

# Bayesian Estimation

Please draw your own subjective distributions for the following events/items.

1. The probability that it will snow at Reed this winter.
2. The probability that, on a given night, the sun has gone supernova.
3. The total number of individual socks that you own.

# Karl Broman's Socks



Karl Broman  
@kwbroman

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That the 1st 11 socks in the laundry are each distinct suggests there are a lot more socks.



RETWEETS  
4

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11



# Classical H test

## Assert a model

$H_0$ : I have  $N_{pairs}$  pairs of socks and  $N_{single}$  singlettons. The first 11 socks that I pull out of the machine are a random sample from this population.

## Decide on a test statistic

The number of singlettons in the sample: 11.

## Construct the sampling distribution

Probability theory or simulation.

## See where your observed stat lies in that distribution

Find the p-value if you like.

*H*<sub>0</sub>



$$N_{pairs} = 9$$

*H*<sub>0</sub>



$$N_{pairs} = 9; \quad N_{singles} = 5$$

# Constructing the sampling dist.

We'll use simulation.

Create the population of socks:

```
sock_pairs <- c("A", "B", "C", "D", "E",
                 "F", "G", "H", "I", "J", "K")
sock_singles <- c("l", "m", "n", "o", "p")
socks <- c(rep(sock_pairs,
                each = 2),
           sock_singles)
```

```
socks
```

```
## [1] "A" "A" "B" "B" "C" "C" "D" "D" "E" "E" "F" "F" "
## [18] "I" "J" "J" "K" "K" "l" "m" "n" "o" "p"
```

# One draw from the machine

```
picked_socks <- sample(socks, size = 11, replace = FALSE)  
picked_socks
```

```
## [1] "K" "D" "F" "F" "H" "J" "A" "E" "B" "H" "o"
```

```
sock_counts <- table(picked_socks)  
sock_counts
```

```
## picked_socks  
## A B D E F H J K o  
## 1 1 1 1 2 2 1 1 1
```

```
n_singles <- sum(sock_counts == 1)  
n_singles
```

```
## [1] 7
```

# Our simulator



## Constructing the sampling dist.

```
pick_socks(N_pairs = 9, N_singles = 5,  
          N_pick = 11)
```

```
## [1] 11
```

```
pick_socks(9, 5, 11)
```

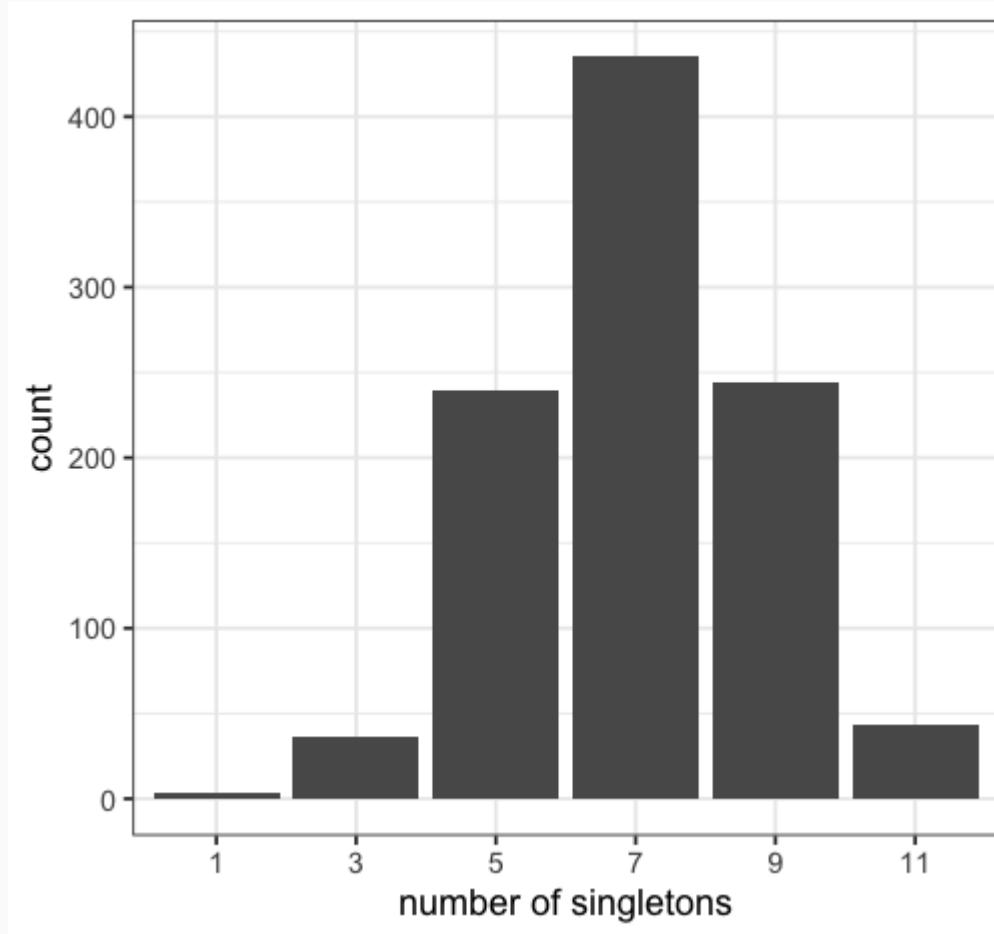
```
## [1] 7
```

```
pick_socks(9, 5, 11)
```

