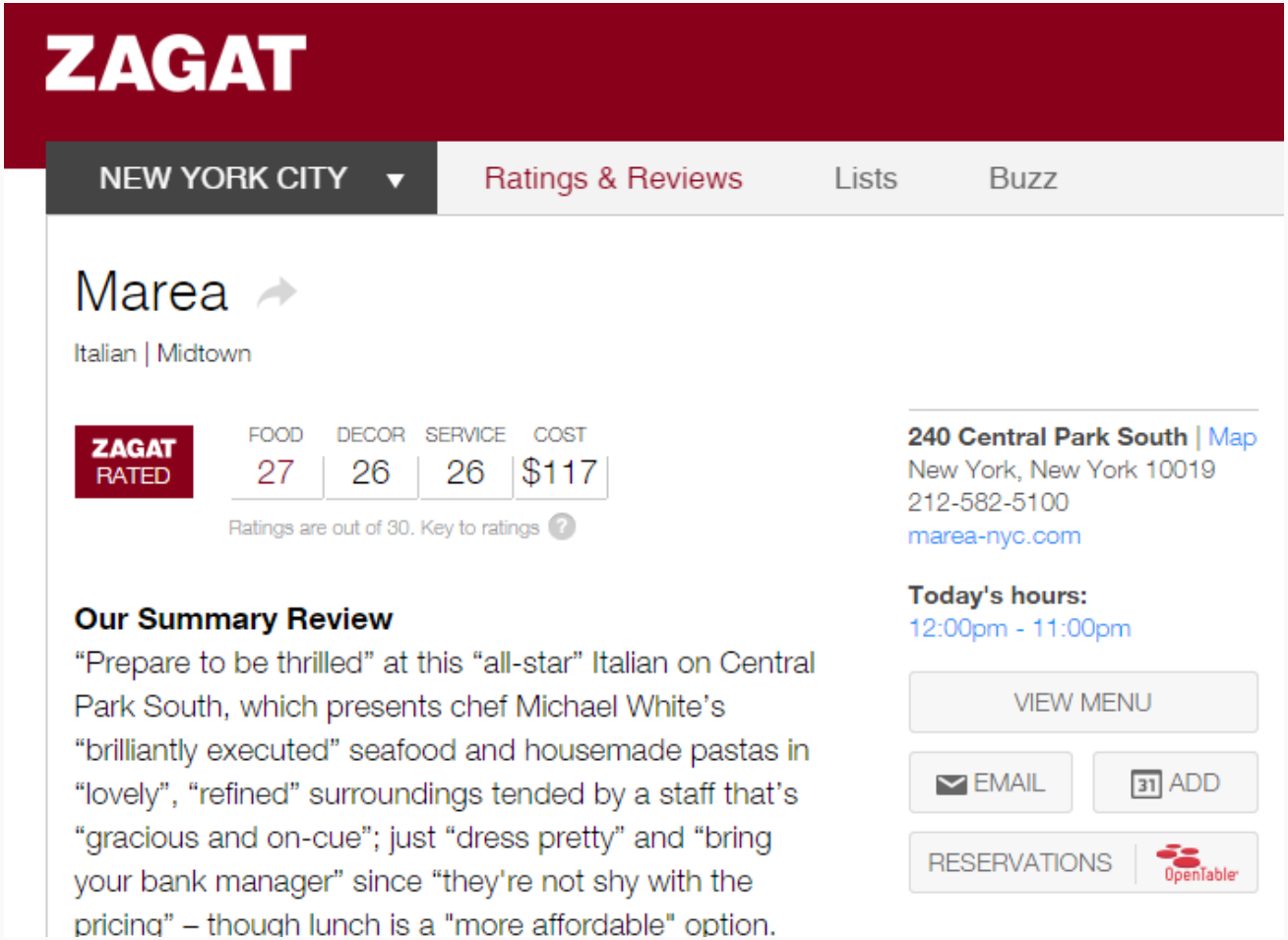
Geometry of Multiple Linear Regression

Ex: Restaurants in NYC



Ex: Restaurants in NYC

```
nyc
```

```
## # A tibble: 168 x 7
##     Case Restaurant           Price  Food Decor Service  East
##    <dbl> <chr>                <dbl> <dbl> <dbl>   <dbl> <dbl>
##  1     1 Daniella Ristorante    43    22    18      20     0
##  2     2 Tello's Ristorante     32    20    19      19     0
##  3     3 Biricchino             34    21    13      18     0
##  4     4 Bottino                41    20    20      17     0
##  5     5 Da Umberto             54    24    19      21     0
##  6     6 Le Madri               52    22    22      21     0
##  7     7 Le Zie                 34    22    16      21     0
##  8     8 Pasticcio              34    20    18      21     1
##  9     9 Belluno                39    22    19      22     1
## 10    10 Cinque Terre           44    21    17      19     1
## # … with 158 more rows
```

What is the unit of observation?

