b. If the required rate of return is 13 percent, what is the anticipated share price at the beginning of 20XZ?

31. Just Tin Coatings has had the following pattern of earnings per share over the last five years:

Year	Earnings per Share
20XU	$2.00
20XV	2.18
20XW	2.28
20XX	2.42
20XY	2.62

The earnings per share have grown over this period and will continue to do so in the future in the same general fashion. Dividends represent 30 percent of earnings.

a. Project earnings and dividends for the next year (20XZ). Round all values in this problem to two places to the right of the decimal point.

b. If the required rate of return is 12 percent, what is the anticipated share price at the beginning of 20XZ?

32. Freudian Slips has just paid a dividend of $2.12 for the year and it is expected to grow at 6 percent per year. With a current share price of $54.50 what is Freudian's required (expected) rate of return?

33. A firm will pay a $4.00 dividend at the end of year one, has a share price of $50, and a constant growth rate of 5 percent. Compute the required (expected) rate of return.

34. Triple Peaks Playhouse will pay a quarterly dividend of $0.40 at the end of the next quarter. It has common share price of $32.00 and a constant growth rate of 4 percent. Compute the required rate of return.

35. A firm pays a $1.50 dividend at the end of year one. It has a share price of $60 ($P_0$) and a constant growth rate (g) of 9 percent.

a. Compute the required (expected) rate of return (K_e). Also indicate whether each of the following changes would make the required rate of return (K_e) go up or down.

(In each question below, assume only one variable changes at a time. No actual numbers are necessary.)

b. The dividend payment increases.

c. The expected growth rate increases.

d. The stock price increases.

36. Hunter Petroleum Corporation paid a $2 dividend last year. The dividend is expected to grow at a constant rate of 5 percent forever. The required rate of return is 12 percent (this will also serve as the discount rate in this problem). Round all values to three places to the right of the decimal point where appropriate.

a. Compute the anticipated value of the dividends for the next three years. That is, compute D_1, D_2, and D_3; for example, D_1 is $2.10 ($2.00 × 1.05). Round all values throughout this problem to three places to the right of the decimal point.

b. Discount each of these dividends back to the present at a discount rate of 12 percent and then sum them.

c. Compute the price of the stock at the end of the third year (P_3).

$$P_3 = \frac{D_4}{K_e - g}$$

(D_4 is equal to D_3 times 1.05)

d. After you have computed P_3, discount it back to the present at a discount rate of 12 percent for three years.

e. Add together the answers in parts ***b*** and ***d*** to get P_0, the current value of the stock. This answer represents the present value of the first three periods of dividends, plus the present value of the price of the stock after three periods (which, in turn, represents the value of all future dividends).

f. Use formula 10–8 to show that it will provide approximately the same answer as part ***e***.

$$P_0 = \frac{D_1}{K_e - g} \quad (10\text{-}8)$$

For formula 10–8, use $D_1 = \$2.10$, $K_e = 12$ percent, and $g = 5$ percent. (The slight difference between the answers to parts ***e*** and ***f*** is due to rounding.)

37. You have been examining three firms in the telecommunications business. You have identified the following P/E ratios for the companies.

a. Which firm appears to be the best value on the P/E ratio alone?

Graham	18
Ted	34
Showus	21

b. Why do you suppose that Ted has the highest P/E ratio?

(And for more fun and review, do these valuation problems.)

38. What is the value of a common share that has just paid a dividend of $2.25, is expecting an indefinite annual growth rate of 5 percent, and requires a return of 17 percent based on perceived market risks?

39. Calculate the price of a bond originally issued six years ago that pays semiannual interest at the rate of 12 percent and matures in nine years at $1,000. The market currently requires an 8 percent return for a bond of this risk.

40. The Tahitian Lottery has promised an annual stipend of $75,000, forever, and permanent residency on Tahiti. Assuming interest rates of 6 percent, how much must the Tahitian authorities set aside today to guarantee this stipend?

41. A bond just purchased pays annual interest of 10 percent. In seven years it matures at its face value of $25,000. What price was paid if current yields on a bond of this risk are 8.5 percent?

42. Burrito Bell issued a series of $1,000 bonds eight years ago with an annual coupon rate of $100. The bonds mature 12 years from now. If an investor requires a 6 percent return on this investment, what would be the price of a Burrito Bell bond?