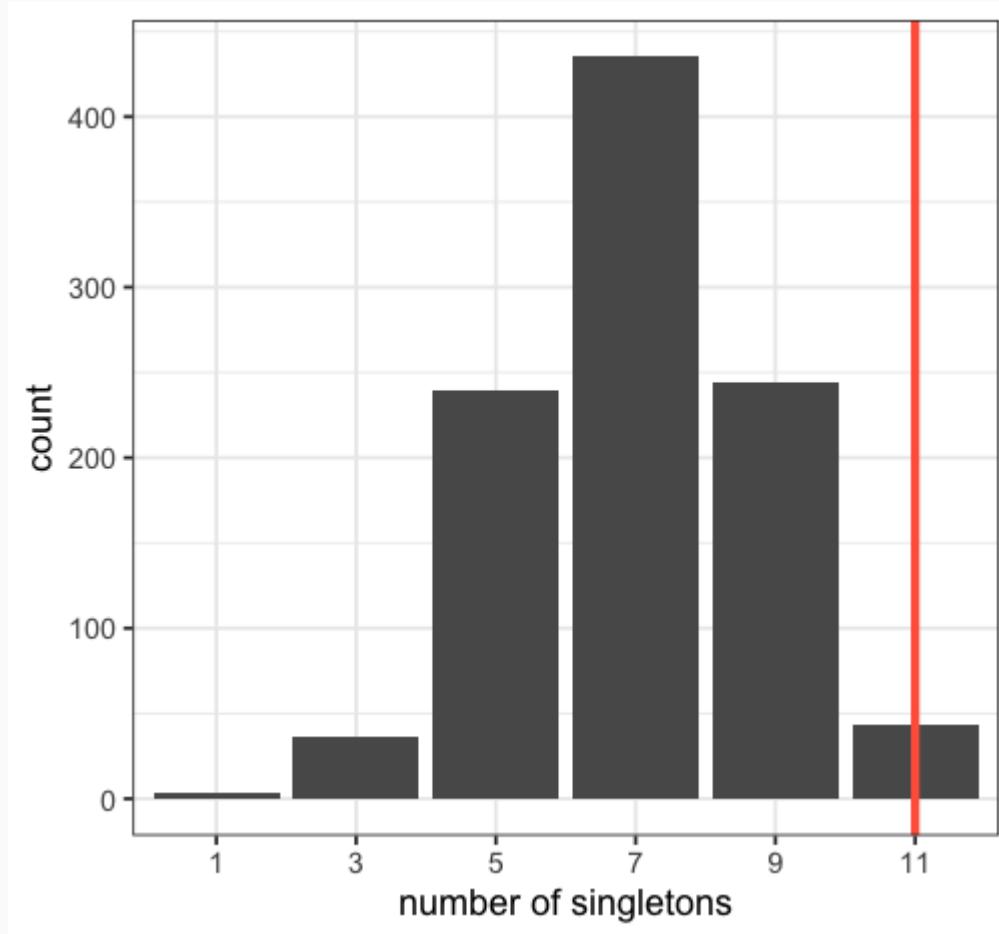
```
## [1] 7
```

Repeat many, many times...

# The sampling distribution



# The sampling distribution



# The p-value

Quantifying how far into the tails our observed count was.

```
table(sim_singles)
```

```
## sim_singles
##   1    3    5    7    9   11
##   3   36  239  435  244   43
```

```
table(sim_singles)[6]/1000
```

```
##      11
## 0.043
```

Our two-tailed p-value is 0.086.

## Question

What is the best definition for our one-tailed p-value in probability notation?

1.  $P(H_0 \text{ is true} | \text{data}) = 0.043$
2.  $P(H_0 \text{ is false} | \text{data}) = 0.043$
3.  $P(H_0 \text{ is true}) = 0.043$
4.  $P(\text{data} | H_0 \text{ is true}) = 0.043$
5.  $P(\text{data}) = 0.043$

## Question

What is the best definition for our one-tailed p-value in probability notation?

1.  $P(H_0 \text{ is true} | \text{data}) = 0.043$
2.  $P(H_0 \text{ is false} | \text{data}) = 0.043$
3.  $P(H_0 \text{ is true}) = 0.043$
4.  **$P(\text{data} | H_0 \text{ is true}) = 0.043$**
5.  $P(\text{data}) = 0.043$

## The challenge with the classical method

The result of a hypothesis test is a probability of the form:

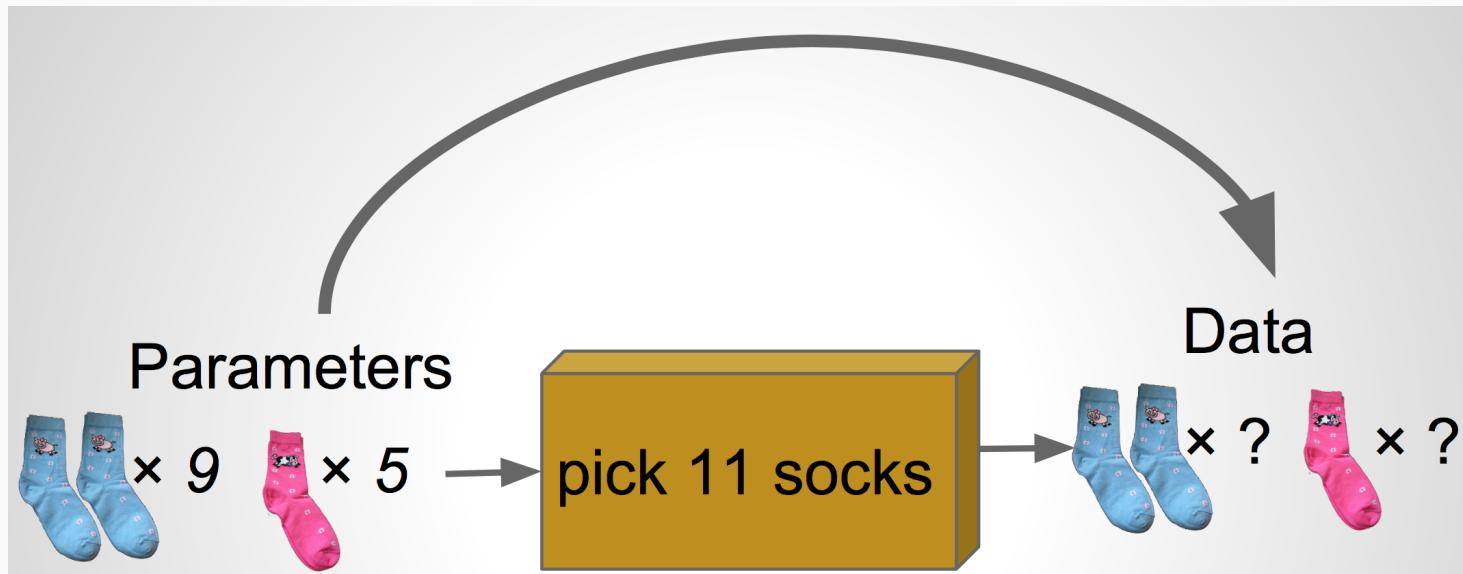
$$P(\text{ data or more extreme} \mid H_0 \text{ true })$$