A restaurant

What determines the price of a meal?

Let's look at the relationship between price, food rating, and decor rating.

$$Price \sim Food + Decor$$

```
m1 <- lm(Price ~ Food + Decor, data = nyc)
```

Model 1: Food + Decor

```
summary(m1)
```

```
##
## Call:
## lm(formula = Price ~ Food + Decor, data = nyc)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -14.945  -3.766  -0.153   3.701  18.757
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -24.500      4.723   -5.19  6.2e-07 ***
## Food           1.646      0.262    6.29  2.7e-09 ***
## Decor          1.882      0.192    9.81  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.79 on 165 degrees of freedom
## Multiple R-squared:  0.617,    Adjusted R-squared:  0.612
## F-statistic:  133 on 2 and 165 DF,  p-value: <2e-16
```

The geometry of regression models

The function for $\hat{y}$ is . . .

- A *line* when you have one continuous x.
- *Parallel lines* when you have one continuous x_1 and one categorical x_2.
- *Unrelated lines* when you have one continuous x_1, one categorical x_2, and an interaction term $x_1 : x_2$.

When you have two continuous predictors x_1, x_2, then your mean function is . . .

a plane

3d plot

interactive code

Location, location, location

Does the price depend on where the restaurant is located in Manhattan?

$$Price \sim Food + Decor + East$$

```
nyc
```

```
## # A tibble: 168 x 7
##     Case Restaurant          Price  Food Decor Service  East
##    <dbl> <chr>               <dbl> <dbl> <dbl>   <dbl> <dbl>
##  1     1 Daniella Ristorante    43    22    18      20     0
##  2     2 Tello's Ristorante     32    20    19      19     0
##  3     3 Biricchino             34    21    13      18     0
##  4     4 Bottino                41    20    20      17     0
##  5     5 Da Umberto             54    24    19      21     0
##  6     6 Le Madri               52    22    22      21     0
##  7     7 Le Zie                 34    22    16      21     0
##  8     8 Pasticcio              34    20    18      21     1
##  9     9 Belluno                39    22    19      22     1
## 10    10 Cinque Terre           44    21    17      19     1
## # … with 158 more rows
```

Model 2: Food + Decor + East

```
m2 <- lm(Price ~ Food + Decor + East, data = nyc)
summary(m2)
```

```
##
## Call:
## lm(formula = Price ~ Food + Decor + East, data = nyc)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -14.045  -3.881   0.039   3.392  17.756
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -24.027      4.673   -5.14  7.7e-07 ***
## Food           1.536      0.263    5.84  2.8e-08 ***
## Decor          1.909      0.190   10.05  < 2e-16 ***
## East           2.067      0.932    2.22    0.028 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.72 on 164 degrees of freedom
## Multiple R-squared:  0.628,    Adjusted R-squared:  0.621
## F-statistic: 92.2 on 3 and 164 DF,  p-value: <2e-16
```

The geometry of regression models

- When you have two continuous predictors x_1, x_2, then your mean function is *a plane*.

- When you have two continuous predictors x_1, x_2, and a categorical predictor x_3, then your mean function represents *parallel planes*.

3d Plot

interactive code

The geometry of regression models

- When you have two continuous predictors x_1, x_2, then your mean function is *a plane*.
- When you have two continuous predictors x_1, x_2, and a categorical predictor x_3, then your mean function represents *parallel planes*.
- When you add in interaction effects, the planes become *tilted*.

Model 3: Food + Decor + East + Decor:East

```
m3 <- lm(Price ~ Food + Decor + East + Decor:East, data = nyc)
summary(m3)
```

```
##
## Call:
## lm(formula = Price ~ Food + Decor + East + Decor:East, data = nyc)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -13.785  -3.665   0.378   3.729  17.636
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -29.397      6.377   -4.61  8.1e-06 ***
## Food           1.663      0.282    5.90  2.1e-08 ***
## Decor          2.070      0.230    9.01  5.4e-16 ***
## East           9.662      6.218    1.55     0.12
## Decor:East    -0.435      0.352   -1.24     0.22
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.71 on 163 degrees of freedom
## Multiple R-squared:  0.631,    Adjusted R-squared:  0.622
## F-statistic: 69.8 on 4 and 163 DF,  p-value: <2e-16
```

3d plot

interactive code

Comparing Models

- The `East` term was significant in model 2, suggesting that there is a significant relationship between location and price.
- That term became nonsignificant when we allowed the slope of `Decor` to vary with location, and that difference in slopes was also nonsignificant.
- Notice that slope estimate for a given variable will almost *always* change depending on the other variables that are in the model.