Determining Yield to Maturity from the Bond Price

Recall from Chapter 6 that our discussion of the term structure of interest rates revealed an investor preference for liquidity. This resulted in higher required yields for longer-term maturities, all other things being equal. The preference for liquidity can be explained by the impact of yield changes on longer-term maturities, in that they experience greater price fluctuations for a given yield change. This subjects the holder of a longer-term security to greater risk and, therefore, a higher expected yield is required.

Until now we have used yield to maturity as well as other factors, such as the interest rate on the bond and number of years to maturity, to determine the price of the bond. We now assume we know the price of the bond, the interest rate on the bond, and the years to maturity, and we wish to determine the yield to maturity. Once we have computed this value, we have determined the rate of return investors are demanding in the marketplace to provide for inflation, risk, and other factors.

We could use formula 10–1, but it looks complicated.

$$P_b = \sum_{t=1}^{n} \frac{I_t}{(1+Y)^t} + \frac{P_n}{(1+Y)^n}$$

We determine the value of Y, the yield to maturity, that equates the interest payments (I_t) and the principal payment (P_n) to the price of the bond (P_b). This is similar to the calculations to determine yield in the previous chapter. It is most easily performed with a business calculator.

Assume a 15-year bond pays $110 per year (11 percent) in interest and $1,000 after 15 years in principal repayment. The current price of the bond is $932.89.

FINANCE IN ACTION

The Ups and Downs of Bond Prices

Unlike Canada savings bonds, government and corporate bonds trade in the markets among investors. Investors are promised a fixed semiannual coupon payment and the face value, or par value, on the maturity of the bond. Since these cash flows are fixed, it is the price of bonds in the markets that must change to reflect investor expectations about the future and required rates of return. The daily dollar trading in bonds exceeds stock market trading by about 10 times.

Let's examine the price changes on government bonds. Between June and December of 1982, long-term bond yields dropped from 16.48 to 11.92 percent. A 20-year bond with a 12 percent coupon rate would have increased in price over this period from $740 to $1,006, a 36 percent return or 72 percent on an annualized basis. These opportunities do not occur very often, however.

In 2008, it was a different story, with GMAC corporate bonds that were used to finance a variety of activities including troublesome automobile leases and residential mortgages. Dominion Bond Rating Service lowered GMAC's bond rating from investment grade at BBB to junk bond status of B in several steps from October 2007 to June 2008. It was considered riskier. GMAC 5.45 percent bonds maturing December 2009 fell from $1,002.60 to $834.40 between February and August 2008. The promised yield (if realized) rose from 5.45 to 21 percent. Any takers?

In 2017, bond yields sat at quite low yields with risk on the upside. If inflation pushed yields higher, bond prices would fall, reducing year-over-year returns on bond investments.

Bond quotes are expressed as follows:

Issuer	Coupon	Maturity Date	Price	Yield	Yield Change
Canada	9.000	2025-Jun-01	164.44	2.31	+0.009

The price is expressed as a percentage of maturity value of $1,000 due June 1, 2025 (or as value per $100 of maturity value). The coupon rate is annual interest, $90, (calculated on the maturity value) that the issuer is obligated to pay, usually every six months ($45). The yield is the current expected return.

Bond quotes can be obtained from *The Financial Post* (financialpost.com). Limited live quotes are available at Perimeter Financial (pfin.ca/canadianfixedincome).

Q1 What are the current price and yield to maturity of Government of Canada bonds?

Q2 What are the current price and yield to maturity of two corporate bonds?

We wish to determine the yield to maturity, or discount rate, that equates future flows with the current price.[3]

FV = $1,000

	$110	$110	...	$110
0	1	2	...	15

PV = −$932.89

N = 15
PMT = 110
I/Y = ?

i = CPT I/Y **11.98%.**

Tables (optional) Tables require a trial-and-error process (as does the calculator when its screen temporarily goes blank). The first step is to choose an initial percentage in the tables to try as the discount rate. Since the bond is trading below the par value of $1,000, we know the yield to maturity (discount rate) must be above the quoted interest rate of 11 percent. By the trial-and-error process, a 12 percent discount rate brings us

Total Present Value	
Present value of interest payments (Appendix D)	$749.21
Present value of principal payment at maturity (Appendix B)	183.00
Total present value, or price, of the bond	$932.21

The answer closely approximates the price of $932.89 for the bond being evaluated. That indicates that the correct yield to maturity for the bond is 12 percent. If the computed value were slightly different from the price of the bond, we could use interpolation to arrive at the correct answer. An example of interpolating to derive yield to maturity is presented in Appendix 10A.

