Disc 4a

Andy

UC Berkeley

July 9, 2018

Disc 4a Andy

Definition

Computability asks can a problem be solved via algorithm (computers)?

I Can we have a program that tells us if another program halts?

Disc 4a

Definition

Computability asks can a problem be solved via algorithm (computers)?

- 1 Can we have a program that tells us if another program halts?
- 2 Is there a perfect antivirus?

Disc 4a

Definition

Computability asks can a problem be solved via algorithm (computers)?

- Can we have a program that tells us if another program halts?
- 2 Is there a perfect antivirus?
- 3 Perfect compression algorithm?

Disc 4a

What is a computer?



Figure: Turing Machine

1 Has memory. Ideally infinite memory.

Disc 4a

What is a computer?



Figure: Turing Machine

- 1 Has memory. Ideally infinite memory.
- Has a head that can read memory

Disc 4a

What is a computer?



Figure: Turing Machine

- 1 Has memory. Ideally infinite memory.
- 2 Has a head that can read memory
- 3 Head can write 1 or 0 to memory

Disc 4a

What is a computer?



Figure: Turing Machine

- Has memory. Ideally infinite memory.
- 2 Has a head that can read memory
- 3 Head can write 1 or 0 to memory
- 4 Head decides what to do based on memory.

Assumptions

- Computers have infinite compute
- Computers have infinite memory
- 3 But computation still takes time

Halting Problem

Disc 4a

Andy

1 "This statement is false"

Halting Problem

- "This statement is false"
- Can we have a program that determines if another program halts?

Halting Problem

- "This statement is false"
- Can we have a program that determines if another program halts?
- 3 Assume we have a program, TestHalt, that takes in P and x, and will tell you if P(x) halts

Disc 4a Andy

```
def Turing(P):
if TestHalt(P, P):
  loopforever();
else:
  halt();
```

We run Turing(Turing)

```
def Turing(P):
if TestHalt(P, P):
  loopforever();
else:
  halt();
```

- We run Turing(Turing)
- 2 If Turing(Turing) halts, then Turing(Turing) loops...

```
def Turing(P):
if TestHalt(P, P):
  loopforever();
else:
  halt();
```

- We run Turing(Turing)
- 2 If Turing(Turing) halts, then Turing(Turing) loops...
- **3** If Turing(Turing) loops, then Turing(Turing) halts...

Disc 4a Andy

```
def Turing(P):
if TestHalt(P, P):
  loopforever();
else:
  halt();
```

- We run Turing(Turing)
- 2 If Turing(Turing) halts, then Turing(Turing) loops...
- **3** If Turing(Turing) loops, then Turing(Turing) halts...
- 4 Paradox

Disc 4a Andy

```
def Turing(P):
if TestHalt(P, P):
  loopforever();
else:
  halt();
```

- We run Turing(Turing)
- 2 If Turing(Turing) halts, then Turing(Turing) loops...
- If Turing(Turing) loops, then Turing(Turing) halts...
- 4 Paradox
- We cannot have a paradox. If you can build a Testhalt -¿ paradox

Disc 4a Andy

```
def Turing(P):
if TestHalt(P, P):
  loopforever();
else:
  halt();
```

- We run Turing(Turing)
- 2 If Turing(Turing) halts, then Turing(Turing) loops...
- If Turing(Turing) loops, then Turing(Turing) halts...
- 4 Paradox
- We cannot have a paradox. If you can build a Testhalt -¿ paradox
- 6 Anything that allows you to build a TestHalt program is impossible