while most people *think* they're getting

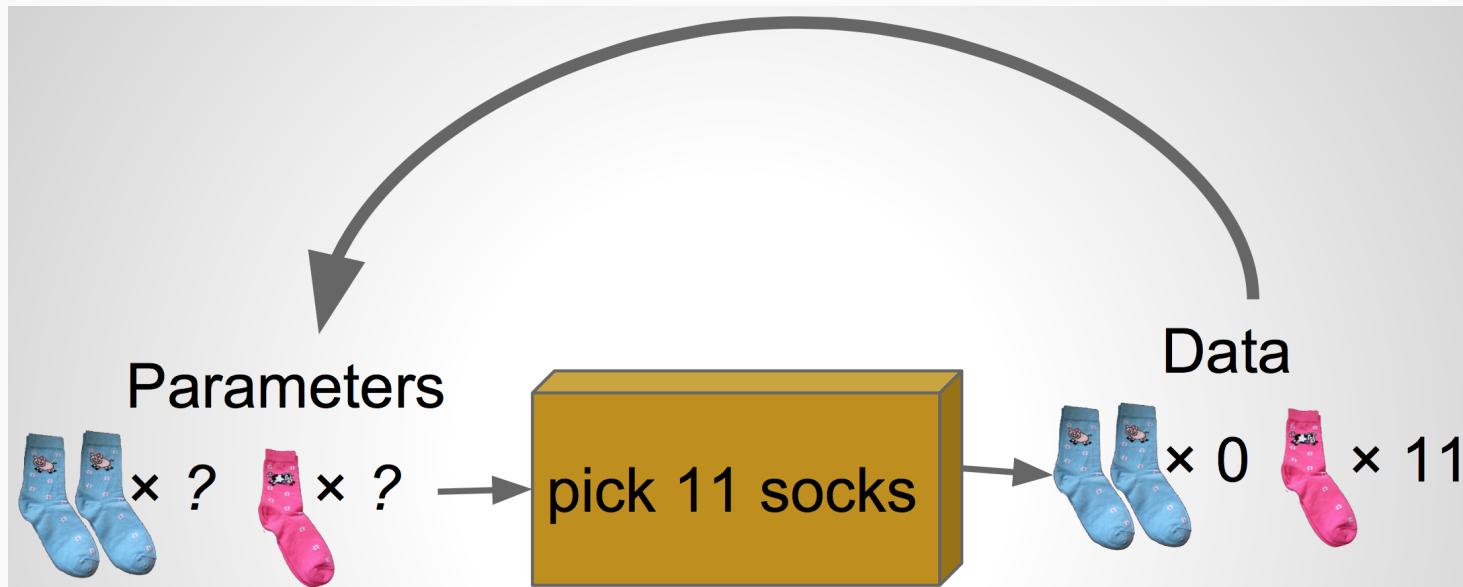
$$P( H_0 \text{ true} \mid \text{ data} )$$

How can we go from the former to the latter?

# What we have



# What we want



# Bayesian modeling via Bayes' rule

$$P(A | B) = \frac{P(\text{A and B})}{P(B)}$$

$$P(A | B) = \frac{P(B | A) P(A)}{P(B)}$$

$$P(\text{model} | \text{data}) = \frac{P(\text{data} | \text{model}) P(\text{model})}{P(\text{data})}$$

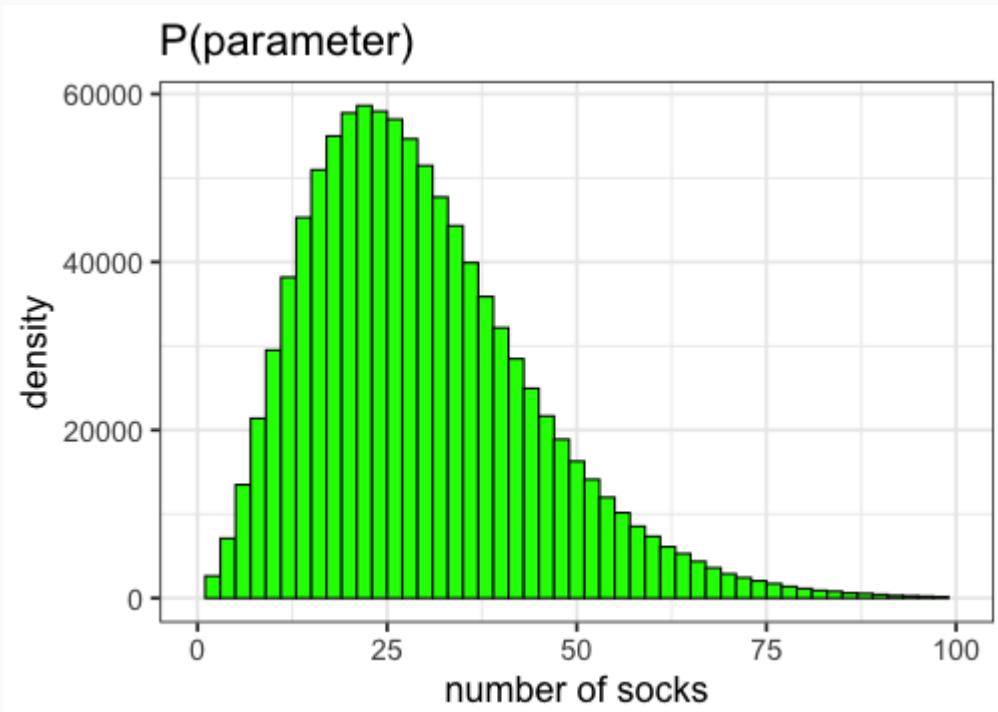
What does it mean to think about  $P(\text{model})$ ?

Please draw your own subjective distributions for the following events/items.

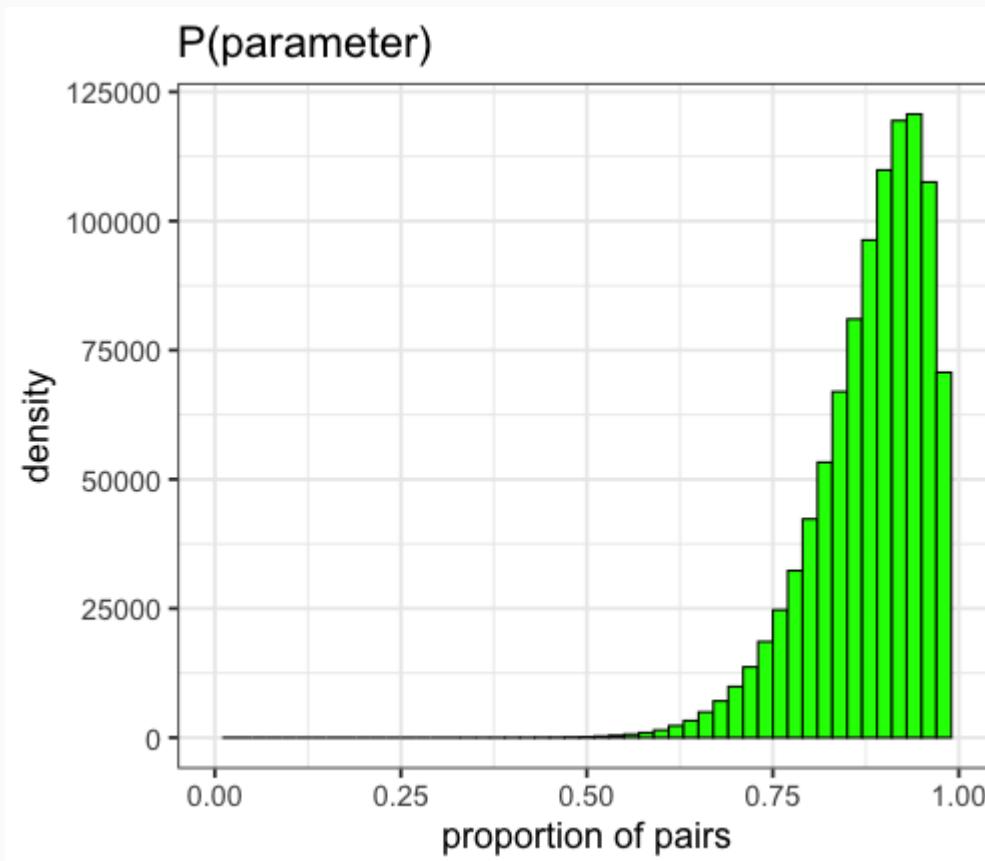
1. The probability that it will snow at Reed this winter.
2. The probability that, on a given night, the sun has gone supernova.
3. The total number of individual socks that you own.

