Relative quantification and equative scope-taking

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UC SANTA CRUZ | **Humanities**

Interpreting proportions

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- Cardinal
- Proportional
- Relative proportional/focus-affected

Interpreting proportions

(1) a. Many [$_S$ Scandinavians] [$_N$ have won the NP].

$\rightsquigarrow |S \cap N| \geqslant n$ (cardinal)

$\rightsquigarrow {}^{|S \cap N|}/_{|S|} \geqslant n$ (proportional)

b. Many [$_S$ Scandinavians$_F$] [$_N$ have won the NP].

$\rightsquigarrow {}^{|S \cap N|}/_{|N|} \geqslant n$ (relative proportional)

. . . for some contextually-determined threshold of quantity n.

Interpreting (precise) proportions

What about precise quantifiers expressing proportions?

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    b. The fruit supplier sold [60% olives$_F$].

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    b. The fruit supplier sold [60% olives$_F$].

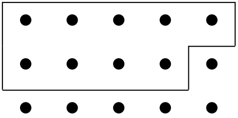
Relative measure (RM) phrases (*one-third*, *a quarter*, *percent*) can admit non-conservative readings too!

(following Ahn & Sauerland 2015a,b, 2017; a.o.)

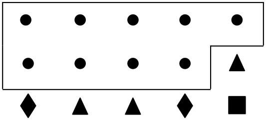
Interpreting (precise) proportions

The restrictor to the RM (*60%*) needn’t be the substance noun (*olives*). (3a) partitions the set of olives, while (3b), the set of everything the fruit supplier sold.

(3) a. 60% of the olives



b. 60% olives$_F$



Modifying proportions

The high-level focus of this talk is what happens when we modify RMs, such as precise percentages:

(4)
a. The university accepted between 20 and 30% transfer$_F$ students.
b. The vet's office saw up to 20% dogs$_F$ last week.
c. Exactly 2 recruiters interviewed exactly 60% women$_F$ (between them).
d. The soda contains as much as 40% sugar$_F$.

Modifying proportions

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b. The vet's office saw up to 20% dogs$_F$ last week.
$\rightsquigarrow$ Exactly 2 recruiters interviewed exactly 60% women$_F$ (between them).
$\rightsquigarrow$ The soda contains as much as 40% sugar$_F$.

Modifying proportions

To capture the behavior of modified RMs, including cumulativity, we'll combine a compositional scope-taking approach with an ontology of negative entities (Bledin 2024; Elliott 2024).

Negating entities

Bledin (2024) observes that there seem to be certain expressions that intuitively express individual exclusion or non-participation:

(5)
- a. [Not Ann but Mary] . . .
- b. [Turingzaal but not Eulerzaal] . . .
- c. [Michel and no one else] . . .

Negating entities

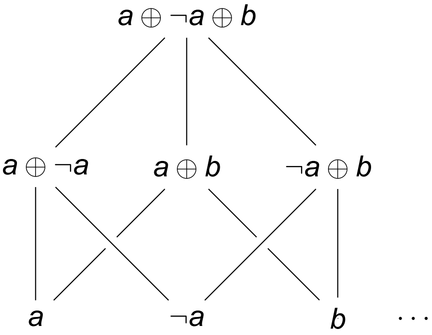
Accordingly, this is taken to reflect the encoding of *negative* entities (akin to falsemakers and falsifiers, but for entities).
(Bledin 2024; Elliott 2024)

(6) a. $[\![\text{[Not Ann but Mary]}]\!] \approx \neg\texttt{Ann} \oplus \texttt{Mary}$
b. $[\![\text{[Turingzaal but not Eulerzaal]}]\!] \approx \texttt{T} \oplus \neg\texttt{E}$
c. $[\![\text{[Ringo and no one else]}]\!] \approx \texttt{Ringo} \oplus \neg\texttt{Paul} \oplus \ldots$

Pluralizing entities

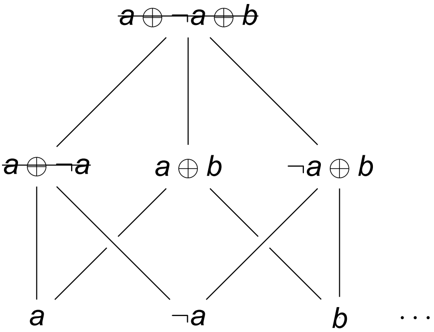
(7) a. If $[\![\text{olive}]\!] = \{a, b, c, \neg a, \neg b, \neg c\}$,

b. then $[\![{}^{*}\text{olive}]\!] = \{a, b, c, \neg a, \neg b, \ldots, a \oplus b \oplus \neg c, \ldots\}$



Pluralizing entities

Sum-combinations including an atom and its negative counterpart are excluded in $[\![{}^{*}P]\!]$ ('incoherence'; Elliott 2024):



Pluralizing entities

In other words,

(8) a. $\forall x \in D_e : \mathtt{at}(x)[\neg x \in D_e]$
b. $\forall X \in D_e[\forall x \sqsubseteq X\,[\neg x \notin X]]$
c. As an example: $\mathtt{max}_{\sqsubseteq}(\{a, \neg a\}) = \{a, \neg a\}$

Sets of entities, then, don't have a unique maximum by default.

