Disc 2b

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UC Berkeley

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Mini Lecture: Euler's Formula

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11. +0 vertices, +1 face, +1 edge.
12. Euler's formula holds under induction!

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9. Sanity check $K_{3,3}$ has 9 edges, 6 vertices $9. \leq 12$ is true?
10. If your graph is planar, $e \leq 3v - 6$ is true. If graph violates $e \leq 3v - 6$, graph is not planar. If you do not violate $e \leq 3v - 6$, graph is not nessecarily planar.

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11. Planar graphs tend to be sparse (number of edges grows porportionally to number of vertices).

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3. Everything except division works like in regular math

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4. 1 = k * a + j * N
5. From here, which variable is the modular inverse of a?

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Problem 2

1. You need to prove both directions!

Problem 3a

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7. gcd(a, b) $=$ k means $k = m * a + n * b$.

7. $= kmeansk = m * a + n * b$. This is e-gcd!

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4. $= 1 = 1 * 9 + (-4) * 2$

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