CS124 Programming Assignment 2

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1. In the standard matrix multiplication algorithm for two n by n matrices, we need n multiplications and $n-1$ additions for each of the n^2 entries. Therefore, the algorithm takes $n^2(n+n-1) = 2n^3 - n^2$ arithmetic operations.

   In Strassen's algorithm for two n by n matrices, we perform 18 additions or subtractions of $n/2$ by $n/2$ matrices, and we perform 7 multiplications of $n/2$ by $n/2$ matrices. Therefore, we get a recurrence of $T(n) = 7T(n/2) + 18(n/2)^2 = 7T(n/2) + 9n^2/2$, $T(1) = 1$, where $T(n)$ is the number of arithmetic operations. Solving this recurrence using Wolfman Alpha, we get that $T(n) = 49 \cdot 7^{\log_2 n - 1} - 6n^2$. To find n_0, we need the smallest n where $49 \cdot 7^{\log_2 n - 1} - 6n^2 < 2n^3 - n^2$. Plotting both, we see that $n_0 = 655$.

2. To find the crossover point experimentally, I used my implementation of Strassen's algorithm to multiply two 1024 by 1024 matrices while varying the values for n_0.

n_0	Strassen's Algorithm Performance (seconds)
8	4.131324
16	3.292821
32	3.017062
64	2.997684
128	3.494401
256	4.635245
512	6.232389
1024	24.759836

   The minimum occurs when n_0 is around 64. This was surprising, as the theoretical crossover point I derived above was $n_0 = 655$. If anything, I would have expected the crossover point in my implementation to be higher than the theoretical. I believe I made a mistake in the theoretical crossover point, but I could not figure out where.

3. The experimentally derived numbers of trianges were fairly close to the expected numbers of triangles for all five values of p.

p	Triangles	Expected
0.01	180	178
0.02	1321	1427
0.03	4654	4818
0.04	11132	11420
0.05	22384	22304