Disc 2a

Andy

UC Berkeley

June 25, 2018

Revist: Disc 1d, Probem 1. Well Ordering Principle

1. WOP says $\exists x : \forall y \in S : x \leq y$

Revist: Disc 1d, Probem 1. Well Ordering Principle

1. WOP says $\exists x : \forall y \in S : x \leq y$
2. In English: There exists a smallest element in your set (of Naturals).

Revist: Disc 1d, Probem 1. Well Ordering Principle

1. WOP says $\exists x : \forall y \in S : x \leq y$
2. In English: There exists a smallest element in your set (of Naturals).
3. $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in Sx \leq y$

Revist: Disc 1d, Probem 1. Well Ordering Principle

1. WOP says $\exists x : \forall y \in S : x \leq y$
2. In English: There exists a smallest element in your set (of Naturals).
3. $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in Sx \leq y$
4. In English: If you set contains an element between $[0, z]$, then your set has a smallest element

Revist: Disc 1d, Probem 1. Well Ordering Principle

1. WOP says $\exists x : \forall y \in S : x \leq y$
2. In English: There exists a smallest element in your set (of Naturals).
3. $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in S x \leq y$
4. In English: If you set contains an element between $[0, z]$, then your set has a smallest element
5. Base: If you set contains 0, your set has a smallest element. This is true because 0 is by definition the smallest natural, so your set contains the smallest possible element.

Revist: Disc 1d, Probem 1. Well Ordering Principle

1. WOP says $\exists x : \forall y \in S : x \leq y$
2. In English: There exists a smallest element in your set (of Naturals).
3. $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in Sx \leq y$
4. In English: If you set contains an element between $[0, z]$, then your set has a smallest element
5. Base: If you set contains 0, your set has a smallest element. This is true because 0 is by definition the smallest natural, so your set contains the smallest possible element.
6. If your set has a 0 or 1, your set has a smallest element.

Revist: Disc 1d, Probem 1. Well Ordering Principle

1. WOP says $\exists x : \forall y \in S : x \leq y$
2. In English: There exists a smallest element in your set (of Naturals).
3. $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in Sx \leq y$
4. In English: If you set contains an element between $[0, z]$, then your set has a smallest element
5. Base: If you set contains 0, your set has a smallest element. This is true because 0 is by definition the smallest natural, so your set contains the smallest possible element.
6. If your set has a 0 or 1, your set has a smallest element.
7. If your set has a 0, 1, or 2, your set has a smallest element.

Revist: Disc 1d, Probem 1. Well Ordering Principle

1. WOP says $\exists x : \forall y \in S : x \leq y$
2. In English: There exists a smallest element in your set (of Naturals).
3. $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in Sx \leq y$
4. In English: If you set contains an element between $[0, z]$, then your set has a smallest element
5. Base: If you set contains 0, your set has a smallest element. This is true because 0 is by definition the smallest natural, so your set contains the smallest possible element.
6. If your set has a 0 or 1, your set has a smallest element.
7. If your set has a 0, 1, or 2, your set has a smallest element.
8. If your set as a number between [0, 100], ...

Revist: Disc 1d, Probem 1. Well Ordering Principle

1. WOP says $\exists x : \forall y \in S : x \leq y$
2. In English: There exists a smallest element in your set (of Naturals).
3. $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in Sx \leq y$
4. In English: If you set contains an element between $[0, z]$, then your set has a smallest element
5. Base: If you set contains 0, your set has a smallest element. This is true because 0 is by definition the smallest natural, so your set contains the smallest possible element.
6. If your set has a 0 or 1, your set has a smallest element.
7. If your set has a 0, 1, or 2, your set has a smallest element.
8. If your set as a number between [0, 100], ...
9. If your set has a number between $[0, \propto)$, ...

Revisit: Disc 1d, Problem 1. WOP (cont.)

1. If your set has a number between $[0, \propto)$, your set has a smallest element.

Revisit: Disc 1d, Problem 1. WOP (cont.)

1. If your set has a number between $[0, \propto)$, your set has a smallest element.
2. If your set is non-empty, your set has a smallest element.

Revisit: Disc 1f, Problem 1. WOP (ignore.)

1. Inductive Hypo:
   $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in Sx \leq y$

Revisit: Disc 1f, Problem 1. WOP (ignore.)

1. Inductive Hypo:
   $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in S x \leq y$
2. Inductive Step: Lets prove
   $(\exists s \in S, 0 \leq s \leq z+1) \implies WOP$.