# Prior distribution

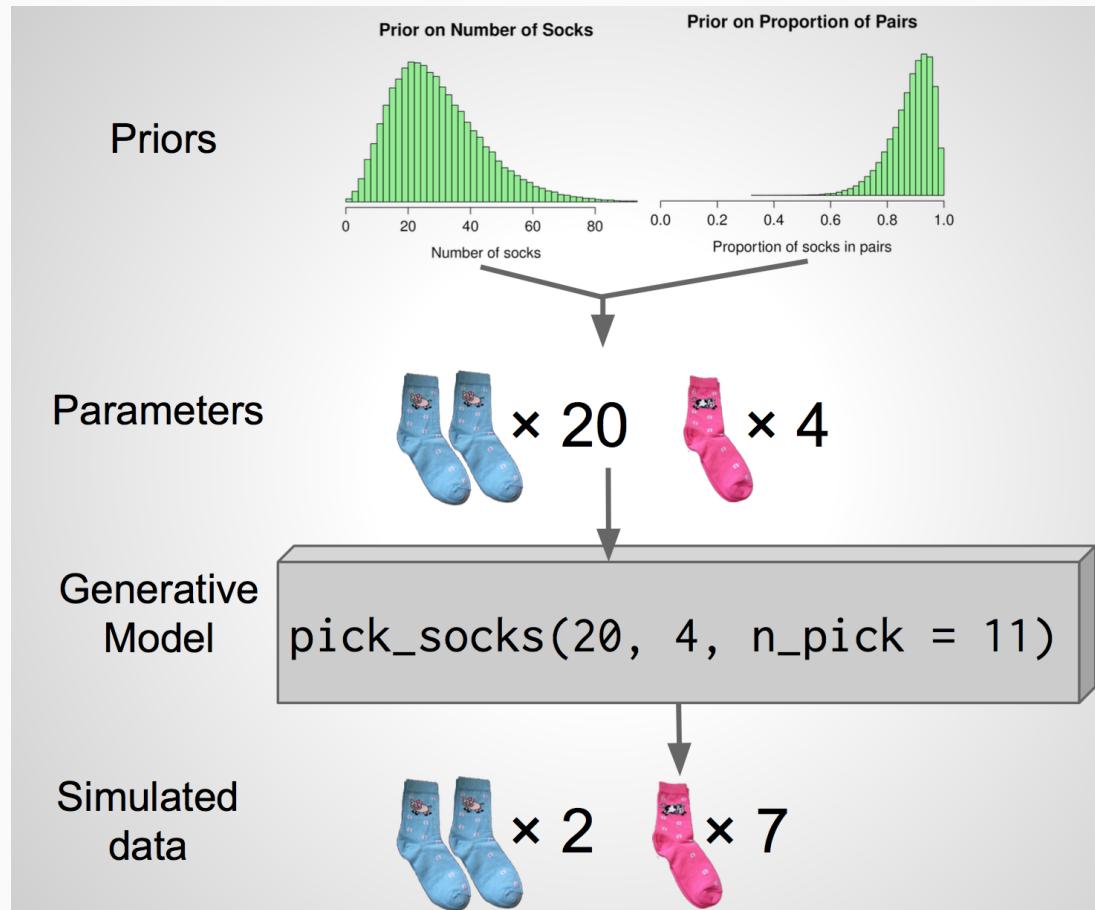
A *prior distribution* is a probability distribution for a *parameter* that summarizes the information that you have before seeing the data. Prior on  $N$ :