[3]An approximate yield formula is given by

$$\text{Approximate yield to maturity } (Y') = \frac{\text{Annual interest payment} + \dfrac{\text{Principal payment} - \text{Price of the bond}}{\text{Number of years to maturity}}}{0.6\ (\text{Price of the bond}) + 0.4\ (\text{Principal payment})}$$

This formula is recommended by Gabriel A. Hawawini and Ashok Vora, "Yield Approximations: A Historical Perspective," ***Journal of Finance* 37** (March 1982), pp. 145–56. It tends to provide the best approximation.

Semiannual Interest and Bond Prices

Until now, in our bond analysis we have been considering examples where interest was paid annually. However, most bonds in Canada and the United States pay interest semiannually. This is not the case in countries such as Germany. Thus, a 10 percent interest rate bond may actually pay $50 twice a year instead of $100 annually. To make the conversion from an annual to semiannual analysis, we follow three steps.

Deutsche Bundesbank
bundesbank.de

1. Divide the annual interest rate by two.
2. Multiply the number of years by two.
3. Divide the annual yield to maturity by two.

Assume a 10 percent, $1,000 par value bond has a maturity of 20 years. The annual yield to maturity is 12 percent. In following the preceding three steps, we would show,

1. 10%/2 = 5% semiannual interest rate; therefore, 5% × $1,000 = $50 semiannual interest
2. 20 × 2 = 40 periods to maturity
3. 12%/2 = 6% yield to maturity, expressed on a semiannual basis

In computing the price of the bond issued, on a semiannual analysis, we show

FV = $1,000

PV = ?

N = 40 (20 × 2)
I/Y = 6% (12%/2)
PMT = 50

CPT PV = **−$849.54.**

The answer of PV = $849.54 is slightly below that which we found previously for the same bond, assuming an annual interest rate ($850.61). In terms of accuracy, the semiannual analysis is a more acceptable method. As is true in many finance texts, the annual interest rate approach is given first for ease of presentation, and then the semiannual basis is given. In the problems at the end of the chapter, you will be asked to do problems on both an annual and semiannual interest payment basis.

Tables (optional) Total Present Value

(PV_A) Present value of interest payments (Appendix D)	$752.30
(PV) Present value of principal payment at maturity (Appendix B) . .	97.00
Total present value, or price, of the bond	$849.30

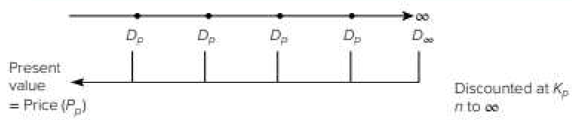
Valuation of Preferred Stock

Preferred stock represents a long-term interest by an investor in a firm. This is another way that a firm hopes to raise long-term capital to invest in its revenue-generating assets. A preferred share, although considered an equity financial asset, is a hybrid security that has neither the ownership privileges of common stock nor the legally enforceable provisions of debt. Preferreds offer regular fixed payments (usually quarterly or every three months) as a dividend, but do not have the binding contractual obligation to pay interest as on debt. Generally, preferred stock is a perpetuity meaning it has no maturity date.

Preferreds are valued in the market on the basis of the expected stream of dividend payments and without any principal payment since there is no ending life. If preferred stock had a maturity date, the analysis would be similar to that for bonds. Preferred stock dividends carry a higher order of precedence than common stock dividends as to payment. To value a perpetuity such as preferred stock, we first consider the formula

$$P_p = \frac{D_p}{(1+K_p)^1} + \frac{D_p}{(1+K_p)^2} + \frac{D_p}{(1+K_p)^3} + \cdots + \frac{D_p}{(1+K_p)^n} + \cdots (n \to \infty) \quad \text{(10–2)}$$

Represented graphically as



Where

P_p = Price of preferred stock
D_p = Annual dividend for preferred stock (a constant value)
K_p = Required rate of return, or discount rate, applied to preferred stock dividends