43. Current yields are 9 percent on a preferred share that pays a perpetual annual dividend of $6.00. What is the appropriate price of one preferred share?

44. With an anticipated dividend of $1.20, continual annual growth of 8 percent, and a market expectation of a 19 percent yield, at what price would a common share sell?

45. You wish to invest $175,000 in a 12-year annuity. Current yields over the same time to maturity are 8 percent. What could you expect as an annual payment?

46. You have purchased a preferred share that promises a $3.00 dividend. If you expect a 14 percent yield, what price did you pay for the preferred?

47. Waterman Company has had a fantastic growth of 22 percent per year, but this growth rate is expected to fall to 6 percent in the near future and then continue at that rate for a long time. Shareholders expect a 17 percent annual rate of return and a dividend of $0.75 next year. What is the share price of Waterman's common stock?

48. Lou Spence bought a stock seven years ago for $15.00 a share. If it is now selling for $42.39 a share, what is the stock's compound annual growth rate? (No dividends were paid.)

49. You are interested in receiving a true yield on a government bond investment of 8 percent. Your broker suggests a 20-year issue with 12 years to maturity, an original coupon rate of 10 percent, payable semiannually, and a face value of $10,000. The bond has had its coupons stripped, so you won't receive the coupon payments. What price will you pay?

50. A national financial institution is currently offering $50,000 a year for life as a special promotion. Current inflation is 2 percent a year, and the real rate of return is assumed to be 3 percent. One could suggest that this financial institution would receive a 2 percent premium for the risk inherent in its long-term investments. Assuming you will live forever, how much will this promise cost the financial institution today when you win?

51. Thunderbay Ltd. has a 20-year bond outstanding that matures in 14 years. The annual coupon rate is 11 percent, paid semiannually. Current annual nominal yields for bonds of similar risk are 9 percent. What is the price of this bond?

52. Royal Blue Bonds were purchased nine years ago at $1,000, with a 13 percent annual coupon. Today they are sold for $1,215. Assuming the coupons were reinvested at 9 percent, what was the annual yield actually received?

53. Baffin College has an endowment that pays one lucky student $7,500 a year forever; of course, it is a different student each year. Long-term yields are 10 percent.
    a. How much is currently required to fund the endowment?
    b. If the endowment will not commence until five years from today, how much is required?

COMPREHENSIVE PROBLEM

54. Allie Reynolds, the chief financial officer of Healthy Products Inc., has been asked to do an evaluation of Fibre Cereal Inc. by the president, Gail Martinez. Healthy Products was planning a joint venture with Fibre Cereal (privately traded) and Gail and Allie needed a better feel for what Fibre's stock is worth because there is interest in buying the firm at some time in the future.

    Fibre Cereal paid a dividend at the end of year one of $1.20, with an anticipated growth rate of 10 percent; the required rate of return is 13 percent.
    a. What is the value of a share based on the dividend valuation model?
    b. Indicate that the value computed in part ***a*** is correct by showing the values of D_1, D_2, and $D_3$and the anticipated share price at the end of year three, as present values. Then sum the computations.
    c. As an alternative measure, examine the value of the firm based on the price-earnings (P/E) ratio times earnings per share. The anticipated P/E ratio can be estimated by taking the average of five publicly traded food industry companies. The P/E ratios were as follows during the time period under analysis:

	P/E ratio
Del Monte	12
General Mills	15
Maple Leaf	14
Kellogg	22
Kraft	17

d. If, in computing the industry average P/E, it is decided to weight Kellogg by 40 percent and the other four firms by 15 percent each, what would be the new weighted average industry P/E? (Kellogg is more similar to Fibre.) What will be the new suggested share price? (Earnings per share remain at $2.45.)

e. By what percent will the share price change as a result of using the weighted average industry P/E in part ***d*** as compared to part ***c***?

MINI CASE

Gilbert Enterprises

Tom Gilbert, founder and chair of the board of Gilbert Enterprises, could not believe his eyes as he read the quote about his firm in *The Globe and Mail*. The stock had closed at $35.25, down $3.75 for the week. He called his vice-president of finance, Jane Arnold, and they agreed to meet on Saturday morning at 9 a.m. for breakfast.

When Jane arrived, they reviewed the stock's performance for the past few months. Although the stock opened the year (20XX) at $28.50 per share, it had reached a high of $50 in March, but had steadily slid in value to its current level of $35.25 in mid-May. Tom and Jane both thought the stock was undervalued in the marketplace and were seriously considering an announcement that the firm was going to repurchase up to one million of its own shares in the open market beginning on June 1, 20XX. They thought this would send a message to investors that the market had placed the stock at an unrealistically low level.