Counting entities

The conventional denotation of a numeral-noun construction is measured based on non-negative parthood.

(9) a. $[\![\text{one olive}]\!] = \lambda x.{}^{*}\texttt{olive}\ x \wedge |x|^{+} \geqslant 1$
$\rightsquigarrow \{a, b, c, a \oplus b, a \oplus \neg b, \ldots\}$ $\qquad \texttt{e} \rightarrow \texttt{t}$

b. $[\![\text{two olives}]\!] = \lambda x.{}^{*}\texttt{olive}\ x \wedge |x|^{+} \geqslant 2$
$\rightsquigarrow \{a \oplus b, \ldots, a \oplus b \oplus \neg c, \ldots\}$ $\qquad \texttt{e} \rightarrow \texttt{t}$

Turning to *percent*

With this sketch, we can return to a semantics for *percent*.
(Pasternak & Sauerland 2022; Spathas 2022)

(10) $[\![\text{percent}]\!] := \lambda d \lambda D. \dfrac{\mathtt{max}\ D}{\mathtt{max}(\mathtt{dom}\ D)} \geqslant \dfrac{d}{100}$ $\quad \mathtt{d} \to \mathtt{D}$

The `max` operator returns the highest degree in *D*.
(Heim 2000; a.o.)

(11) $[\![\mathtt{max}]\!] := \lambda D \iota d. D\ d \wedge \forall d' [D\ d' \to d' \leqslant d]$ $\quad (\mathtt{d} \to \mathtt{t}) \to \mathtt{d}$

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I take the numeral argument to be type $\mathtt{d}$, with modificational uses (*two olives*) preceded by type-shifting.
(see Bylinina & Nouwen 2020 for an overview)

(13) $[\![\mathtt{size}]\!] := \lambda d \lambda x. |x|^{+} \geqslant d$ $\qquad \mathtt{d} \rightarrow \mathtt{e} \rightarrow \mathtt{t}$

Turning to *percent*

The desired consequence is that the RM will occupy a higher scope in the clause (type-driven).

(14) 60% [λd [the fruit supplier sold d-$\texttt{meas}$ olives$_F$]]

$\texttt{Meas}$ is the off-the-shelf measure operator shifting predicates to a gradable denotation. (Rett 2014; Solt 2015; a.o.)

Turning to *percent*

For, e.g., count nouns, the contextual measure function μ will amount to the non-negative cardinality $|\cdot|^{+}$ we saw earlier.

(15)

MeasP :: $\texttt{e} \rightarrow \texttt{t}$
$\lambda x.{}^{*}\texttt{olive}\ x \wedge \mu\ x \geqslant d$

d

Meas′ :: $\texttt{d} \rightarrow \texttt{e} \rightarrow \texttt{t}$
$\lambda d \lambda x.{}^{*}\texttt{olive}\ x \wedge \mu\ x \geqslant d$

$\texttt{meas}$

olives$_F$

Turning to *percent*

Now, we're quantifying over measurements of (pluralities of) polarized entities, still expressing a proportion.

(16)

$$\begin{array}{c} \text{TP} :: \texttt{t} \\ \dfrac{\texttt{max}(\lambda d.\exists x[{}^{*}\texttt{olive}\ x \wedge |x|^{+} \geqslant d \wedge \texttt{buy}\ x\ \texttt{Aldi}])}{\texttt{max}(\texttt{dom}(\lambda d.\exists x[{}^{*}\texttt{olive}\ x \wedge |x|^{+} \geqslant d \wedge \texttt{buy}\ x\ \texttt{Aldi}]))} \geqslant \dfrac{60}{100} \end{array}$$

$$60\% :: \texttt{D} \qquad \begin{array}{c} \text{TP} :: \texttt{d} \rightarrow \texttt{t} \\ \lambda d.\exists x[{}^{*}\texttt{olive}\ x \wedge \\ \mu\ x \geqslant d \wedge \texttt{buy}\ x\ \texttt{Aldi}] \end{array}$$

Modifying with *exactly*

We can now return to cumulativity for RMs modified by *exactly*:

(17) Exactly two recruiters interviewed exactly 60% women$_F$.

