Homework 2 feedback

13/20

1. (1/2) For part (c), the derivative of 2^x is not $2x$. The derivative of x^2 is $2x$. The way we compute the derivative of exponential functions is via e^x and the chain rule:

$$(2^x)' = ((e^{\log(2)})^x)' = (e^{x\log 2})' = e^{x\log 2} \cdot \log 2.$$

2. Good!

3. (1/2) The calculation of the limit wasn't really finished here. . . Part of the purpose of this problem is to actually verify that the power rule works, since in-class we only verified the power rule works for $x^{1/2}$. The way we calculate the limit is via

$$\begin{aligned}\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}\cdot\frac{\sqrt[3]{x+h}^2+\sqrt[3]{x+h}\sqrt[3]{x}+\sqrt[3]{x}^2}{\sqrt[3]{x+h}^2+\sqrt[3]{x+h}\sqrt[3]{x}+\sqrt[3]{x}^2} &= \frac{\sqrt[3]{x+h}^3-\sqrt[3]{x}^3}{h\cdot(\sqrt[3]{x+h}^2+\sqrt[3]{x+h}\sqrt[3]{x}+\sqrt[3]{x}^2)}\\ &= \frac{1}{\sqrt[3]{x+h}^2+\sqrt[3]{x+h}\sqrt[3]{x}+\sqrt[3]{x}^2},\end{aligned}$$

and taking $h \to 0$ we find that the limit is equal to $\frac{1}{3\sqrt[3]{x^2}} = \frac{1}{3}x^{-2/3}$.

4. For part (c), it isn't actually possible to evaluate $\frac{x\cos(x)-\sin(x)}{x^2}$ at $x = 0$, since then we get $\frac{0}{0}$. There is a solution to this problem using the squeeze theorem, and although it only uses material we've covered so far in class, this problem was harder than I thought. (I had a mistake in the original solution that made it seem easier.) I can discuss the solution in detail, upon request.

5. Good!

6. (1/2) When implicitly differentiating the equation, we should get

$$\cos(\pi xy)\cdot(\pi y+\pi xy') = \pi(1+y'),$$

where we used the chain rule for $\sin(\pi xy)$, and then we have to use product rule to compute $(\pi xy)' = \pi y + \pi xy'$. With this formula we should get the answer $y' = 0$. (If you graph this equation in Desmos or something then you'll also see this is the case.)

7. (0/2) I don't understand equations such as

$$\arcsin(0) = 1/\sqrt{1}(1 - u^2).$$

Apparently the chain rule is in mind here, but we have to fully compute the derivatives first, and then evaluate them at specific values. For example, the derivative of $\arccos(2\sqrt{x})$ is

$$\frac{-1}{\sqrt{1-(2\sqrt{x})^2}} \cdot (2\sqrt{x})' = \frac{-1}{\sqrt{1-4x}} \cdot (2 \cdot \frac{1}{2}x^{-1/2}) = \frac{-1}{\sqrt{1-4x}\sqrt{x}}.$$

For part (b), the derivative of $\arctan(x)$ is $\frac{1}{1+x^2}$, and so the derivative of $\arctan(x^2)$ is

$$\frac{1}{1+(x^2)^2} \cdot (x^2)' = \frac{2x}{1+x^4}.$$

For part (c), the derivative of $\arcsin(x)$ is $\frac{1}{\sqrt{1-x^2}}$, and the derivative of $\log(x) = \frac{1}{x}$, and so the derivative of $\arcsin(\log(x))$ is

$$\frac{1}{\sqrt{1-(\log(x))^2}} \cdot (\log(x))' = \frac{1}{\sqrt{1-(\log(x))^2}} \cdot \frac{1}{x}.$$

8. (0/2) The way this problem should be done is by finding the general equation of a line tangent to the parabola $y = x^2$ at some point (a, a^2), and finding when this line passes through the point $(0, -1)$. The slope of the graph at $x = a$ is $2a$, and so the equation of the tangent line is

$$(y - a^2) = (2a)(x - a) \Leftrightarrow y = 2ax + (a^2 - 2a^2).$$

In order for the y-intercept to be -1, we need $-a^2 = -1$, or in other words $a^2 = 1$. Thus, $a = \pm 1$, and the two tangent lines are given by

$$y = 2x - 1, y = -2x - 1.$$