Before taking any action, they decided to consult with their investment banking representative, Albert Roth, senior vice-president at the investment firm of Baker, Green and Roth. Roth had aided the firm in initially selling its stock to the public ("going public") five years ago, and was quite familiar with its operations. Although he was surprised to receive their call during an early Saturday morning round of golf at the country club, he promised to get back to them in the next few days with his recommendation on a stock repurchase.

Gilbert Enterprises was the third-largest firm in the auto parts replacement industry, specializing in brake parts, power transmissions, batteries, cables, and other products related to used automobiles. Although most of the auto industry advertising relates to flashy new cars, Albert Roth knew that the auto parts replacement industry was becoming increasingly important.

His research indicated that the average age of an automobile life had reached eight years in 20XX, up from a mere 6.8 years from 10 years previously. Why? New vehicle price increases had far surpassed the rise in consumer income. People are forced to keep their old cars longer whether they want to or not. Furthermore, environmental legislation mandated more emission inspections and maintenance programs. Consumers were being forced to spend more money to update older automobiles to meet these standards.

Gilbert Enterprises had the most advanced just-in-time (JIT) inventory management system in the industry. For that reason, Albert Roth believed the firm would enjoy supernormal growth, beyond industry standards, for the next three years. His best estimate was that a 15 percent growth rate during that time period was entirely

reasonable. After that time span, a more normal growth rate of 6 percent was expected. Current dividends were 1.20 per share, and he decided to use a discount or required rate of return of 10 percent.

He discussed this approach with his partners, and although they generally agreed, they suggested that he also consider a more traditional approach of comparing the firm's P/E ratio to other firms in the industry. P/E data along with other information are shown in Table 1 for Gilbert Enterprises and three other firms in the industry.

	Gilbert Enterprises	Reliance Parts	Standard Auto	Allied Motors
Annual growth in EPS (past five years)	12.0%	8.0%	7.0%	9.0%
Return on shareholders' equity	18.0%	25.3%	14.0%	15.3%
Return on total assets	12.1%	8.1%	10.5%	9.8%
Debt to total assets	33.0%	68.0%	25.0%	36.0%
Market value	$35.25	$70.50	$24.25	$46.75
Book value	$16.40	$50.25	$19.50	$50.75
Replacement value	$43.50	$68.75	$26.00	$37.50
Dividend yield	3.40%	2.18%	5.26%	3.12%
P/E ratio	16.8	24.1	14.2	18.1

Table 1 Comparative data for auto parts replacement firms

What recommendation would you suggest that Albert Roth make? Do you suggest the firm is under- or overvalued?

APPENDIX 10A

The Bond Yield to Maturity Using Interpolation

As demonstrated in the body of the chapter, this calculation is much easier and more accurate if a financial calculator is used. However, here we use a numerical example to demonstrate this process. Assume a 20-year bond pays $118 per year (11.8 percent) in interest and $1,000 after 20 years in principal repayment. The current price of the bond is $1,085. We wish to determine the yield to maturity, or discount rate, that equates the future flows with the current price.

Because the bond is trading above par value at $1,085, we can assume the yield to maturity must be below the quoted interest rate of 12 percent (the yield to maturity would be the full 12 percent at a bond price of $1,000). As a first approximation, we try 10 percent. Annual analysis is used.

Present value of interest payments

$PV_A = A \times PV_{IFA}(n = 20, i = 10\%)$ (Appendix D)

$PV_A = \$118 \times 8.514 = \$1{,}004.65$

Present value of principal payment at maturity

$PV = FV \times PV_{IF}(n = 20, i = 10\%)$ (Appendix B)

$PV = \$1{,}000 \times 0.149 = \$149$

Total present value

Present value of interest payments	$1,004.65
Present value of principal payment at maturity.	149.00
Total present value, or price, of the bond.	$1,153.65

The discount rate of 10 percent gives us too high a present value in comparison to the current bond price of $1,085. Let's try a higher discount rate to get a lower price. We will use 11 percent.

