MIT CSAIL

6.869 Advances in Computer Vision

Fall 2014

Problem Set 3: Neural Nets and Color

Posted: Thursday, September 18, 2014 **Due:** Thursday, September 25, 2014

You should submit a hard copy of your work in class, and upload your code (and all files needed to run it, images, etc) to Stellar.

Your report should include images and plots showing your results, as well as pieces of your code that you find relevant.

Problem 1 *Color*

For this problem you should use Matlab for computations and plotting. We say that a set of primaries $p_i(\lambda)$ is associated with a set of color matching functions $c_i(\lambda)$ if

$$\sum_i (p_i(\lambda) \sum_{\lambda_1} c_i(\lambda_1) s(\lambda_1)) \tag{1}$$

is a perceptual match to $s(\lambda)$ (in words, we project an input spectrum onto the color matching function associated with each primary to determine the amount of that primary needed to give a perceptual match to the input spectrum). In all data provided in this example, the data points are sampled at wavelengths $[360 : 5 : 730]nm$.

(a) We can transform the color coordinate in CIE XYZ space to RGB space using a transformation matrix T:

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = T \times \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, \text{where } T = \begin{bmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.207 & 1.06 \end{bmatrix}.$$

(i) Using the transformation matrix T (*CIE2RGB.mat*) and the color matching functions for CIE XYZ color space (*CIEMatch.mat*), compute the color matching functions associated with those specified by RGB primaries.

(ii) Find a valid set of primary light spectra associated with the RGB color space and verify that they are a valid set. Plot them as function of wavelengths. Comment on the positivity of the power spectra.

(iii) Find a valid set of primary light spectral associated with the CIE color space and verify that they are a valid set. Plot them as a function of wavelengths. Comment on the positivity of the power spectra.

(b) Figure 1 shows the spectral response curves for eye photoreceptors (we have also provided the response curves in *LMSResponse.mat*). Find a set of primary lights that correspond to the spectral sensitivity curves of the eye, and verify that they satisfy the condition in Equation 1. Comment on the positivity of the power spectra.

(c) **(Graduate students only)** Show that if the spectral response curves of the eye (assumed to be non-negative) were orthogonal to each other (with a zero dot product), there would exist a corresponding set of primaries with power spectra that were non-negative.

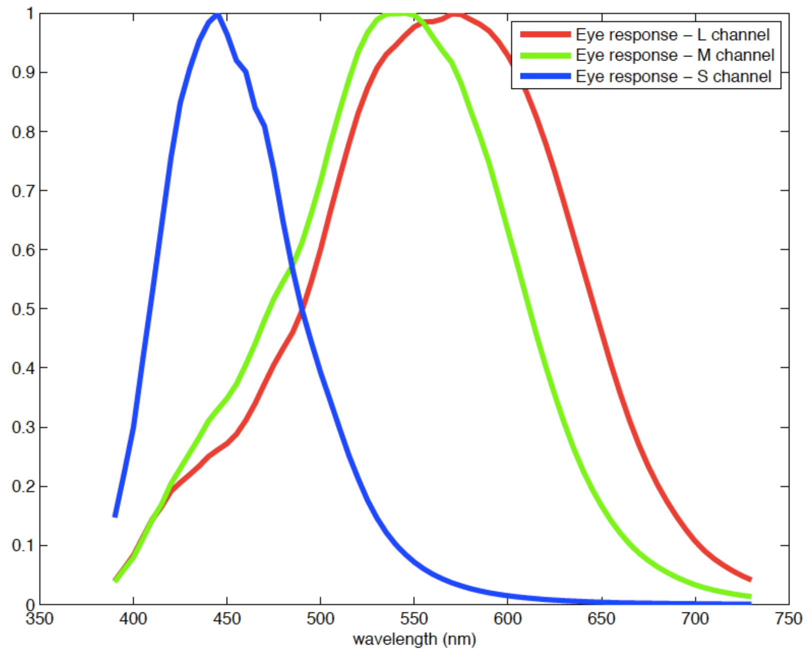


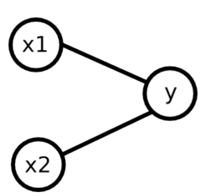
Figure 1: The spectral response curves for eye photoreceptors.

Problem 2 *Neural Networks*

In this problem, we consider the XOR dataset to explore neural networks with training examples $x \in \mathbb{R}^2$ and labels $y \in \{0, 1\}$:

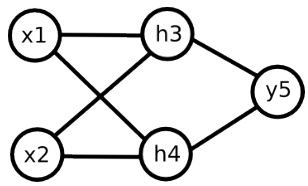
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

(a) Below we show a fully connected neural network without any hidden layers:



Assume the network has a sigmoid $\sigma(\cdot)$ activation function. If the output of the network is greater than 0.5, then we say it is class 1, otherwise class 0. Why is this network unable to correctly classify the XOR dataset? What is the theoretical best classification error it could obtain?

(b) We can add hidden units h_3 and h_4 to this network to improve its accuracy:



We define the inputs to each unit to be a weighted sum of their predecessors:

$$a_3 = w_{13}x_1 + w_{23}x_2 + b_3$$
$$a_4 = w_{14}x_1 + w_{24}x_2 + b_4$$
$$a_5 = w_{35}z_3 + w_{45}z_4 + b_5$$

where w_{ij} is the weight from unit i to unit j and b_i is the bias for unit i. Note that a_i is the input to unit i. Then, we can define the outputs of the units to be:

$$z_3 = \sigma(a_3)$$
$$z_4 = \sigma(a_4)$$
$$z_5 = a_5$$

where the hidden units are passed through a sigmoid activation function $\sigma(x) = \frac{1}{1+e^{-x}}$. Note that z_i is the output from unit i.

We wish to have a machine *learn* the parameters w_{ij} and b_i for this network on our simple dataset using backpropagation. We will learn these weights so that they minimize the squared error $E = \frac{1}{2}(z_5 - t)^2$ where t is the target output and z_5 is the predicted output.

(i) Suppose that we initialize all the weights w_{ij} to ones and all the biases b_i to zeros. What does this network predict for the output of the first example ($x_1 = 0$ and $x_2 = 0$)? What is the error for this example?

(ii) Backpropagation works by iteratively adjusting the weights of the network so as to minimize E. To do this efficiently, we must calculate the gradient of the error with respect to the network weights. Calculate the gradient for the output units $\frac{\delta E}{\delta w_{35}}$.

(iii) Calculate the gradient for the hidden unit, $\frac{\delta E}{\delta w_{13}}$.

(iv) Once we have calculated the gradients, we are able to update the weights. A common way to adjust the weights is through the formula:

$$w_{ij} \leftarrow w_{ij} - \eta \frac{\delta E}{\delta w_{ij}} \tag{2}$$

where η is called the learning rate, and it is often a constant between 0 and 1. For this problem, assume $\eta = 0.1$. What are the new weights after learning from the second data point ($x_1 = 0$ and $x_2 = 1$ with $y = 1$)?

(c) Is backpropagation always guaranteed to find the optimal solution? Why or why not?