Revisit: Disc 1f, Problem 1. WOP (ignore.)

1. Inductive Hypo:
   $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in S x \leq y$
2. Inductive Step: Lets prove
   $(\exists s \in S, 0 \leq s \leq z+1) \implies WOP$.
3. Case 1. The smallest element in S is $z+1$. Then $z+1$ is the minimum element.

Revisit: Disc 1f, Problem 1. WOP (ignore.)

1. Inductive Hypo:
   $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in S x \leq y$
2. Inductive Step: Lets prove
   $(\exists s \in S, 0 \leq s \leq z + 1) \implies WOP.$
3. Case 1. The smallest element in S is $z + 1$. Then $z + 1$ is the minimum element.
4. Case 2. The smallest element is less than $z + 1$, or between $[0, z]$. Then we know a smallest element in S exists by inductive hypothesis.

Revisit: Disc 1f, Problem 1. WOP (ignore.)

1. Inductive Hypo:
   $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in S x \leq y$
2. Inductive Step: Lets prove
   $(\exists s \in S, 0 \leq s \leq z+1) \implies WOP$.
3. Case 1. The smallest element in S is $z+1$. Then $z+1$ is the minimum element.
4. Case 2. The smallest element is less than $z+1$, or between $[0, z]$. Then we know a smallest element in S exists by inductive hypothesis.
5. What should you change so that the proof works by simple induction (as opposed to strong induction)?

Revisit: Disc 1f, Problem 1. WOP (ignore.)

1. Inductive Hypo:
   $(\exists s \in S, 0 \leq s \leq z) \implies \exists x : \forall y \in Sx \leq y$
2. Inductive Step: Lets prove
   $(\exists s \in S, 0 \leq s \leq z+1) \implies WOP.$
3. Case 1. The smallest element in S is $z+1$. Then $z+1$ is the minimum element.
4. Case 2. The smallest element is less than $z+1$, or between $[0, z]$. Then we know a smallest element in S exists by inductive hypothesis.
5. What should you change so that the proof works by simple induction (as opposed to strong induction)?
6. For all sets S that contain a number z' such that $z' \leq z$, then S contains a smallest element.

Problem 1a

1. Problem: Consider a tree with $n \geq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.

Problem 1a

1. Problem: Consider a tree with $n \geq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. Hint: Why can't a tree have n leaves? Pretend there does exist a tree with n leaves. What is each vertice's degree? What is the definition of a tree?

Problem 1a

1. Problem: Consider a tree with $n \geq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. Hint: Why can't a tree have n leaves? Pretend there does exist a tree with n leaves. What is each vertice's degree? What is the definition of a tree?
3. Hint: A tree, by defintion, has a unique, finite length path between 2 vertices.

Problem 1a soln

1. Problem: Consider a tree with $n \leq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.

Problem 1a soln

1. Problem: Consider a tree with $n \leq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. We know we can construct a tree with $n \geq 3$ nodes and make it have $n-1$ leaves. Do a star graph!

Problem 1a soln

1. Problem: Consider a tree with $n \leq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. We know we can construct a tree with $n \geq 3$ nodes and make it have $n - 1$ leaves. Do a star graph!
3. We can't have a tree with n leaves.

Problem 1a soln

1. Problem: Consider a tree with $n \leq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. We know we can construct a tree with $n \geq 3$ nodes and make it have $n - 1$ leaves. Do a star graph!
3. We can't have a tree with n leaves.
4. Assume such a tree with n leaves exists.

Problem 1a soln

1. Problem: Consider a tree with $n \leq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. We know we can construct a tree with $n \geq 3$ nodes and make it have $n-1$ leaves. Do a star graph!
3. We can't have a tree with n leaves.
4. Assume such a tree with n leaves exists.
5. Each vertice has 1 edge, since each vertex has degree 1.

Problem 1a soln

1. Problem: Consider a tree with $n \leq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. We know we can construct a tree with $n \geq 3$ nodes and make it have $n - 1$ leaves. Do a star graph!
3. We can't have a tree with n leaves.
4. Assume such a tree with n leaves exists.
5. Each vertice has 1 edge, since each vertex has degree 1.
6. Choose an arbitrary x vertex, it has a neighbor, y.

Problem 1a soln

1. Problem: Consider a tree with $n \leq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. We know we can construct a tree with $n \geq 3$ nodes and make it have $n - 1$ leaves. Do a star graph!
3. We can't have a tree with n leaves.
4. Assume such a tree with n leaves exists.
5. Each vertice has 1 edge, since each vertex has degree 1.
6. Choose an arbitrary x vertex, it has a neighbor, y.
7. y also has only 1 edge, and it connects to x.