Present value of interest payments

$PV_A = A \times PV_{IFA}(n = 20, i = 11\%)$ (Appendix D)

$PV_A = \$118 \times 7.963 = \$939.63$

Present value of principal payment at maturity

$PV = FV \times PV_{IF}$ $(n = 20, i = 11\%)$ (Appendix B)

$PV = \$1{,}000 \times 0.124 = \$124$

Total present value

Present value of interest payments	$ 939.63
Present value of principal payment at maturity.	124.00
Total present value, or price, of the bond.	$1,063.63

The discount rate of 11 percent gives us a value slightly lower than the bond price of $1,085. The rate for the bond must fall between 10 and 11 percent. Using linear interpolation, the answer is 10.76 percent.

$1,153.65	PV @ 10%	$1,153.65	PV @ 10%
1,063.63	PV @ 11%	1,085.00	bond price
$ 90.02		$ 68.65	

$$10\% + \frac{\$68.65}{\$90.02}(1\%) = 10\% + 0.76(1\%) = 10.76\%$$

PROBLEMS

1. Bonds issued by the Peabody Corporation have a par value of $1,000, are selling for $890, and have 18 years to maturity. The annual interest payment is 8 percent. Find yield to maturity by combining the trial-and-error approach with interpolation, as shown in this appendix. (Use an assumption of annual interest payments.)
2. Bonds issued by the Bullwinkle Corporation have a par value of $1,000, are selling for $1,100, and have 7 years to maturity. The annual interest payment is 9 percent, payable semiannually. Find yield to maturity by combining the trial-and-error approach with interpolation, as shown in this appendix.

APPENDIX 10B

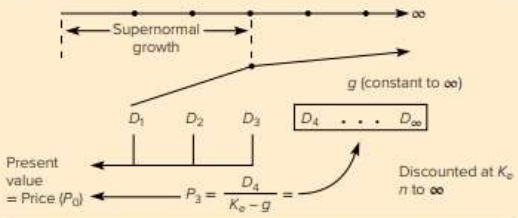
Valuation of a Supernormal Growth Firm

The formula for the valuation of a supernormal growth firm is

$$P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+K_e)^t} + P_n\left(\frac{1}{1+K_e}\right)^n \quad \text{(10B-1)}$$

(Supernormal growth period) (After supernormal growth period)

Represented graphically as



The formula is not difficult to use. The first term calls for determining the present value of the dividends using the supernormal growth period. The second term calls for computing the present value of the future stock price as determined at the end of the supernormal growth period. If we add the two together, we arrive at the current stock price. We are adding together the two benefits the shareholder will receive: (1) a future stream of dividends during the supernormal growth period and (2) the future stock price.

Let's assume that the firm paid a dividend over the last 12 months of $1.67; this represents the current dividend rate. Dividends are expected to grow by 20 percent per year over the supernormal growth period (***n***) of three years. They will then grow at a normal constant rate (***g***) of 5 percent. The required rate of return (discount rate) as represented by K_e is 9 percent. We first find the present value of the dividends during the supernormal growth period.

1. Present Value of Supernormal Dividends

D_0 = $1.67. We allow the value to grow at 20 percent per year over the three years of supernormal growth.

$D_1 = D_0 (1 + 0.20) = \$1.67\ (1.20) = \$2.00$
$D_2 = D_1 (1 + 0.20) = \$2.00\ (1.20) = \$2.40$
$D_3 = D_2 (1 + 0.20) = \$2.40\ (1.20) = \$2.88$

We then discount these values back at 9 percent to find the present value of dividends during the supernormal growth period.

	Supernormal Dividends	Present Value of Dividends during the Supernormal Period (K_e = 9%)
D_1	$2.00	$1.83
D_2	2.40	2.02
D_3	2.88	2.22
		$6.07

The present value of the supernormal dividends is $6.07. We now turn to the future stock price.

2. **Present Value of Future Stock Price** We first find the future stock price at the end of the supernormal growth period. This is found by taking the present value of the dividends that will be growing at a normal, constant rate after the supernormal period. This will begin ***after*** the third (and last) period of supernormal growth.

Since after the supernormal growth period the firm is growing at a normal, constant rate ($g = 5$ percent) and K_e (the discount rate) of 9 percent exceeds the new, constant growth rate of 5 percent, we have fulfilled the two conditions for using the constant dividend growth model after three years. That is, we can apply formula 10–8 (without subscripts for now).

$$P = \frac{D}{K_e - g}$$

In this case, however, D is really the dividend at the end of the fourth period, because this phase of the analysis starts at the beginning of the fourth period, and D is as of the end of the first period of analysis in the formula. Also, the price we are solving for now is the price at the beginning of the fourth period, which is the same concept as the price at the end of the third period (P_3).