Note that the calls for taking the present value of an infinite stream of constant dividend payments at a discount rate equal to K_p. This discount rate of K_p also consists of the three factors influencing yield that were discussed under bond valuation. Formula 10–2 can be reduced to a much more usable form, as indicated in formula 10–3.

$$P_p = \frac{D_p}{K_p} \quad \text{(10–3)}$$

According to formula 10–3, all we have to do to find the price of preferred stock (P_p) is to divide the constant annual dividend payment (D_p) by the required rate of return that preferred shareholders are demanding (K_p). For example, if the annual dividend were \$10 and the shareholder required a 10 percent rate of return, the price of preferred stock would be \$100.

$$P_p = \frac{D_p}{K_p} = \frac{\$10}{0.10} = \$100$$

We could achieve the same with a calculator using a large number, such as 1,000, for N.

I/Y = 10%	FV = 0
PMT = \$10	N = 1,000

CPT PV = **−\$100.**

As was true in our bond valuation analysis, if the rate of return required by security holders' changes, the value of the financial asset (in this case, preferred stock) changes. You may also recall that the longer the life of an investment, the greater the impact of a change in required rate of return. It is one thing to be locked into a low-paying security for one year when the rate goes up; it is quite another to be locked in for 10 or 20 years. With preferred stock, you have a ***perpetual*** security, so the impact is at a maximum. Assume in the prior example that because of higher inflation or increased business risk, K_p (the required rate of return) increases to 12 percent. The new value for the preferred stock shares then becomes

$$P_p = \frac{D_p}{K_p} = \frac{\$10}{0.12} = \$83.33$$

If the required rate of return were reduced to 8 percent, the opposite effect would occur. The preferred stock price would be recomputed as

$$P_p = \frac{D_p}{K_p} = \frac{\$10}{0.08} = \$125$$

It is not surprising that preferred stock is now trading well above its original price of \$100. It is still offering a \$10 dividend (10 percent of original offering price of \$100), while the market is demanding only an 8 percent yield. To match the \$10 dividend with the 8 percent rate of return, the market price will advance to \$125.

Determining the Required Rate of Return (Yield) from the Market Price

In our analysis of preferred stock, we have used the value of the annual dividend (D_p) and the required rate of return (K_p) to solve for the price of preferred stock (P_p). We could change our analysis to solve for the required rate of return (K_p) as the unknown, given that we knew the annual dividend (D_p) and the preferred stock price (P_p). We take formula 10–3 and rewrite it as formula 10–4, where the unknown is the required rate of return (K_p).

$$P_p = \frac{D_p}{K_p} \quad \text{(reverse the position of } K_p \text{ and } P_p\text{)} \qquad (10\text{–}3)$$

$$K_p = \frac{D_p}{P_p} \qquad (10\text{–}4)$$

Using formula 10–4, if the annual preferred dividend (D_p) is \$10 and the price of preferred stock (P_p) is \$100, the required rate of return (yield) would be 10 percent.

$$K_p = \frac{D_p}{P_p} = \frac{\$10}{\$100} = 10\%$$

If the price goes up to \$130, the yield will be only 7.69 percent.

$$K_p = \frac{\$10}{\$130} = 7.69\%$$

We see that the rise in market price causes quite a decline in the yield.

VALUATION OF COMMON STOCK

Common stock also represents a long-term investment in a firm, again as a means of raising long-term capital for the firm's operations. It represents an ownership interest referred to as equity and entitles a common shareholder to the firm's profits after all contractual obligations (wages, interest) are satisfied. The value of a common share to the shareholder is the claim on these residual earnings of the firm. These earnings can be retained and reinvested in the firm's operations or paid out as dividends.

Investors place value on common shares based on the firm's ability to generate cash flow or earnings and the risks attached to those expected earnings. These earnings will eventually flow to the shareholder as dividends in current periods or at some time in the future, possibly as a liquidating dividend at the end of the corporation's life. Therefore, a share of common stock can be valued based on the present value of

- An expected stream of future dividends (dividend valuation model)
- The expected future earnings (price/earnings model)

Shareholders will be influenced by a change in earnings, a change in the risks faced by the firm, or other variables, but the ultimate value of any holding rests with the distribution of earnings in the form of dividend payments. Though the shareholder may benefit from the retention and reinvestment of earnings by the corporation, at some point the earnings must be translated into cash flow for the shareholder.

