

Disc 3b

Andy

UC Berkeley

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Secret Sharing

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- 7 Kind of like the 2 man rule on nuclear submarines

Packet Loss

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- 4 Thus if we drop k from $n + k$, we are still good!

Berlekamp Welch

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- 2 Berlekamp is a former UC Berkeley Professor (emeritus)
- 3 We need a way to recover from errors in messages
- 4 If original message is size n , and we have k errors. We need $n + 2k$ packets

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- 6 $E(i)$ has degree k . First coeff is 1, thus it has k unknown coeffs.
- 7 $P(i)E(i) = r_i * E(i)$ has $n + 2k$ unknowns that require $n + 2k$ equations

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- 7 Get back the original message!!

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