

Volumes and areas. Sphere of radius r : $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$.

Cylinder of radius r and height h : $V = \pi r^2 h$, $A = 2\pi r^2 + 2\pi r h$.

Cone of base radius r and height h : $V = \frac{1}{3}\pi r^2 h$, $A = \pi r \sqrt{r^2 + h^2}$.

Equation of a circle and line. The equation of a circle of radius r centred at the point (a, b) is $(x - a)^2 + (y - b)^2 = r^2$.

The slope of a line through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The equation of a line with slope m through the point $P_1(x_1, y_1)$ is $y - y_1 = m(x - x_1)$.

Summations.

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

Trigonometric identities.

$$1 = \sin^2(x) + \cos^2(x), \quad 1 + \tan^2(x) = \sec^2(x), \quad 1 + \cot^2(x) = \csc^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y), \quad \sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y), \quad \cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}, \quad \tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x), \quad \tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}, \quad \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

Inverse trigonometric functions.

$$\sin^{-1}(y) = x \iff x = \sin(y) \text{ and } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\cos^{-1}(y) = x \iff x = \cos(y) \text{ and } 0 \leq x \leq \pi$$

$$\tan^{-1}(y) = x \iff x = \tan(y) \text{ and } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

	θ	$\sin(\theta)$	$\cos(\theta)$
	0	0	1
	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
	$\frac{\pi}{2}$	1	0

Useful trigonometric values.

Derivatives of elementary functions. We assume that f and g are differentiable functions, and that the various combinations of functions are defined; a, b, c are real numbers.

$$\text{Linearity: } \frac{d}{dx} (af(x) + bg(x)) = af'(x) + bg'(x)$$

$$\text{Chain rule: } \frac{d}{dx} (f \circ g(x)) = f'(g(x)) g'(x)$$

$$\text{Product rule: } \frac{d}{dx} ((fg)(x)) = f(x)g'(x) + f'(x)g(x)$$

$$\text{Quotient rule: } \frac{d}{dx} \left(\frac{f}{g}(x) \right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\text{Power rule: } \frac{d}{dx} (x^n) = nx^{n-1}, \quad \frac{d}{dx} (c) = 0$$

$$\text{Exponential functions: } \frac{d}{dx} (e^x) = e^x, \quad \frac{d}{dx} (b^x) = b^x \ln(b)$$

$$\text{Logarithmic functions: } \frac{d}{dx} (\ln|x|) = \frac{1}{x}, \quad \frac{d}{dx} (\log_b(x)) = \frac{1}{x \ln(b)}$$

Trigonometric functions:

$$\frac{d}{dx} (\sin(x)) = \cos(x), \quad \frac{d}{dx} (\cos(x)) = -\sin(x), \quad \frac{d}{dx} (\tan(x)) = \sec^2(x)$$

$$\frac{d}{dx} (\csc(x)) = -\csc(x) \cot(x), \quad \frac{d}{dx} (\sec(x)) = \sec(x) \tan(x), \quad \frac{d}{dx} (\cot(x)) = -\csc^2(x)$$

Inverse trigonometric functions:

$$\frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx} (\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx} (\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} (\csc^{-1}(x)) = -\frac{1}{x\sqrt{x^2-1}}, \quad \frac{d}{dx} (\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}, \quad \frac{d}{dx} (\cot^{-1}(x)) = -\frac{1}{1+x^2}$$