Problem 1a soln

1. Problem: Consider a tree with $n \leq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. We know we can construct a tree with $n \geq 3$ nodes and make it have $n - 1$ leaves. Do a star graph!
3. We can't have a tree with n leaves.
4. Assume such a tree with n leaves exists.
5. Each vertice has 1 edge, since each vertex has degree 1.
6. Choose an arbitrary x vertex, it has a neighbor, y.
7. y also has only 1 edge, and it connects to x.
8. The there cannot be a path to z, a third arbitrary vertex.

Problem 1a soln

1. Problem: Consider a tree with $n \leq 3$ vertices. What is the largest possible number of leaves the tree could have? Prove that this maximum m is possible to achieve, and further that there cannot exist a tree with more than m leaves.
2. We know we can construct a tree with $n \geq 3$ nodes and make it have $n - 1$ leaves. Do a star graph!
3. We can't have a tree with n leaves.
4. Assume such a tree with n leaves exists.
5. Each vertice has 1 edge, since each vertex has degree 1.
6. Choose an arbitrary x vertex, it has a neighbor, y.
7. y also has only 1 edge, and it connects to x.
8. The there cannot be a path to z, a third arbitrary vertex.
9. Defintion of tree has been violated!

Problem 1b

1. Problem: Prove that every tree with $n \geq 2$ has at least 2 leaves.

Problem 1b

1. Problem: Prove that every tree with $n \geq 2$ has at least 2 leaves.
2. Hint: A tree, by defintion, has a unique, finite length path between 2 vertices. We can assume (by WOP) that there is a longest path.

Problem 1b soln

1. Problem: Prove that every tree with $n \geq 2$ has at least 2 leaves.

Problem 1b soln

1. Problem: Prove that every tree with $n \geq 2$ has at least 2 leaves.
2. Assume the longest path is from x to y (if there are many longest paths, choose one).

Problem 1b soln

1. Problem: Prove that every tree with $n \geq 2$ has at least 2 leaves.
2. Assume the longest path is from x to y (if there are many longest paths, choose one).
3. x and y are leaves.

Problem 1b soln

1. Problem: Prove that every tree with $n \geq 2$ has at least 2 leaves.
2. Assume the longest path is from x to y (if there are many longest paths, choose one).
3. x and y are leaves.
4. Assume that x is not a leaf. It must have degree at least 2. Thus it has a neighbor (we'll call z).

Problem 1b soln

1. Problem: Prove that every tree with $n \geq 2$ has at least 2 leaves.
2. Assume the longest path is from x to y (if there are many longest paths, choose one).
3. x and y are leaves.
4. Assume that x is not a leaf. It must have degree at least 2. Thus it has a neighbor (we'll call z).
5. z must not appear in the path from x to y. Why?

Problem 1b soln

1. Problem: Prove that every tree with $n \geq 2$ has at least 2 leaves.
2. Assume the longest path is from x to y (if there are many longest paths, choose one).
3. x and y are leaves.
4. Assume that x is not a leaf. It must have degree at least 2. Thus it has a neighbor (we'll call z).
5. z must not appear in the path from x to y. Why?
6. We have a cycle. x to a to ... to z to x makes a cycle.

Problem 1b soln

1. Problem: Prove that every tree with $n \geq 2$ has at least 2 leaves.
2. Assume the longest path is from x to y (if there are many longest paths, choose one).
3. x and y are leaves.
4. Assume that x is not a leaf. It must have degree at least 2. Thus it has a neighbor (we'll call z).
5. z must not appear in the path from x to y. Why?
6. We have a cycle. x to a to ... to z to x makes a cycle.
7. Thus x must be a leaf!

Problem 2

1. a. Draw a cube and its dual in a planar graph form.

Problem 2

1. a. Draw a cube and its dual in a planar graph form.
2. b. Draw a spanning tree for a cube in planar graph form. Do the same for the dual.

Problem 3

1. Hint: Try to prove this for a graph on 7 vertices.

Problem 3

1. Hint: Try to prove this for a graph on 7 vertices.
2. Hint: Where are there definetly not any edges?

Problem 4

1. a. Hint: What size is (VxV)?

Problem 4

1. a. Hint: What size is (VxV)?
2. b. Hint: A planar graph has at most 3v-6 edges.