- Motivating example: Meter reader starts at town hall and wants to check all meters (on foot) as efficiently as possible.
  1. Start and end at same place
  2. Walks on every side of the street with meters
  3. Repeats streets as little as possible.
- A **graph** is a mathematical object which consists of two pieces of information.
  1. Vertex information: A finite set of **vertices**.
  2. Edge information: A finite set of **edges** which connect pairs of vertices.

  In the future we may deal with **directed graphs** (**digraphs** for short) in which edges point from their start vertex to their end vertex.
- Examples of graphs and their visual representations. Note that *one graph may be drawn in many, many different ways – these are still the same graph.*
- Definitions:
  1. A **vertex** x **is connected** to a vertex y if there is a sequence of vertices, each connected to the next, which starts at x and ends with y.
  2. A **graph is connected** if every vertex is connected to every other vertex.
- Examples of real-world graphs:
  1. Facebook / Real-World friends. Vertices are people. Edges connect those who are friends.
  2. Airports and Direct flights. Vertices are airports. Edges correspond to direct flights. This is a digraph.
- Reality check:
  1. Should the airport graph be connected?

*Last updated June 5, 2017.

2. If you built a friendship graph out of your graduating class from high school, would it be connected? Why or why not?

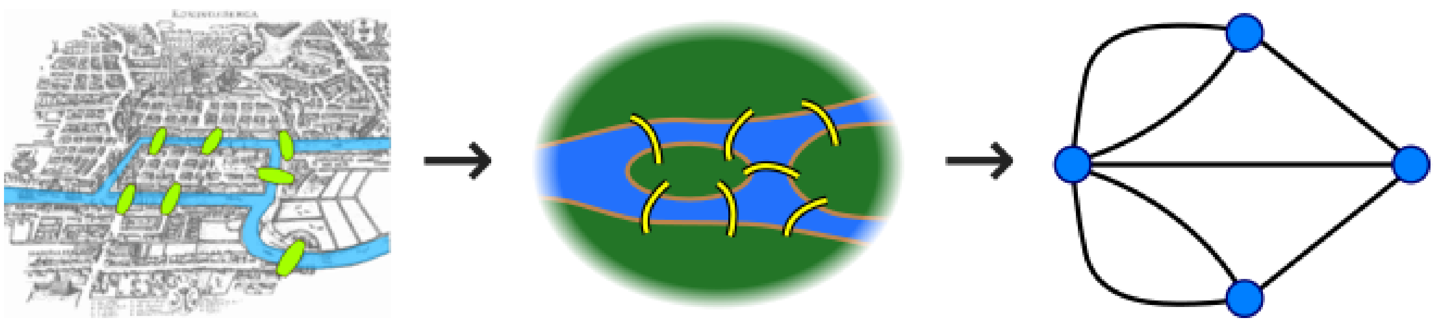
- Definitions:
  1. A **path** is a sequence of vertices, each connected to the next. We already saw this implicitly in the definition of *connected*.
  2. A **circuit** is a path that starts and ends at the same vertex.

  You'd want to find a circuit if you're planning a pilot's daily route. You'd want to find a path if you're looking for someone to introduce you to someone who knows Elon Musk.

- Back to the meter checker. Build this into a graph. Translate the problem into this:

  We are looking for a *circuit* which *passes through each edge* at least once, but no more than necessary.

- Definition: An **Euler circuit** (pronounced Oy-ler) is a circuit which passes through every edge exactly once.
- Example: the seven bridges of Königsberg. Real world problem → math problem



- Think about:
  1. Does the graph from class have an Euler circuit?
  2. Why would the graph of Königsberg not have an Euler circuit?

1.2 Chapter 2: Business efficiency

1.3 Chapter 3: Planning and scheduling

2 Voting and Social Choice

2.1 Chapter 9: Social choice: thie impossible dream

2.2 Chapter 10: The manipulability of voting systems

2.3 Chapter 11: Weighted voting systems

3 Fairness and Apportionment

3.1 Chapter 13: Fair division

3.2 Chapter 14: Apportionment

4 Further Topics

4.1 Identification Numbers

4.2 Information Science