PART 4 THE CAPITAL BUDGETING PROCESS

CHAPTER 9

The Time Value of Money

LEARNING OBJECTIVES

- **LO1** Explain the concept of the time value of money.
- **LO2** Calculate present values, future values, and annuities based on the number of time periods involved and the going interest rate.
- **LO3** Calculate yield based on the time relationships between cash flows.

Establishing the value of assets (capital budgeting) that produce cash flows for future periods is a major consideration of finance. Estimating the cash flows, determining returns required by investors, considering the risks involved, and placing a current price (present worth) on these cash flows are significant endeavours. Both the cost of capital and the capital asset pricing model will assist us in this endeavour, as will the financial markets.

FINANCE MANAGEMENT

Capital Growth

Indigenous people sold Manhattan Island in 1624 for the ridiculously low figure of $24. But was it really ridiculous? If they had merely taken the $24 and reinvested it at 6 percent annual interest up to 2016, they would have had $237 billion, an amount sufficient to repurchase most of New York City. If the Indigenous people had been slightly more astute and had invested the $24 at 7.5 percent compounded annually, they would now have over $61 trillion—and Indigenous American chiefs would now rival oil sheiks and Bill Gates as the richest people in the world. However, at 4 percent they would have accumulated only about $128 million.

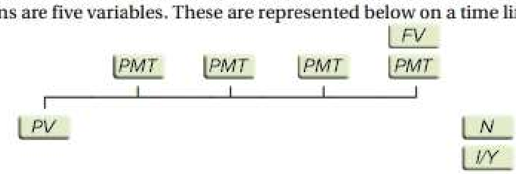
This is a dramatic example of the time value of money. Money, or "capital," has an opportunity cost related to time. Money received today is considered more valuable than the same money received at some time in the future, because today the money could be spent or, alternatively, invested at some interest rate to earn additional money. The investor/lender essentially requires that a financial "rent" be paid on his or her funds as current dollars are set aside today in anticipation of higher returns in the future.

The time value of money applies to many decisions. Understanding the effective rate on a business loan, the mortgage payment in a real estate transaction, or the value of an investment is dependent on understanding the time value of money. The mathematical concepts and calculations of the time value of money are developed in this chapter through several methods (your choice):

- Financial calculators (Appendix E for additional guidance on calculator functions)
- Tables (Appendixes A through D)
- Mathematical formulas

A calculator, set of tables, or computer are equivalent tools for time value calculations. However, to fully understand and be able to calculate values and interest rates, the student must visualize the timing patterns of the cash flows. This will provide the basis for solid financial decision making that will increase shareholder or investor value.

Visualization A time line to identify expected cash flows and when they will occur is a helpful visualization technique to simplify a problem. Common to all time-value-of-money calculations are five variables. These are represented below on a time line:



PV	Present value	Single amount at the beginning of a time period
FV	Future value	Single amount at the end of a time period
PMT	Payment or Annuity (ANN)	Series of equal amounts, at equal intervals (***n***), over a period of time
N	Number	Number of time periods to which interest rate applies or is "compounded" (month, year, etc.)

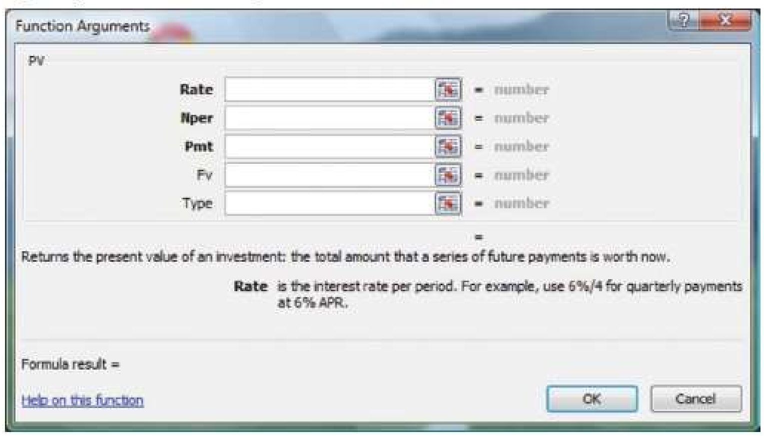
I/Y	Interest rate (in %)	Interest, or rate of return, per period or per compounding period (***n***)
CPT	Compute (or ***COMP***)	Initiates computation of one of the time value variables

Calculator A business calculator can capture any cash flow or series of cash flows over a period of time and be used to evaluate their value at a particular time (usually the present) or the relationship between the different cash flows expressed as an interest or rate of return. All business calculators have five keys corresponding to the variables identified on the above time line.

