

Homework 2 feedback

18/20

1. For (b), there was an arithmetic error. $\frac{\pi}{3} \cos(\pi/3) = \pi/6$, not $1/6$.
2. Good!
3. (1/2) The problem was quite clear about how you should compute the derivative from the limit definition. It goes as follows:

$$\begin{aligned} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} \cdot \frac{\sqrt[3]{x+h}^2 + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x}^2}{\sqrt[3]{x+h}^2 + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x}^2} &= \frac{\sqrt[3]{x+h}^3 - \sqrt[3]{x}^3}{h \cdot (\sqrt[3]{x+h}^2 + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x}^2)} \\ &= \frac{1}{\sqrt[3]{x+h}^2 + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x}^2}, \end{aligned}$$

and taking $h \rightarrow 0$ we find that the limit is equal to $\frac{1}{3\sqrt[3]{x}^2} = \frac{1}{3}x^{-2/3}$.

4. Good!
5. Good!
6. Good!
7. Good!
8. (1/2) Why is the slope 2? More detail is needed.
Here's is summary of the solution: the equation of the tangent line passing through an arbitrary point (a, a^2) is

$$(y - a^2) = (2a)(x - a) \Leftrightarrow y = 2ax - a^2.$$

Thus, in order for this line to pass through the point $(0, -1)$, we need $a^2 = 1$, hence $a = \pm 1$ and the two equations are given by $y = 2x - 1$, $y = -2x - 1$.