TEST 3 REVIEW

- You are also responsible for understanding all homework, quizzes, collabs and lectures.
- The following problems will NOT be graded. Do NOT turn in these problems.

1. $f(x) = 2x + 4$ and $g(x) = 2x^2 + 3$. Find $(f \circ g)(2)$ and $(g \circ f)(2)$.

2.

For $f(x) = \frac{x-3}{x+4}$ and $g(x) = \frac{x+5}{x-6}$, find the following composite functions and state the domain of each.

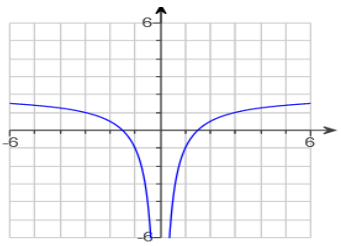
a. f o g **b.** g o f

3.

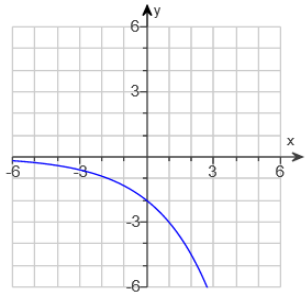
The surface area S (in square inches) of a cylindrical pipe with length 12 inches is given by $S(r) = 2\pi r^2 + 24\pi r$, where r is the radius of the piston (in inches). If the radius is increasing with time t (in minutes) according to the formula $r(t) = \frac{1}{6}t^2$, $t \geq 0$, find the surface area S of the pipe as a function of the time t.

4. For each function, determine whether it is one-to-one.

a.



b.



5.

The function $f(x) = x^2 - 3$, $x \geq 0$ is one-to-one.
(a) Find the inverse of f and check the answer.
(b) Find the domain and the range of f and f^{-1}.
(c) Graph f, f^{-1}, and $y = x$ on the same coordinate axes.

6.

The function $f(x) = \frac{2x+7}{x+2}$ is one-to-one.

(a) Find its inverse and check your answer. (b) Find the domain and the range of f and f^{-1}.

7.

The ideal body weight W for men (in kilograms) as a function of height h (in inches) is given by the following function.

$$W(h) = 50 + 2.2(h - 60)$$

(a) What is the ideal weight of a 6-foot male?

(b) Express the height h as a function of weight W.

(c) What is the height of a male who is at his ideal weight of 80 kilograms?

8.

Determine whether the function given by the table is linear, exponential, or neither. If the function is linear, find a linear function that models the data; if it is exponential, find an exponential function that models the data.

x	f(x)
-1	$\frac{8}{7}$
0	8
1	56
2	392
3	2744

9.

Determine whether the function given by the table is linear, exponential, or neither. If the function is linear, find a linear function that models the data; if it is exponential, find an exponential function that models the data.

x	f(x)
-1	6
0	8
1	10
2	12
3	14

10.

Use transformations to graph the function. Then determine its domain, range, and horizontal asymptote.

$$g(x) = 2 + 5^{x+1}$$

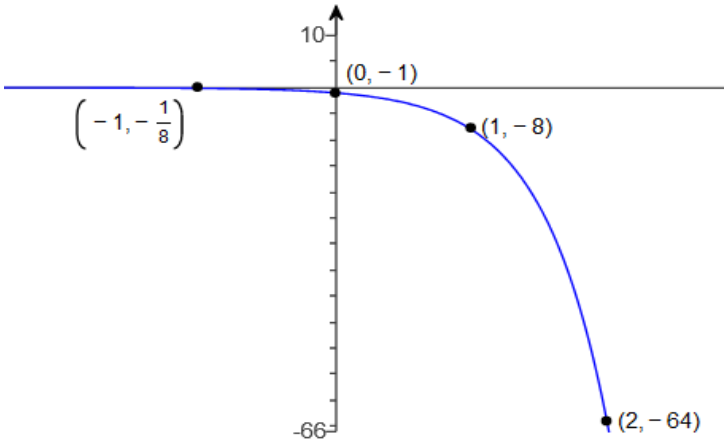
11.