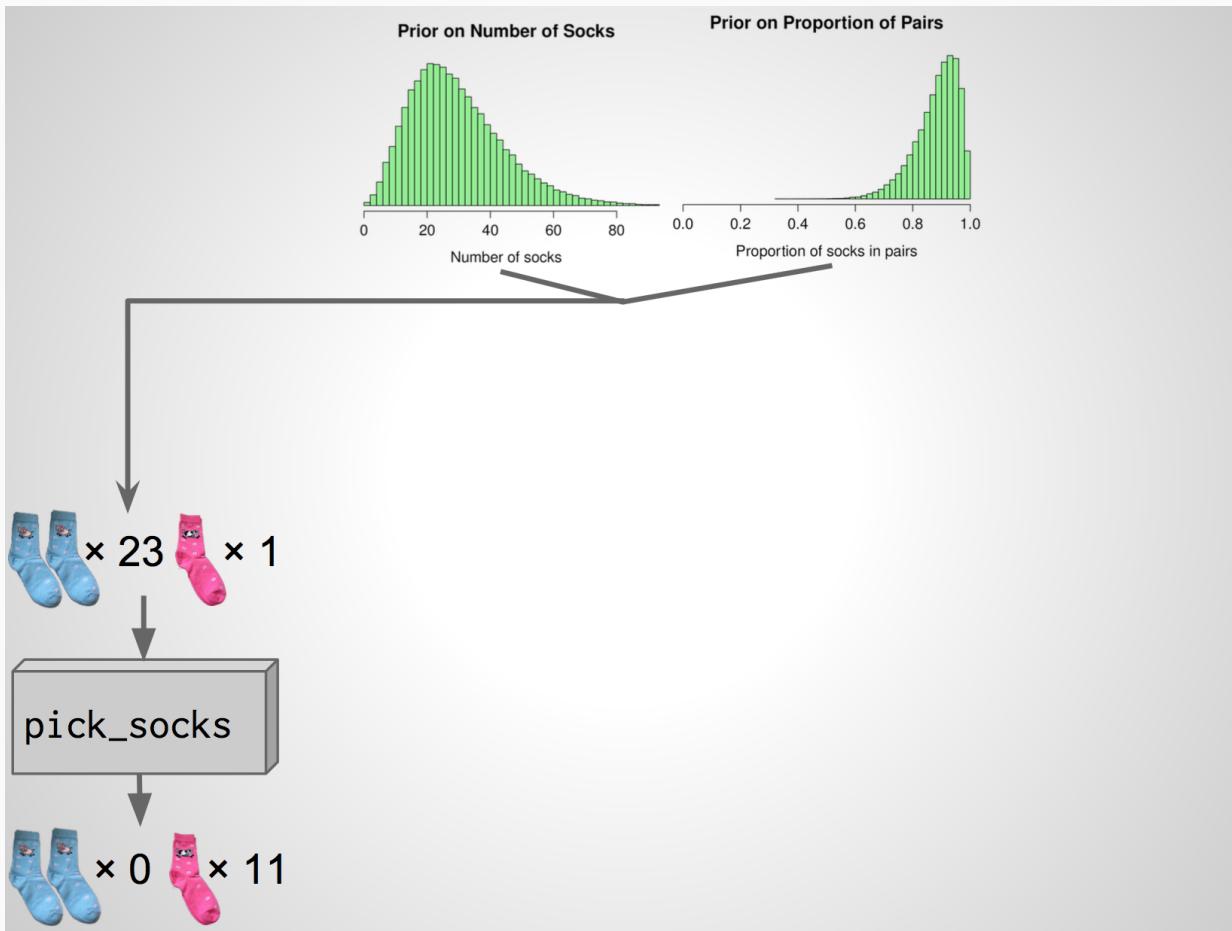
# Prior on proportion pairs



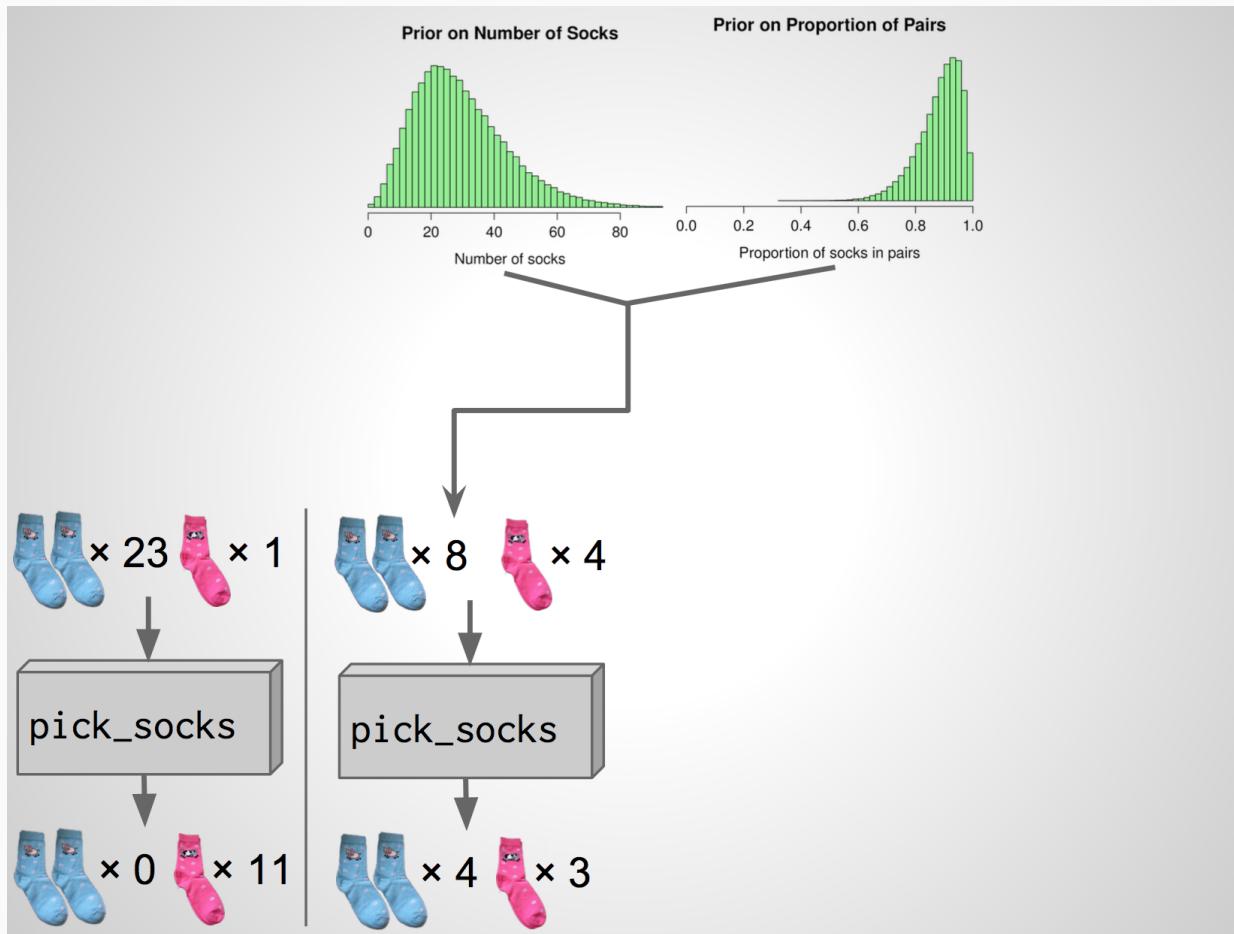
# Our scheme



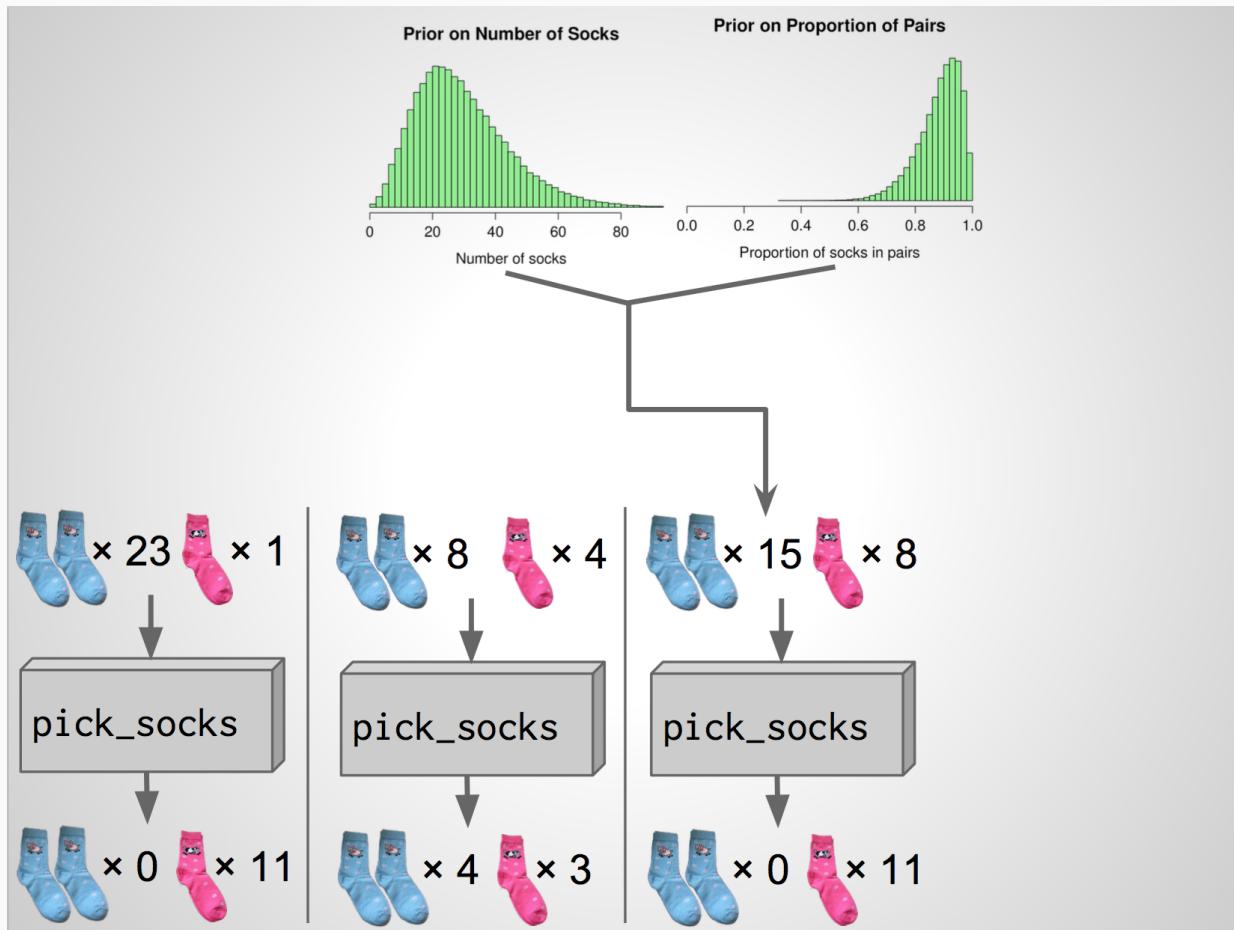
# One simulation



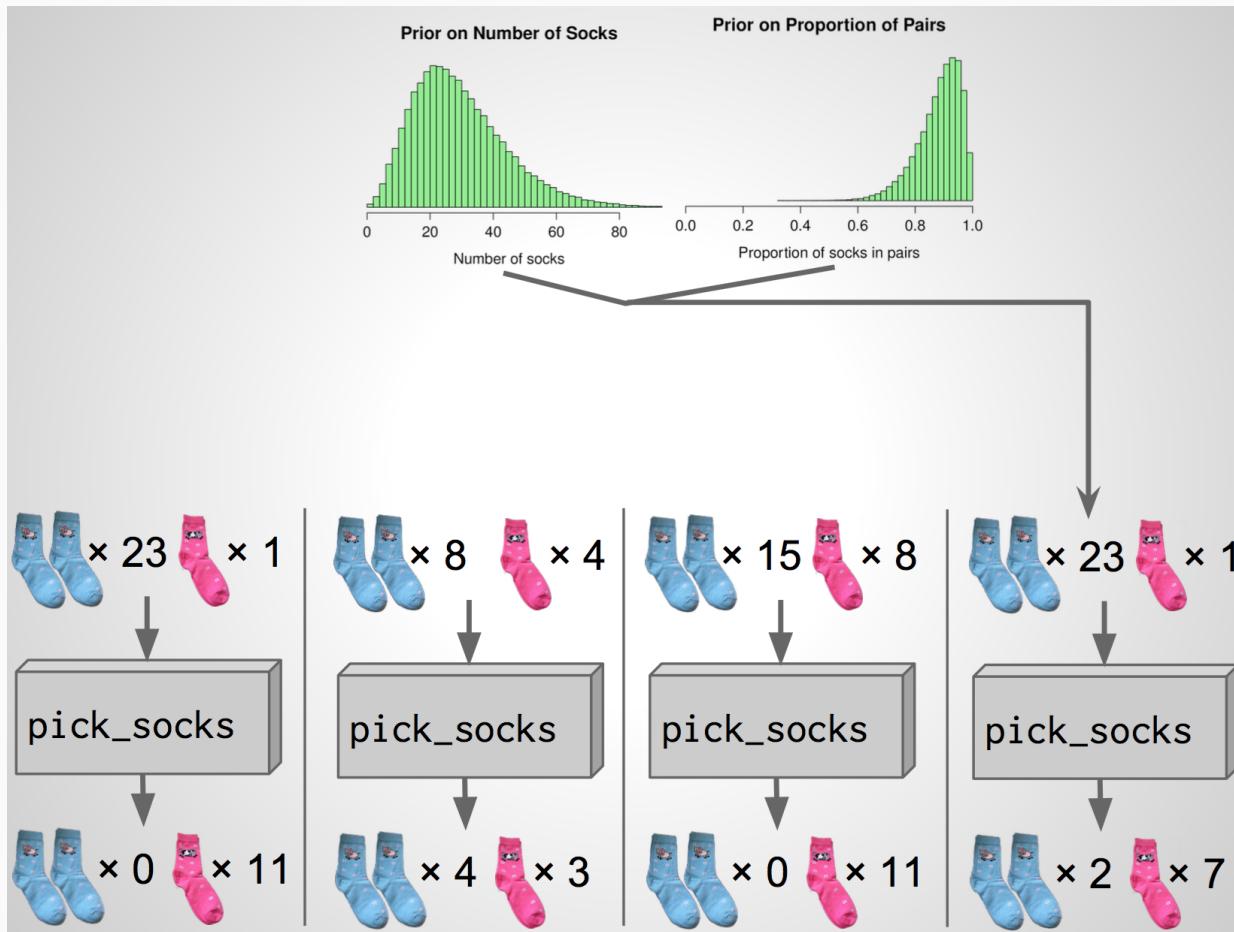
# A second simulation



# A third simulation



# A fourth simulation



# The actual data



# The actual data



# Full simulation

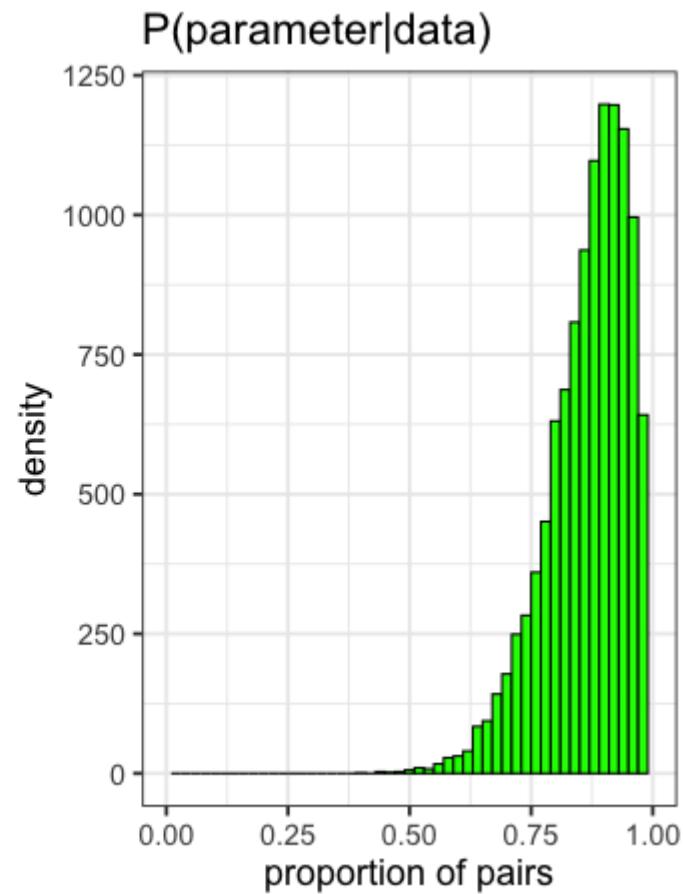