Additionally, calculators have a "begin" (BGN) or (DUE) key, which is used when cash flows occur at the beginning of a time period, such as with leases. Normally, tables (as in our appendices) or calculators assume that any cash flows occur at the end of time periods.

The +/− sign used for a cash flow will also be important for proper results. Calculators may vary in their "thinking," but there are some general considerations. It is helpful to think in terms of whether the investor (borrower) is receiving or paying out capital. For example, if an investor purchases an investment this is an outflow (negative cash flow) from which the investor expects inflows in the future (positive cash flows). The inputs for your calculation should reflect this consideration to achieve the appropriate answer.

Spreadsheet An electronic spreadsheet can also be utilized for these calculations, especially when we want to repeat them with different variables.



Type: Inserting a (1) is the instruction similar to the BGN or DUE key of the calculator
Rate is the ***%I/Y*** key of the calculator.

APPLICATION TO THE CAPITAL BUDGETING DECISION AND THE COST OF CAPITAL

Time-value-of-money concepts and the calculations (present values and yields) of this chapter form the foundation for two of the most important considerations in finance and, ultimately, for the decisions of the firm. The ***capital budgeting decision*** involves the commitment (or not) of capital for an extended period of time and, therefore, focuses on the time value of money in today's terms (present value). The ***capital structure decision***, which involves the appropriate mix of debt and equity for the firm, determines the ***cost of capital*** that is often used (with some adjustments) as the discount rate (yield) to evaluate the financial decisions of the firm.

Capital budgeting is essentially a cost-benefit analysis in that the costs (capital for new plant, equipment, or products) provide benefits (cash flows and earnings) over several future time periods. Decisions and analysis must evaluate whether the future benefits from these projects are sufficient to justify the current outlays. The mathematical tools of the time value of money are the first step toward making capital allocation decisions. This technique allows the evaluation of the present worth of these future benefits on the same terms as the current capital cost outlays.

LO1 To equate values that occur at different points in time in time-value-of-money calculations, a discount rate is required. The discount rate is also referred to as an interest rate, rate of return, yield, opportunity cost, or the cost of capital. It specifies a relationship between a value or series of values tomorrow and a value today (effectively, the future values are discounted). The interest rate or yield (cost) is the evaluation yardstick, often determined from the firm's cost of capital, which is employed to determine value and provide criteria in the acceptance or rejection of an investment proposal.

The Right Yardstick From our Manhattan Island example, the choice of an appropriate discount rate, 6 or 7.5 percent, produces a significant difference in value, $200 billion versus $49 trillion. However, at 1 percent it would have amounted to only $1,186 ($56,000 at 2 percent). With no banks to pay interest, the $24 worth of trinkets was probably a good deal.

When working with the time-value-of-money, only values that are specified at the same time can be added or subtracted. (Doing otherwise is a common mistake.) The formulas and concepts to apply time-value-of-money considerations are now developed.

FINANCE IN ACTION

Greece: Like Theseus Lifting the Boulder of Debt

In 2014, Greece had a debt level (approximately US$425 billion) of about 170 percent of GDP (approximately US$250 billion). Many were expressing concerns, although the economy appeared to be on the road to recovery.

However, the time value of money reveals that the significance of Greece's debt load was overestimated. About half of the debt required no interest payments for 10 years. Greek bonds at that time called for a 5.8 percent interest rate over 10 years. In present value terms, this debt was worth about $121 billion, not $213 billion, effectively reducing the debt load to about $333 billion, or about 133 percent of GDP. Still significant but less severe!

By extending the maturities and lowering interest rates (sometimes to zero) on its debt, Greece was working to make its debt manageable. The time value of money helps us to identify the real value or cost of the debt.

Q1 What is the Greek debt situation today?

FUTURE VALUE (COMPOUND VALUE)—SINGLE AMOUNT

LO2 A future value is a measure of an amount that is allowed to grow at a given interest rate over a time period. The future value is also referred to as the compound value. Assume an investor has $1,000 and wishes to know its worth after four years if it grows at 10 percent per year. Each year the investor is credited with the interest earned so that, in subsequent years, interest is earned on interest. This is known as compounding, and the more frequently it occurs, the higher the future value. At the end of the first year, the investor will have $1,000 × 1.10, or $1,100. By the end of year two, the $1,100 will have grown to $1,210 ($1,100 × 1.10). The four-year pattern is indicated below.