A stock valuation model based on future expected dividends can be stated as

$$P_0 = \frac{D_1}{(1+K_e)^1} + \frac{D_2}{(1+K_e)^2} + \frac{D_3}{(1+K_e)^3} + \cdots + \frac{D_n}{(1+K_e)^n} + \cdots (n \to \infty) \quad \text{(10–5)}$$

where

P_0 = Price of the stock today
D = Dividend for each year
K_e = Required rate of return for common stock (discount rate)

With modification, this dividend valuation model formula, for shares, is generally applied to three different circumstances:

1. No growth in dividends
2. Constant growth in dividends
3. Variable growth in dividends (Appendix 10B)

No Growth in Dividends

Under the no-growth circumstance, common stock is similar to preferred stock. The common stock pays a constant dividend each year. For that reason we merely translate the terms in formula 10–4, which applies to preferred stock, to apply to common stock. This is shown as new formula 10–6.

$$P_0 = \frac{D_1}{K_e} \quad \text{(10–6)}$$

Where

P_0 = Price of common stock today
D_1 = Current annual common stock dividend (a constant value) ($D_1 = D_2 = D_3 \ldots D_\infty$)
K_e = Required rate of return for common stock

Assume D_0 = \$1.86 and K_e = 12 percent; the price of stock would be \$15.50.

$$P_0 = \frac{\$1.86}{0.12} = \$15.50$$

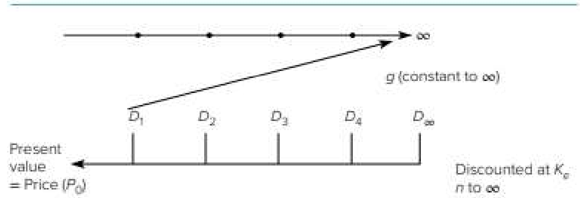
A no-growth policy for common stock dividends does not hold much appeal for investors and so is seen infrequently in the real world.

Constant Growth in Dividends

A firm that increases dividends at a constant rate is a more likely circumstance. Perhaps a firm decides to increase its dividends by 5 or 7 percent per year. Under such a circumstance, formula 10–5 converts to formula 10–7.

$$P_0 = \frac{D_0(1+g)^1}{(1+K_e)^1} + \frac{D_0(1+g)^2}{(1+K_e)^2} + \frac{D_0(1+g)^3}{(1+K_e)^3} + \cdots + \frac{D_0(1+g)^n}{(1+K_e)^n} + \cdots (n \to \infty) \quad \text{(10–7)}$$

Represented graphically as



Where

P_0 = Price of common stock today
$D_0(1 + g)^1$ = Dividend in year 1, D_1
$D_0(1 + g)^1$ = Dividend in year 2, D_2, and so on
g = Constant growth rate in dividends
K_e = Required rate of return for common stock (discount rate)

In other words, the current price of the stock is the present value of the future stream of dividends growing at a constant rate. If we can anticipate the growth pattern of future dividends and determine the discount rate, we can ascertain the price of the stock.

For example, assume the following information:

D_0 = Latest 12-month dividend (assume \$1.87)
D_1 = First year, \$2 (growth rate, 7%)
D_2 = Second year, \$2.14 (growth rate, 7%)
D_3 = Third year, \$2.29 (growth rate, 7%) etc.
K_e = Required rate of return (discount rate), 12%

Then

$$P_0 = \frac{\$2}{(1.12)^1} + \frac{\$2.14}{(1.12)^2} + \frac{\$2.29}{(1.12)^3} + \cdots + \frac{\$2(1.07)^n}{(1.12)^n} + \cdots (n \rightarrow \infty)$$

To find the price of the stock, we take the present value of each year's dividend. This is no small task when the formula calls for us to take the present value of an ***infinite*** stream of growing dividends. Fortunately, formula 10–7 can be compressed into a much more usable form if two circumstances are satisfied.