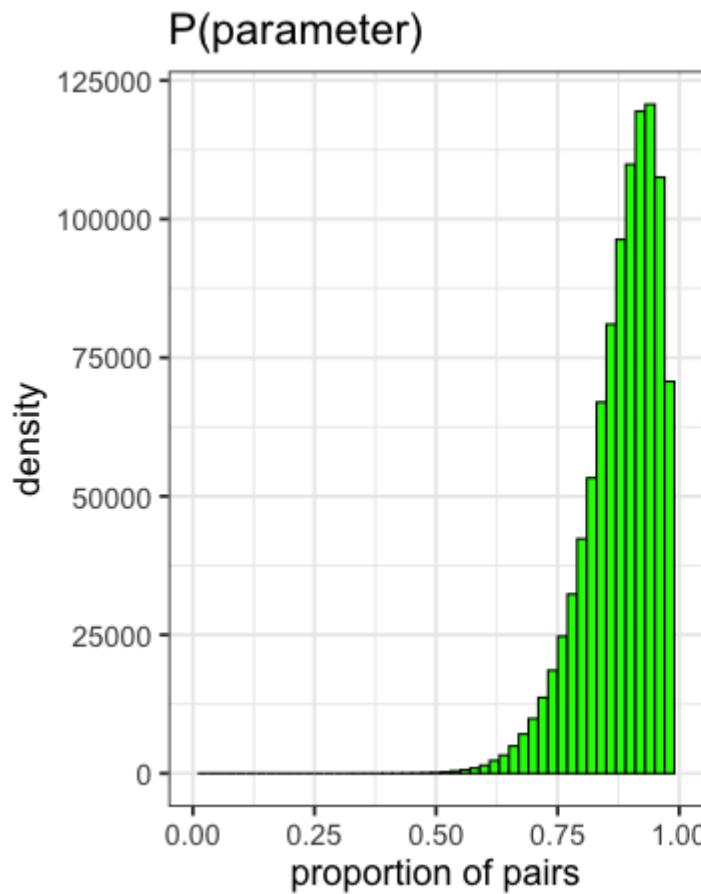
```
head(sock_sim)
```

```
##   singletons pairs n_socks prop_pairs
## 1          5     3     18      0.826
## 2         11     0     53      0.715
## 3          9     1     27      0.973
## 4          7     2     35      0.724
## 5          9     1     31      0.869
## 6          9     1     33      0.758
```

```
sock_sim %>%