We thus say

$$P_3 = \frac{D_4}{K_e - g} \quad \text{(10B-2)}$$

D_4 is equal to the previously determined value for D_3 of \$2.88 moved forward one period at the constant growth rate of 5 percent.

$$D_4 = \$2.88(1.05) = \$3.02$$

Also,

$K_e = 0.09$ discount rate (required rate of return)
$g = 0.05$ constant growth rate

$$P_3 = \frac{D_4}{K_e - g} = \frac{\$3.02}{0.09 - 0.05} = \frac{\$3.02}{0.04} = \$75.50$$

This is the value of the stock at the end of the third period. We discount this value back to the present.

Stock Price After Three Years	Present Value of Future Price ($K_e = 9\%$)
\$75.50	\$58.30

The present value of the future stock price (P_3) of \$75.50 is \$58.30.

By adding together the answers in part 1 and part 2 of this appendix, we arrive at the total present value, or price, of the supernormal growth stock.

1.	Present value of dividends during the normal growth period	\$ 6.07
2.	Present value of the future stock price	58.30
	Total present value, or price	\$64.37

The process we have just completed is presented in Figure 10B–1. Students who wish to develop skills in growth analysis should work the problems at the end of this appendix.

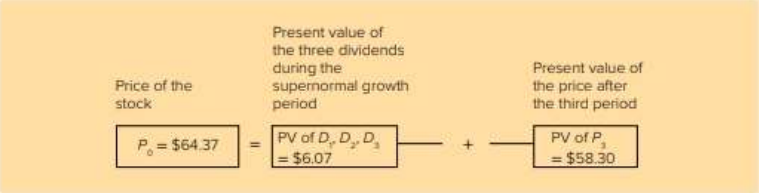


Figure 10B–1 Stock valuation under supernormal growth analysis

REVIEW OF FORMULAS

1. $$P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+K_e)^t} + P_n\left(\frac{1}{1+K_e}\right)^n \quad \text{(10B-1)}$$

2. $$P_3 = \frac{D_4}{K_e - g} \quad \text{(10B-2)}$$

PROBLEMS

1. Surgical Supplies Corporation paid a dividend of $1.12 over the last 12 months. The dividend is expected to grow at a rate of 25 percent over the next three years (supernormal growth). It will then grow at a normal, constant rate of 7 percent for the foreseeable future. The required rate of return is 12 percent (this will also serve as the discount rate).
   a. Compute the anticipated value of the dividends for the next three years (D_1, D_2, and D_3).
   b. Discount each of these dividends back to the present at a discount rate of 12 percent and then sum them.
   c. Compute the price of the stock at the end of the third year (P_3). (Review Appendix 10B for the definition of D_4.)
   d. After you have computed P_3, discount it back to the present at a discount rate of 12 percent for three years.
   e. Add together the answers in parts ***b*** and ***d*** to get the current value of the stock. (This answer represents the present value of the first three periods of dividends plus the present value of the price of the stock after three periods.)
2. You are considering investing in Black Tie Co., a holding enterprise that will pay a dividend of $2.00, which will increase by 12 percent each year over the following three years and then grow at an annual rate of 5 percent indefinitely. You expect a 22 percent return on your invested capital. What price would you pay for a share in this company?
3. Ninja Co. will pay a dividend of $5.00, which will increase by 9 percent each year over the following three years and then grow at an annual rate of 6 percent forever. You expect a 16 percent return on your invested capital. What price would you pay for a share in this company?

4. Sleepy Ltd. expects its present $1.25 dividend (just paid) to grow by 20 percent over the next three years, after which it will remain the same with no growth, forever. If an investor requires a return of 16 percent for investing in Sleepy, what would be its current price?
5. Clarinet Inc. has an expected yield of 18 percent. It anticipates paying the same dividend of $1.10 for four more years, after which the dividend will grow at 7 percent a year indefinitely. Based on the dividend valuation (capitalization) model, at what price should Clarinet currently sell?
6. March Hair Ltd. just paid a dividend of $1.80, which it expects to be $2.90 next year and $4.00 the next year. After that time, the dividend will likely decline to 5 percent per year, forever. With required rates of return at 14 percent, what should investors pay for March Hair?