1st year	$1,000 × 1.10 = $1,100
2nd year	$1,100 × 1.10 = $1,210
3rd year.............	$1,210 × 1.10 = $1,331
4th year.............	$1,331 × 1.10 = $1,464

After the fourth year, the investor has accumulated $1,464. Because compounding problems often cover a long time period, a more generalized formula is necessary to describe the compounding procedure. We shall let

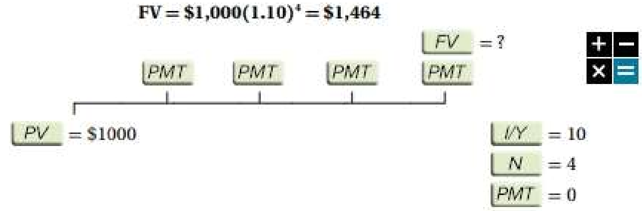
FV = Future value
PV = Present value
i = Interest rate
n = Number of periods

The formula is[1]

$$FV = PV(1 + i)^n \quad (9\text{–}1)$$

In this case, PV = $1,000, i = 10 percent, and n = 4, so we have

$$FV = \$1{,}000(1.10)^4 = \$1{,}464$$



CPT FV = **−$1,464.10.**

With the known variables input into the calculator, we compute FV = −$1,464.10. The result is negative (dependent on calculator) to indicate that $1,464.10 must be given up in the future to receive $1,000 today, or vice versa. We will often ignore the negative sign in our illustrations, as the meaning should be clear.

[1]All formulas are developed at the end of this chapter, in Appendix 9A.

Tables (optional) The term $(1.10)^4$ is found to equal 1.464 by multiplying 1.10 by itself four times (the fourth power) or by using logarithms. Using an interest rate table, such as presented at the back of the text in Appendix A, can also reveal the future value of a dollar. With $n = 4$ and $i = 10$ percent, the value is also found to be 1.464.

The table also tells us the amount that $1 would grow to if it was invested for any number of periods at a given interest rate. We multiply this factor times any other amount to determine the future value.

In determining the future value, we will change our formula from $FV = PV(1 + i)^n$ to

$$FV = PV \times FV_{IF}$$
$$FV = 1{,}000 \times 1.464 = 1{,}464$$

where FV_{IF} equals the interest factor found in the table.

As another example, suppose $10,000 was invested for 10 years at 8 percent. The future value would be

FV = ?

PMT = 0 … PMT PMT

PV = $10,000

N = 10

I/Y = 8

CPT FV = **$21,589.25.**

ANNUAL INTEREST RATES—EFFECTIVE AND NOMINAL

Interest rates are most commonly expressed on an annual basis and we will accept that convention unless specified otherwise. However, it is not always clear whether or not an expressed interest rate has incorporated the effects of compounding. Again, unless specified otherwise, we will assume that compounding effects are included in an expressed interest rate. However, we should be able to adjust interest rates for compounding effects.

In the previous future value example, the investor earned an annual rate of interest of 10 percent. If we had simply multiplied the 10 percent annual rate of interest by the four years the monies were invested, we would get a 40 percent rate of return. This would be a return of $400. However, we would have missed the compounding effects of interest on interest. The 40 percent rate of return is referred to as a nominal rate of interest, an interest rate that does not capture the effects of compounding. Generally, at the end of a period of time, often a year, the investor receives interest and can reinvest it, along with the original investment, for another year. The investor will earn interest on interest as well as on the original investment.

In our example, after four years of reinvestment, $464 in interest was earned. Over the four-year period, this represents a 46.4 percent rate of return. This is the effective rate of interest, an interest rate that includes any compounding effects. An effective rate of interest is more informative because we can calculate the actual interest earned or, if we are borrowing, the actual cost of the loan.