  filter(singletons == 11, pairs == 0) %>%
  head()
```

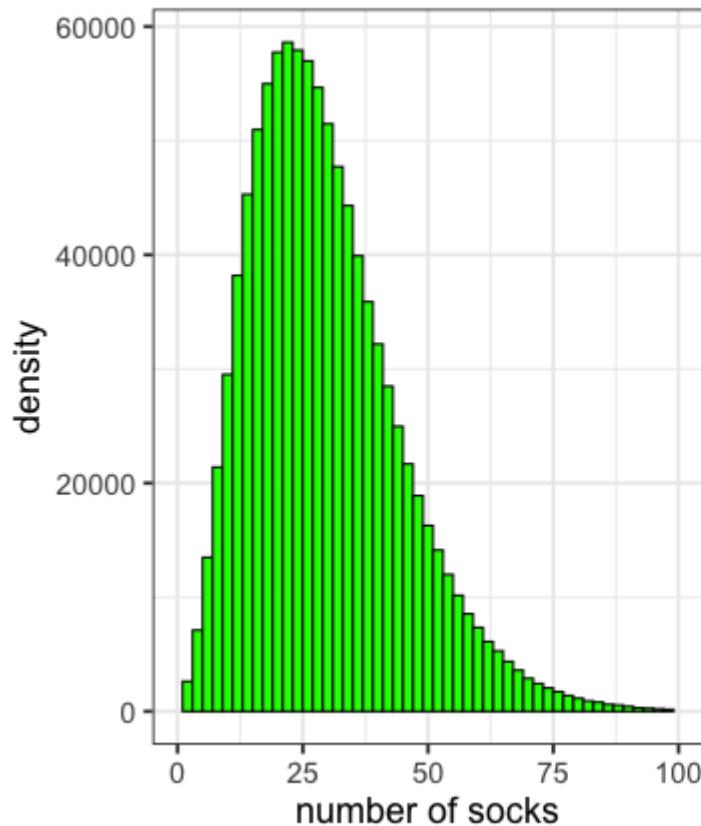
```
##   singletons pairs n_socks prop_pairs
## 1         11     0     53      0.715
## 2         11     0     41      0.885
## 3         11     0     53      0.957
## 4         11     0     37      0.773
## 5         11     0     45      0.880
## 6         11     0     51      0.754
```