Solve the equation.

$$16^{-x+18} = 32^{x}$$

12.

Determine the exponential function whose graph is given.



13.

The percentage of patients P who have survived t years after initial diagnosis of a certain disease is modeled by the function $P(t) = 100(0.6)^t$.

(a) According to the model, what percent of patients survive 1 year after initial diagnosis?
(b) What percent of patients survive 4 years after initial diagnosis?
(c) Explain the meaning of the base 0.6 in the context of this problem.

14. Graph the functions $f(x) = 2^x$ and $f(x) = \log_2 x$ on the same set of axes. For each graph, you should find and plot at least 5 key points. Symmetry should be clearly shown.

15. Find the exact solution of $5e^{0.2x} = 12$

16. Find the exact solution of $4 \bullet 10^{7-x} = 5$

17.

The atmospheric pressure p on a balloon or an aircraft decreases with increasing height. This pressure, measured in millimeters of mercury, is related to the height h (in kilometers) above sea level by the formula $p = 760\, e^{-0.145h}$.

Find the height of an aircraft if the atmospheric pressure is 298 millimeters of mercury.

18. Expand the following.

(a) $\ln \frac{x+7}{e^{2x}}$

(b)

$$\ln \frac{6x\sqrt{1+5x}}{(x-4)^{11}}, \quad x > 4$$

19.

Write the expression as a single logarithm. Express powers as factors.

$$\ln\left(\frac{x}{x-7}\right) + \ln\left(\frac{x+7}{x}\right) - \ln\left(x^2 - 49\right)$$

20.

Use common logarithms or natural logarithms and a calculator to evaluate the expression.

$$\log_{15} 12$$

21.

Solve the following logarithmic equation.

$$\log(4x+9) = 1 + \log(x-6)$$

22.

Solve the following exponential equation. Express irrational solutions in exact form and as a decimal rounded to three decimal places.

$$8\left(5^{5x}\right) = 3$$

23.

Solve the following exponential equation. Express irrational solutions in exact form and as a decimal rounded to three decimal places.

$$4^{1-7x} = 3^{x}$$

24.

The population of a certain country in 1997 was 288 million people. In addition, the population of the country was growing at a rate of 1.0% per year. Assuming that this growth rate continues, the model $P(t) = 288(1.010)^{t-1997}$ represents the population P (in millions of people) in year t. According to this model, when will the population of the country reach

(a) 304 million people?

(b) 385 million people?

25.

Solve the system of equations. If the system has no solution, say that it is inconsistent. Graph the lines of the system.

$$\begin{cases} 3x - y = 7 \\ 5x + 3y = 21 \end{cases}$$

26.

Solve the given system of equations.

$$\begin{cases} x - 3y + 4z = 22 \\ 2x + y + z = 9 \\ -2x + 3y - 3z = -21 \end{cases}$$

Answers.

Reminder: The following are just answers. ON TESTS, YOU ARE REQUIRED TO SHOW WORK.

1. 26, 131

2.

a. $(f \circ g)(x) = \dfrac{-2x+23}{5x-19}$

The domain is $\left\{x \mid x \neq 6, x \neq \dfrac{19}{5}\right\}$

b. $(g \circ f)(x) = \dfrac{6x+17}{-5x-27}$

The domain is $\left\{x \mid x \neq -4, x \neq -\dfrac{27}{5}\right\}$

3.

$$S(t) = \frac{1}{18}\pi t^4 + 4\pi t^2$$

4. a. not one-to-one b. one-to-one

5.