When compounding is called for, a formula to calculate the effective rate of interest can be developed. At the end of the first compounding period, the return on the original investment plus the interest earned is given by the principal (1.00), representing

100 percent of the investment, and the interest rate (0.10) added together (1.00 + 0.10 = 1.10). This suggests 110% of the original investment value. This value is then raised to an exponent (4) representing the number of compounding periods. The original principal (1.00), which does not represent any return of interest, is then subtracted to isolate the effective interest rate or return of 46.4%. To demonstrate the increasing value of the investment (principal plus interest),

$$1.00 \times (1 + \boldsymbol{i}) = (1 + \boldsymbol{i})^1$$
$$(1 + \boldsymbol{i})^1 \times (1 + \boldsymbol{i}) = (1 + \boldsymbol{i})^1$$
$$(1 + \boldsymbol{i})^1 \times (1 + \boldsymbol{i}) = (1 + \boldsymbol{i})^1$$
$$(1 + \boldsymbol{i})^1 \times (1 + \boldsymbol{i}) = (1 + \boldsymbol{i})^1$$

Note the similarity to the future-value development in the previous section.

By formula, the effective interest rate is

$$(1 + \boldsymbol{i})^{n-1} = \textbf{Effective interest rate} \quad \textbf{(9–2)}$$

$\boldsymbol{i}$ = Interest rate per compounding period

PMT = 0 FV = ?

PV = −1

N = 4
I/Y = 10

PV = 1 PMT = 0
N = 4 I/Y = 10

CPT FV **= 1.4641 (includes principal).**

If we multiply our future value of 1.4641 by the original investment of $1,000 we get the value of $1,464.10, the same amount as derived in the previous section on future values. The $464.10 is 46.4 percent of the $1,000.

Interest rates are usually expressed as annual rates, but not all annual rates are equal. Quite often annual rates of interest are expressed as nominal rates and do not include the compounding effects that may be in effect. For example, an institution may quote a rate of 10 percent, compounded quarterly. Each quarter an investor will receive 2.5 percent on the investment. By formula,

$$\mathbf{(1 + \boldsymbol{i}/\boldsymbol{m})^{m} - 1 = Effective\ annual\ interest\ rate}$$

where

$\boldsymbol{m}$ = Number of compounding periods per year

For this example,

$$\mathbf{(1 + 0.10/4)^4 - 1 = 0.1038,\ or\ 10.38\%}$$

PV = (1) FV = 1.025
PMT = 0 N = $\frac{1}{4}$

CPT I/Y **= 10.38%**

To find the accumulated future value with compounding,

PV = −1 PMT = 0
N = 4 I/Y = $\frac{10}{4}$

CPT FV **= 1.1038 (includes principal).**

The calculator will convert nominal annual interest rates to effective annual interest rates. Effective annual rates will be larger than nominal annual rates.

4 2nd EFF→ 10 = 10.38
or
2nd {ICONV}
NOM = 10 ENTER
C/Y = 4 ENTER
EFF CPT = **10.38**

FINANCE IN ACTION

Starting Salaries 50 Years from Now—Will $355,334 Be Enough?

The answer is probably yes if inflation averages 4 percent over the next 50 years. Over the last 50 years the inflation rate was in the 4 to 5 percent range, so $355,334 might allow a college graduate to pay his or her bills in 50 years if inflation rates stay about the same. The $355,334 is based on a starting salary of $50,000 today and the future value of a dollar for 50 periods at 4 percent. Of course, $50,000 may be too low for some majors and too high for others.

Inflation in Canada actually was as high as 12.7 percent in 1981, although it has averaged slightly less than 2 percent since 2000. Conversely, there were declining prices during the depression of the 1930s. Suppose inflation averaged 6 percent over the next 50 years; then, it would require $921,008 to replace a $50,000 salary today. At 10 percent inflation, the college graduate would need to ask an employer for a starting salary of $5,869,543 in 50 years to be as well off as his or her predecessor of today. However, at 2 percent the salary would be only $134,579. Those in more popular majors would certainly not take a penny under $6 million if inflation was 10 percent. Although 10 percent inflation seems high for Canada, in some countries it might be a happy occurrence. Bolivia's estimated inflation rate in 1985 was 3,400 percent. In 2008, Zimbabwe's exceeded 1 million percent.

The intent of this discussion is to demonstrate the effect of the time value of money. So far, all of the discussion has been forward looking. Now let's look back. How much would one of your grandparents have had to make 50 years ago to equal a $50,000 salary today, assuming a 4 percent rate of inflation? The answer is $7,036. The Bank of Canada's inflation target and the core inflation rate are provided at its website.

Q1 What is the latest inflation rate?

Q2 In 50 years, with this inflation rate, what will be equivalent to $80,000 today?

bankofcanada.ca

It is important that we distinguish between nominal and effective interest rates because, over time, they can represent a significant difference in the time value of money. Effective interest rates that include compounding effects give accurate results and allow us to better compare interest rates from different investments.