# Proportion of pairs

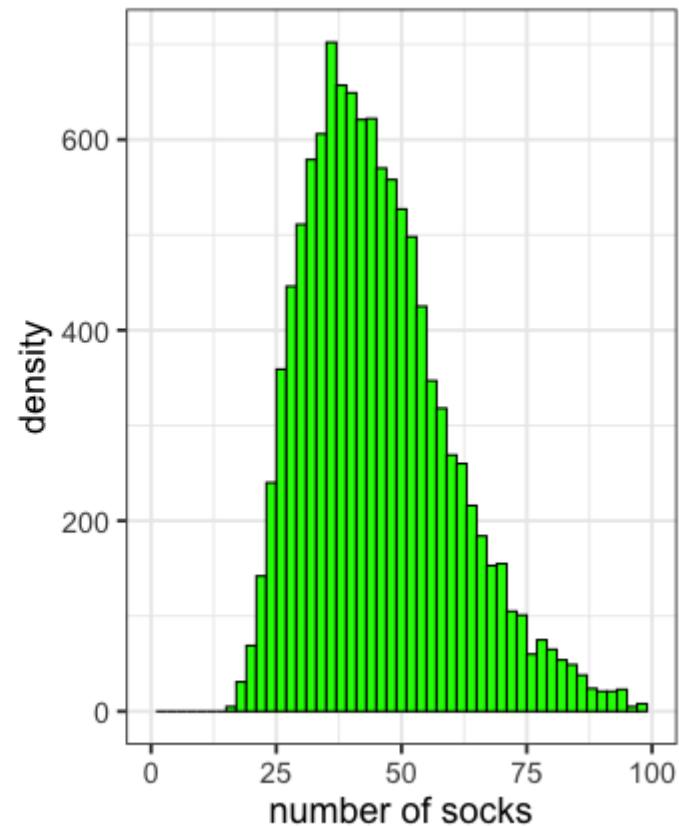


# Number of socks

$P(\text{parameter})$



$P(\text{parameter}|\text{data})$



# Karl Broman's Socks



Karl Broman  
@kwbroman

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That the 1st 11 socks in the laundry are each distinct suggests there are a lot more socks.

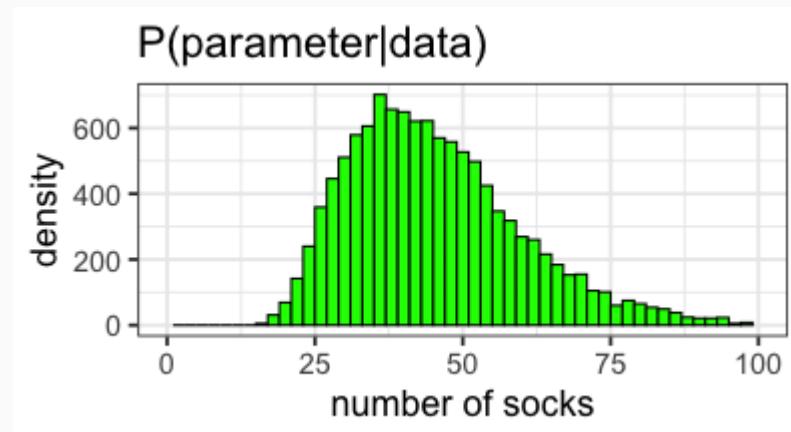


RETWEETS  
4

LIKES  
11



# The posterior distribution

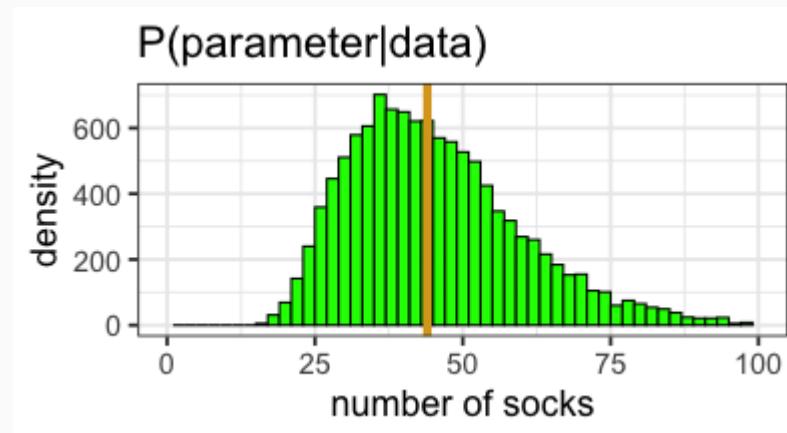


- Distribution of a parameter after conditioning on the data
- Synthesis of prior knowledge and observations (data)

## Question

What is your best guess for the number of socks that Karl has?

# Our best guess



- The posterior median is 44 socks.

# Karl Broman's Socks

A screenshot of a Twitter post from user @kwbroman. The post contains the following text: "@rabaath @sgrifter There were 21 pairs and 3 singletons. Will spend the rest of the evening working out what my est would have been." Below the text are standard Twitter interaction icons (retweet, favorite, etc.) and the timestamp "3:00 PM - 17 Oct 2014". At the top right of the tweet card, there is a "Following" button.

@rabaath @sgrifter There were 21 pairs  
and 3 singletons. Will spend the rest of the  
evening working out what my est would  
have been.

3:00 PM - 17 Oct 2014

$$21 \times 2 + 3 = 45 \text{ socks}$$

# Summary

Bayesian methods . . .

- Require the subjective specification of your prior knowledge
- Provide a posterior distribution on the parameters
- Are usually computationally intensive
- Have strong intuition

## DID THE SUN JUST EXPLODE? (IT'S NIGHT, SO WE'RE NOT SURE.)

THIS NEUTRINO DETECTOR MEASURES WHETHER THE SUN HAS GONE NOVA.

THEN, IT ROLLS TWO DICE. IF THEY BOTH COME UP SIX, IT LIES TO US. OTHERWISE, IT TELLS THE TRUTH.

LET'S TRY.

DETECTOR! HAS THE SUN GONE NOVA?

(ROLL)

YES.



BAYESIAN STATISTICIAN:

BET YOU \$50 IT HASN'T.

FREQUENTIST STATISTICIAN:

THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS  $\frac{1}{36} = 0.027$ . SINCE  $p < 0.05$ , I CONCLUDE THAT THE SUN HAS EXPLODED.