a) $f^{-1}(x) = \sqrt{x+3}$

b) For $f(x)$, $D = [0, \infty)$, $R = [-3, \infty)$

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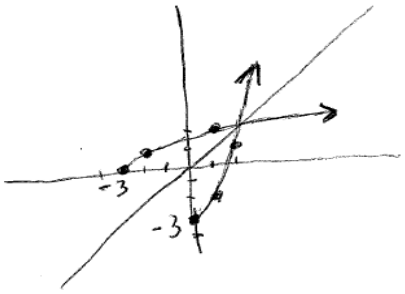
c)



6.

a) $f^{-1}(x) = \dfrac{-2x+7}{x-2}$

b) For $f(x)$, $D = \{x \mid x \neq -2\}$, $R = \{y \mid y \neq 2\}$

For $f^{-1}(x)$, $D = \{x \mid x \neq 2\}$, $R = \{y \mid y \neq -2\}$

7. (a) 76.4 kg (b) $h(W)=\frac{W-50}{2.2}+60$ (c) 74 inch.

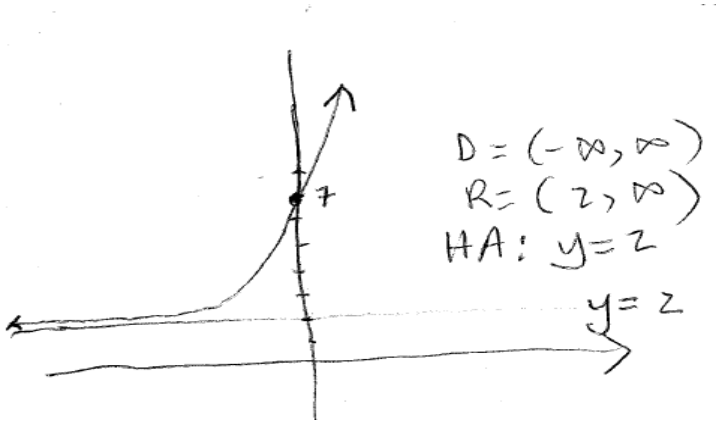
8.

The function is exponential. An exponential function that models the data is f(x) = $8 \cdot 7^x$.

9.

The function is linear. A linear function that models the data is f(x) = $2x+8$.

10.



11. 8

12. -8^x

13. (a) 60% (b) 12.96% (c) 60% of the previous year's survivors survived.

14. For this problem, please try to show all work and answer on your own.

15. $\frac{\ln 2.4}{0.2}$

16. $7-\log\frac{5}{4}$

17. 6.46 km

18 (a) $\ln(x+7)-2x$

(b)

$\ln 6+\ln x+\frac{1}{2}\ln(1+5x)-11\ln(x-4)$

19. $-2\ln(x-7)$

20. 0.9176

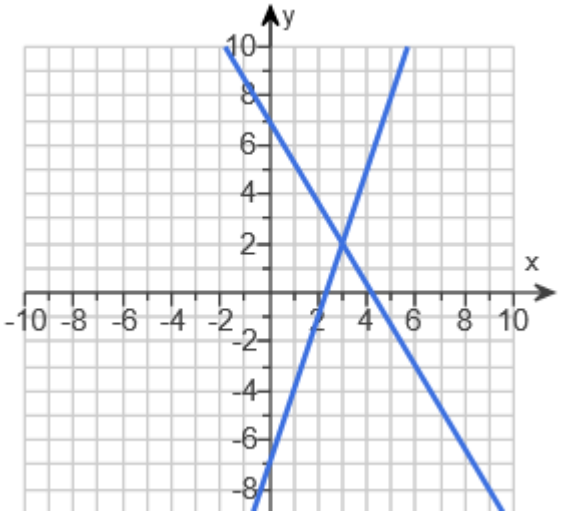
21. 23/2

22. $\frac{\ln\left(\frac{3}{8}\right)}{5\ln 5} \approx -0.122$

23. $\frac{\ln 4}{7\ln 4 + \ln 3} \approx 0.128$

24. 2026

25. (3, 2)



26. (3, - 1, 4)