Homework 2 feedback

19/20

1. Good!

2. Good!

3. Good!

4. For part (c), L'Hospital's Rule doesn't apply directly because we don't know that the *derivative* of $\sin(x)/x$ is continuous; that is, we don't know that
$$f'(0) = \lim_{h\to 0} f'(h).$$
You computed the right hand side, but we don't know if it is equal to the left hand side. This problem was accidentally harder than I thought (I thought there was a simpler solution but I messed up), but for the sake of completeness, here's a potential solution that only uses material we've covered so far.

   In order to compute $f'(0)$, we can go back to the definition. Then, since $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$, this means
$$f'(0) = \lim_{h\to 0} \frac{\frac{\sin(h)}{h} - 1}{h}.$$
Recalling that $\cos(h) \leq \frac{\sin(h)}{h} \leq \frac{1}{\cos(h)}$, we can squeeze the above quantity:
$$\lim_{h\to 0} \frac{\cos(h) - 1}{h} \leq \lim_{h\to 0} \frac{\frac{\sin(h)}{h} - 1}{h} \leq \lim_{h\to 0} \frac{\frac{1}{\cos(h)} - 1}{h} = \lim_{h\to 0} \frac{1 - \cos(h)}{h\cos(h)}.$$
I also mentioned once in class that $\lim_{h\to 0} \frac{\cos(h)-1}{h} = 0$, and therefore both the left and right terms above go to 0. By the squeeze theorem, $f'(h) = 0$. This wasn't the solution I intended, and I don't expect you to find this on your own. Sorry about that, but I hope you can learn something from this problem!

5. Good!

6. Good!

7. (1/2) For part (c), I don't see the chain rule being used appropriately. The derivative should be $\frac{1}{\sqrt{1-\log(x)^2}} \cdot \frac{1}{x}$, and when evaluated at $x = 1$ this gives $\frac{1}{1-0} \cdot \frac{1}{1}$. (Concerning the use of log and ln, I believe I said in lecture that I always mean the natural logarithm when I write log.)

8. Good!