PRESENT VALUE (DISCOUNTED VALUE)—SINGLE AMOUNT

A present value, the opposite of a future value, is today's worth of a future amount. The concept of present value is that a sum payable in the future is worth less today than the stated amount.

Earlier, we determined that the future value of $1,000 for four periods at 10 percent was $1,464. We could reverse the process to state that $1,464 received four years into the future, with a 10 percent interest or discount rate, is worth only $1,000 today—its present value. The relationship is depicted in Figure 9–1.

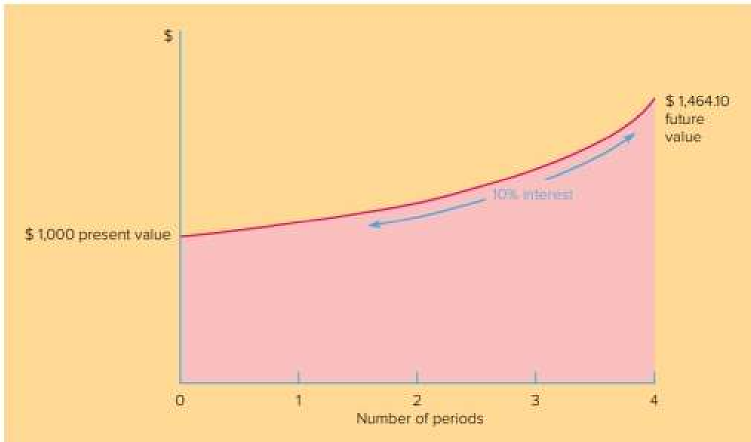


Figure 9–1 Relationship of present value and future value

The formula for present value is derived from the original formula for future value.

$$\text{FV} = \text{PV}(1+i)^n \quad \textbf{Future value}$$

$$\text{PV} = \text{FV}\left[\frac{1}{(1+i)^n}\right] = \text{FV}(1+i)^{-n} \quad \textbf{Present value} \quad \text{(9–3)}$$

FV = −$1,464.10

PV = ?

I/Y = 10

N = 4

PMT = 0

CPT PV = **$1,000.**

Tables (optional) The present value can be determined by using Appendix B, the present value of a dollar. In the latter instance, we restate the formula for present value as

$$PV = FV \times PV_{IF}$$

Once again, PV_{IF} represents the interest factor found in Appendix B.

Let's demonstrate that the present value of $1,464, based on our assumptions, is $1,000 today.

$$PV = FV \times PV_{IF}\ (n = 4, i = 10\%) \quad \text{(from Appendix B)}$$
$$PV = \$1{,}464 \times 0.683 = \$1{,}000$$

An inheritance of $50,000 might be expected 15 years from today at a time when interest rates for longer periods are 6 percent. The present value would be

FV = −$50,000

PV = ?

I/Y = 6
N = 15
PMT = 0

CPT PV = **$20,863.25.**

FUTURE VALUE (CUMULATIVE FUTURE VALUE)—ANNUITY

An annuity may be defined as a series of consecutive payments or receipts of equal amount (generally assumed to occur at the end of each period). The future value of an annuity (FV_A) is a measure of the amount to which a series of consecutive payments grow, at a given interest rate over a time period.

If we invest $1,000 at the end of each year for four years and our funds grow at 10 percent, we find the future value of the annuity (Figure 9–2) to be $4,641. In a sense, we find the future value for each payment and then total them.

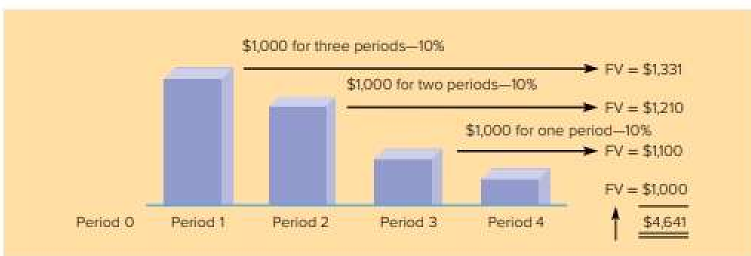


Figure 9–2 Compounding process for annuity

Although this is a four-period annuity, the first $1,000 comes at the end of the first period and has but three periods to run, the second $1,000 at the end of the second period, with two periods remaining—and so on down to the last $1,000 at the end of the fourth period. The final payment (period 4) is not compounded at all.