Homework 3 feedback

18/20

1. Good!

2. Good!

3. In the same way that we can calculate the derivative of $\arctan(x) = \frac{1}{1+x^2}$, we can calculate that the derivative of $\operatorname{arccot}(x) = \frac{-1}{1+x^2}$. If we let $y = \operatorname{arccot}(x)$ then $\cot y = x$, and since $1 + \cot^2 y = \csc^2 y$, we have $\csc^2 y = \csc^2(\operatorname{arccot} x) = 1 + \cot^2(\operatorname{arccot} x) = 1 + x^2$.

4. Good!

5. Good!

6. Good!

7. (1/2) We discussed part (c) in office hours, but here's another way to see why this case cannot happen. Notice that a concave up function (namely, a function where $f''(x) > 0$ for all x) will always lie above any tangent line. However, basically any tangent line will eventually go above the x-axis, and thus the function must eventually do this also. Thus, the condition $f''(x) > 0$ is incompatible with the condition $f(x) < 0$.

8. (1/2) I don't recognize the method used to calculate the derivative. It should be

$$f'(x) = 1 - \left(\frac{4(x+1) - (4x)(1)}{(x+1)^2} \right) = 1 - \frac{4}{(x+1)^2},$$

and then solving for critical points yields $(x+1)^2 = 4 \Leftrightarrow x = -3, 1$. However, only $x = 1$ is in the domain, and then we can check that $f(1) = -1$, while $f(0) = f(3) = 0$, so -1 is the minimum value of the function on $[0, 3]$.

9. Good!