1. The dividend growth rate (g) must be constant forever.
2. The discount rate (K_e) must exceed the growth rate (g).

These assumptions are usually made to reduce the complications in the analytical process. They then allow us to reduce or rewrite formula 10–7 as formula 10–8. Formula 10–8 is the basic formula for finding the value of common stock and is referred to as the dividend valuation model.

$$P_0 = \frac{D_1}{K_e - g} \quad (10\text{–}8)$$

This is an extremely easy formula to use in which[4]

P_0 = Price of the stock today
D_1 = Dividend at the end of the first year (or period)
K_e = Required rate of return (discount rate)
g = Constant growth rate in dividends

[4]To derive this relationship we multiply both sides of formula 10–7 by $\frac{(1 + K_e)}{(1 + g)}$ and subtract formula 10–7 from the product. The result is

$$\frac{P_0(1 + K_e)}{(1 + g)} - P_0 = D_0$$

Therefore,

$$P_0\left[\frac{1 + K_e}{1 + g} - 1\right] = D_0$$

$$P_0\left[\frac{(1 + K_e) - (1 + g)}{1 + g}\right] = D_0$$

$$P_0(K_e - g) = D_0(1 + g)$$

$$P_0 = \frac{D_1}{K_e - g}$$

Based on the current example,

$$D_1 = \$2$$
$$K_e = 0.12$$
$$g = 0.07$$

P_0 is computed as

$$P_0 = \frac{D_1}{K_e - g} = \frac{\$2}{0.12 - 0.07} = \frac{\$2}{0.05} = \$40$$

Thus, given that the stock has a \$2 dividend at the end of the first period, a discount rate of 12 percent, and a constant growth rate of 7 percent, the current price of the stock is \$40.

Let's take a closer look at formula 10–8 and the factors that influence valuation. For example, what is the anticipated effect on valuation if K_e (the required rate of return, or discount rate) increases as a result of inflation or increased risk? Intuitively, we would expect the stock price to decline if investors demand a higher return and the dividend and growth rate remain the same. This is precisely what happens.

If D_1 remains at \$2 and the growth rate (g) is 7 percent but K_e increases from 12 percent to 14 percent, using formula 10–8, the price of the common stock would now be \$28.57. This is considerably lower than its earlier value of \$40.

$$P_0 = \frac{D_1}{K_e - g} = \frac{\$2}{0.14 - 0.07} = \frac{\$2}{0.07} = \$28.57$$

Similarly, if the growth rate (g) increases and D_1 and K_e remain constant, the stock price can be expected to increase. Assume $D_1 = \$2$, K_e is set at its earlier level of 12 percent, and g increases from 7 percent to 9 percent. Using formula 10–8 once again, the new price of the stock would be \$66.67.

$$P_0 = \frac{D_1}{K_e - g} = \frac{\$2}{0.12 - 0.09} = \frac{\$2}{0.03} = \$66.67$$

We should not be surprised to see that an increasing growth rate has enhanced the value of the stock.

Determining the Inputs for the Dividend Valuation Model

Our model for valuation based on future dividends seems reasonable, but where do we find the numbers for the model that allow us to determine the share price? Our Finance in Action box demonstrates. Dividends are fairly accessible, if they are paid, and are found in annual reports or at various investment sites. An appropriate required return for the common shares (K_e) can be estimated using CAPM, examined further in Appendix 11A, or by using the current yield for long-term Government of Canada bonds to which a risk premium is added, based on the riskiness of the common shares. This yield to maturity concept is discussed earlier in this chapter.

FINANCE IN ACTION

Estimating Value with the Dividend Capitalization Model

Historical earnings are available in annual reports and accessible on company websites or at the SEDAR site (sedar.com) for publicly traded companies. Several investment sites provide earnings estimates for Canadian companies, although the information is often in U.S. dollars (look carefully). An example is reuters.com/finance/stocks.

We will estimate the share value of the Royal Bank of Canada (RY) using the dividend capitalization model as of March 2017.

D_0 = \$3.48 (from TSX site: financial snapshot)
= K_e = 7.34%
5% (for risk) plus 2.34% (Government of Canada log-term bond rate from Bank of Canada site (bankofcanada.ca)

g = 3.78 (5-year dividend growth through Reuters)

$D_1 = D_0 \times (1 + g)$
$= \$3.48 \times (1.0378)$
$= \$3.61$

$$P_0 = \frac{D_1}{K_e - g} = \frac{\$3.61}{0.0734 - 0.0378} = \$101.40$$

The actual price of Royal Bank shares at this time was \$96.55. This may suggest the shares are slightly underpriced, or we should revisit the inputs to our model.