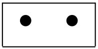
(18) *{Exactly, at least, at most, less than} many/few . . .

Under a cumulative reading, (17) is true just in case . . .

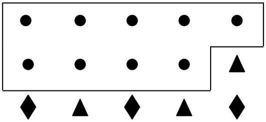
- The maximum number of interviewing recruiters is 2, and
- The maximum proportion of women interviewed by recruiters, out of all interviewees, is 60%.

Modifying with *exactly*

(19) a. ex. two recruiters



b. 60% women$_F$



Modifying with *exactly*

The cumulative intuition doesn't fall out from the subject being simply existential with an at-least interpretation.

We also need to prevent (17) from yielding truth when there are multiple possible combinations of interviewing recruiters s.t. each combination yields the 60%-40% split.

This challenge is a version of 'van Benthem's problem'.
(van Benthem 1986; Krifka 1999; Brasoveanu 2013; Charlow 2021; a.o.)

Modifying with *exactly*

Enrichment with entity negation allows for a straightforward understanding of *exactly two recruiters*.

(Differs from Elliott 2024 in that predicates don't already denote maximums.)

(20) a. $[\![\texttt{size two}]\!] = \lambda x.|x|^{+} \geqslant 2$

$\rightsquigarrow \{a \oplus b \oplus c, a \oplus b \oplus \neg c, a \oplus \neg b \oplus c, \neg a \oplus b \oplus c, a \oplus b, a \oplus c, b \oplus c\}$

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b. $[\![\text{exactly}]\!]([\![\text{two}]\!]) = \lambda P \lambda x.x \in \mathbf{M}(P) \wedge |x|^+ = 2$

$\rightsquigarrow \{a \oplus b \oplus \neg c, a \oplus \neg b \oplus c, \neg a \oplus b \oplus c\}$

(**M** abbreviates $\texttt{max}_{\sqsubseteq}$ to differentiate from degree-$\texttt{max}$)

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(21) $[\![\text{exactly}]\!]([\![\text{two}]\!])([\![\text{recruiters}]\!]) =$
$\lambda x.x \in \mathbf{M}({}^*\texttt{recruiter}) \wedge |x|^+ = 2$

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Percent and *exactly*

Exactly 60% women$_F$ applies the same idea, but with the modifier applying to the degree variable that the RM abstracts over. The modifier undertakes the task of `meas` operator.

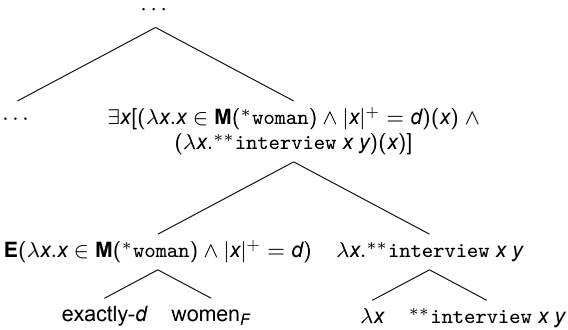
(22) 60% λd [. . . *ex. two recruiters* [. . . *ex.-d women*$_F$]]

We can now consider the full composition.

Scoping a modified proportion

E is the canonical predicates-to-quantifiers (existential) type-shifter (Partee 1989). ** is the cumulation operator on an *n*-ary relation (Sternefeld 1998; Beck & Sauerland 2001; see Elliott 2024 for a polarity-sensitive version).

(23)



Scoping a modified proportion

So far, (24) yields truth when there exists a y that is a maximal plurality of recruiters (and $|x|^{+} = 2$), and y interviewed exactly d women$_F$.

(24)

$$\exists y[\Big(\lambda y.\exists x[(\lambda x.x \in \mathbf{M}({}^{*}\texttt{woman}) \wedge |x|^{+} = d)(x) \wedge (\lambda x.{}^{**}\texttt{interview}\ x\ y)(x)]\Big)(y) \wedge \Big(\lambda x.x \in \mathbf{M}({}^{*}\texttt{recruiter}) \wedge |x|^{+} = 2\Big)(y)]$$

$$\mathbf{E}(\lambda x.x \in \mathbf{M}({}^{*}\texttt{recruiter}) \wedge |x|^{+} = 2) \qquad \lambda y.\exists x[(\lambda x.x \in \mathbf{M}({}^{*}\texttt{woman}) \wedge |x|^{+} = d)(x) \wedge (\lambda x.{}^{**}\texttt{interview}\ x\ y)(x)]$$

$$\lambda y \qquad \ldots$$

Scoping a modified proportion

The RM phrase scopes over the two quantifiers to express the proportion of female interviewees given the existence of exactly two recruiters:

