Homework 2 feedback

15/20

1. (0/2) For (b), the product rule must be used. This means the derivative is computed as

$$(x)' \sin x + x(\sin x)' = \sin x + x \cos x,$$

and then substituting $x = \pi/3$ gives the answer $\sin(\pi/3) + \pi/3 \cos(\pi/3) = \frac{\sqrt{3}}{2} + \frac{\pi}{3} \cdot \frac{1}{2}$.
For (c), the power rule does not apply for exponential functions, only monomials (such as x^2, which is totally different from 2^x). The way we computed the derivative of 2^x in class was by writing it in terms of e^x and using the chain rule:

$$2^x = (e^{\log 2})^x = e^{x \log 2},$$

and then the derivative of this is $e^{x \log 2} \cdot \log 2 = 2^x \log 2$. So, the answer should be $16 \log 2$.

2. Good!

3. Good!

4. There are some arithmetic errors in (b): the denominator is $(1 + x)^2$, not $(1 + x^2)$ which was written in the quotient rule calculation. Also, the derivative of $\cos(x)^3$ is $3 \cos(x)^2 \cdot (-\sin(x))$, not $-\sin(3x)$.

5. Good!

6. (1/2) There are several arithmetic errors/miscopies from line to line. The implicit differentiation is done correctly, but most of the calculations after that don't make sense to me. When substituting $x = 1, y = -1$, we should get

$$\cos(-\pi)\pi(-1 + y') = \pi(1 + y')$$

and then

$$\pi - \pi y' = \pi + \pi y',$$

so that $y' = 0$.

7. There is a typo in the derivative of arccos, since it should be $\frac{-1}{\sqrt{1-x^2}}$, but it appears that $\frac{-1}{\sqrt{1+x^2}}$ was used instead? Otherwise, all looks good.

8. (0/2) I don't understand the steps taken. It is true that $2x$ is the derivative of x^2, but this does not immediately lead to $y = 2x - 1$ being the correct line. Here is a full solution.

   The equation for the tangent line at a given point (a, a^2) is

   $$(y - a^2) = 2a(x - a) \Leftrightarrow y = 2ax - a^2.$$

   Thus, in order for the line to pass through $(0, -1)$, we must have $-a^2 = -1$, i.e. $a^2 = 1$ and so $a = \pm 1$. Then we find that the two tangent lines that pass through $(0, -1)$ are $y = 2x - 1$, $y = -2x - 1$.