With the same methodology, the dividend capitalization model, you should be able to calculate a value for a Royal Bank's share today.

Q1 Can you suggest any reasons for a difference between the market share price and the model's share value?

tmx.com
Symbol: RY

It is the value for growth (***g***) that will require some effort on our part. This is the long-run growth rate for the firm. What we can do is examine the historical growth rate of the firm and project it into the future, adjusting for any micro (firm-related) or macro (overall economy) factors that we believe will cause the growth rate to change. New technologies, changing government regulations, economic slowdowns, and external shocks such as wars are some of the events that might alter our growth estimates.

The historical growth rates are best estimated from dividends, but we could also use the growth in earnings per share, revenues per share, or cash flow per share if one or the other of these items is not readily available. We might also use an alternative entry if there is possible distortion of one or more of the historical growth rates. In fact, determining the growth rates for all of these entries would give us a broader picture of the probable growth rate for the firm.

If, for example, the earnings per share figure five years ago was \$1.50 and the last reported earnings per share was \$2.10, the historical growth rate would be 7 percent.

PV = (1.50) FV = 2.10
PMT = 0 N = 5

CPT I/Y **= 6.96% (round to 7 percent).**

Stock Valuation Based on Future Stock Value The discussion of stock valuation to this point has related to the concept of the present value of future dividends. This is a valid concept, but suppose we wish to approach the issue from a slightly different viewpoint. Assume we are going to buy a stock and hold it for three years and then sell it. We wish to know the present value of our investment. This is somewhat like the bond valuation analysis. We receive a dividend for three years (D_1, D_2, D_3) and then a price (payment) for the stock at the end of three years (P_3). What is the present value of the benefits? What we do is add the present value of three years of dividends and the present value of the stock price after three years. Assuming a constant growth dividend analysis, the stock price after three years is simply the present value of all future dividends after the third year (from the fourth year on). Thus, the current price of the stock in this case is nothing other than the present value of the first three dividends, plus the present value of all future dividends (which is equivalent to the stock price after the third year). Saying the price of the stock is the present value of all future dividends is also the equivalent of saying it is the present value of a dividend stream for a number of years plus the present value of the price of the stock after that time period. The appropriate formula is still $P_0 = D_1/(K_e - g)$, which we have been using throughout this part of the chapter.

Determining the Required Rate of Return from the Market Price

In our analysis of common stock, we have used the first year's dividend (D_1), the required rate of return (K_e), and the growth rate (g) to solve for the stock price (P_0) based on formula 10–8.

We could change the analysis to solve for the required rate of return (K_e) as the unknown, given that we know the first year's dividend (D_1), the stock price (P_0), and the growth rate (g). We take formula 10–8 and algebraically rearrange it to provide formula 10–9.

$$P_0 = \frac{D_1}{K_e - g} \quad (10\text{–}8)$$

$$K_e = \frac{D_1}{P_0} + g \quad (10\text{–}9)$$

This formula allows us to compute the required return (K_e) from the investment. Returning to the basic data from the common stock example,

K_e = Required rate of return (to be solved)
D_1 = Dividend at the end of the first year \$2
P_0 = Price of the stock today \$40
g = Constant growth rate 0.07

$$K_e = \frac{\$2}{\$40} + 0.07 = 0.05 + 0.07 = 0.12 = 12\%$$

In this instance, we would say that the shareholder demands a 12 percent return on the common stock investment. Of particular interest are the individual parts of the formula for K_e that we have been discussing. Let's write out formula 10–9 again.

$$K_e = \frac{\text{First year's dividend}}{\text{Common stock price}}\left(\frac{D_1}{P_0}\right) + \text{Growth } (g)$$

The first term represents the dividend yield the shareholder receives, and the second term represents the anticipated growth in dividends, earnings, and stock price. Though we have been describing the growth rate primarily in terms of dividends, it is assumed that the earnings and stock price also grow at that same rate over the long term if all else holds constant. Observe that the preceding formula represents a total return concept. The shareholder is receiving a current dividend plus anticipated growth in the future. If the dividend yield is low, the growth rate must be high to provide the necessary return. Conversely, if the growth rate is low, a high dividend yield is expected. The concepts of dividend yield and growth are clearly interrelated.