(25)

$$\frac{\mathtt{max}\ \lambda d.\exists y[\ldots]}{\mathtt{max}(\mathtt{dom}\ \lambda d.\exists y[\ldots])} \geqslant \frac{60}{100}$$

$$\lambda D.\frac{\mathtt{max}\ D}{\mathtt{max}(\mathtt{dom}\ D)} \geqslant \frac{60}{100}$$

60 percent

$$\lambda d.\exists y\Big[\Big(\lambda y.\exists x[(\lambda x.x \in \mathbf{M}({}^{*}\mathtt{woman}) \wedge |x|^{+} = d)(x) \wedge (\lambda x.{}^{**}\mathtt{interview}\ x\ y)(x)]\Big)(y) \wedge \Big(\lambda x.x \in \mathbf{M}({}^{*}\mathtt{recruiter}) \wedge |x|^{+} = 2\Big)(y)\Big]$$

Scoping a modified proportion

We also get accurate results when we consider modifiers that aren't non-monotone, e.g., quantity equatives.

(26) a. Fanta contains as much as 30% sugar$_F$.
b. The price fell by as much as 30%.

For (26a), we still get an 'at-least' interpretation, at least for the semantics.

(See proc. paper for details on (26b), and Spathas 2024)

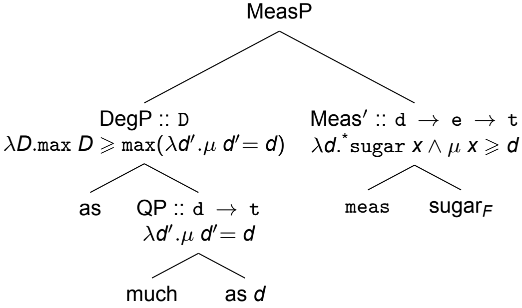
Scoping a modified proportion

The lower bound is vacuously enforced:

(see Rett 2014; Coppock & Bogal-Allbritten 2018)

(27) a. 30% $\lambda d_1 \ldots$ [as much as-d_1] $\lambda d_2 \ldots d_2 - \texttt{meas}$ sugar$_F$

b.



Scoping a modified proportion

We can preserve a scope-taking denotation for *percent*:
(contra Gobeski & Morzycki 2018)

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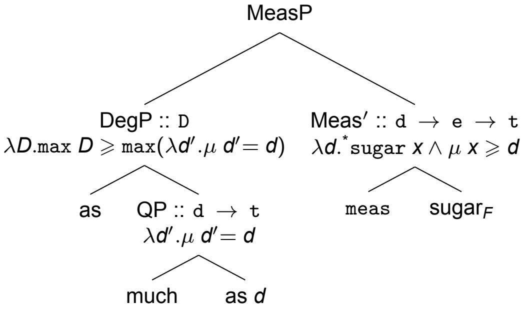
b.



Going forward

Doesn't a scopal approach violate the Heim-Kennedy Generalization (HKG) (i.e., *$\texttt{D} \gg \texttt{Q}$) ? (Kennedy 1997; Heim 2000)

Depends on who you ask, if we liken *exactly* to `shift` in (28):

(28) Adapted from Crnič (2017)

a. Modified HKG: If the scope of an `e`-type quantifier contains the trace *d* of a degree quantifier, *d* must be an argument to `shift`.

b. $[\![\texttt{shift}]\!] := \lambda d \lambda A \lambda x.\texttt{max}(\lambda d'.A\ d'\ x) \sqsubseteq d$

$::= \texttt{d} \rightarrow (\texttt{d} \rightarrow \texttt{e} \rightarrow \texttt{t}) \rightarrow \texttt{e} \rightarrow \texttt{t}$

Going forward

Just as degrees may be pluralized, we could also consider what arises from polarizing them (if even possible), just as Bledin (2024) and Elliott (2024) do for entities.

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I'll leave this to future work, but there seem to be some promising avenues:

(29) Corrective-*but* for degrees

a. The fruit supplier sold not 20 but 30% olives$_F$.
b. Mary is not 4 but 5 inches taller than Jane.
c. The team lost by not 20 but 30 points.

Recap

So, we've devised an approach to modified proportions that incorporates entity negation (Bledin 2024; Elliott 2024) with a scopal analysis of degree quantifiers (Pasternak & Sauerland 2022).

This also captures the novel observation that non-conservative RM phrases can exhibit van Benthem's problem.

Recap

Thank you!

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(see paper for full references)