

1. The American Community Survey is an ongoing survey that provides data every year to give communities the current information they need to plan investments and services. The 2010 American Community Survey estimates that 14.6% of Americans live below the poverty line, 20.7% speak a language other than English (foreign language) at home, and 4.2% fall into both categories.
  - (a) Are living below the poverty line and speaking a foreign language at home disjoint?
  - (b) Draw a Venn diagram summarizing the variables and their associated probabilities.
  - (c) What percent of Americans live below the poverty line and only speak English at home?
  - (d) What percent of Americans live below the poverty line or speak a foreign language at home?
  - (e) What percent of Americans live above the poverty line and only speak English at home?
  - (f) Is the event that someone lives below the poverty line independent of the event that the person speaks a foreign language at home?
2. In parts (a) and (b), identify whether the events are disjoint, independent, or neither (events cannot be both disjoint and independent).
  - (a) You and a randomly selected student from your class both earn A's in this course.
  - (b) You and your class study partner both earn A's in this course.
  - (c) If two events can occur at the same time, must they be dependent?
3. Data collected at elementary schools in DeKalb County, GA suggest that each year roughly 25% of students miss exactly one day of school, 15% miss 2 days, and 28% miss 3 or more days due to sickness.
  - (a) What is the probability that a student chosen at random doesn't miss any days of school due to sickness this year?
  - (b) What is the probability that a student chosen at random misses no more than one day?
  - (c) What is the probability that a student chosen at random misses at least one day?
  - (d) If a parent has two kids at a DeKalb County elementary school, what is the probability that neither kid will miss any school? Note any assumption you must make to answer this question.
  - (e) If a parent has two kids at a DeKalb County elementary school, what is the probability that both kids will miss some school, i.e. at least one day? Note any assumption you make.
  - (f) If you made an assumption in part (d) or (e), do you think it was reasonable? If you didn't make any assumptions, double check your earlier answers.
4.  $\mathbb{P}(A) = 0.3$ ,  $\mathbb{P}(B) = 0.7$ 
  - (a) Can you compute  $\mathbb{P}(A \text{ and } B)$  if you only know  $\mathbb{P}(A)$  and  $\mathbb{P}(B)$ ?
  - (b) Assuming that events  $A$  and  $B$  arise from independent random processes,
    - (i) what is  $\mathbb{P}(A \text{ and } B)$ ?
    - (ii) what is  $\mathbb{P}(A \text{ or } B)$ ?
    - (iii) what is  $\mathbb{P}(A | B)$ ?
  - (c) If we are given that  $\mathbb{P}(A \text{ and } B) = 0.1$ , are the random variables giving rise to events  $A$  and  $B$  independent?
  - (d) If we are given that  $\mathbb{P}(A \text{ and } B) = 0.1$ , what is  $\mathbb{P}(A | B)$ ?

5. A 2010 Pew Research poll asked 1,306 Americans “From what you’ve read and heard, is there solid evidence that the average temperature on earth has been getting warmer over the past few decades, or not?”. The table below shows the distribution of responses by party and ideology, where the counts have been replaced with relative frequencies.

		<i>Response</i>			Total
		Earth is warming	Not warming	Don’t Know Refuse	
<i>Party and Ideology</i>	Conservative Republican	0.11	0.20	0.02	0.33
	Mod/Lib Republican	0.06	0.06	0.01	0.13
	Mod/Cons Democrat	0.25	0.07	0.02	0.34
	Liberal Democrat	0.18	0.01	0.01	0.20
	Total	0.60	0.34	0.06	1.00

- Are believing that the earth is warming and being a liberal Democrat mutually exclusive?
  - What is the probability that a randomly chosen respondent believes the earth is warming or is a liberal Democrat?
  - What is the probability that a randomly chosen respondent believes the earth is warming given that he is a liberal Democrat?
  - What is the probability that a randomly chosen respondent believes the earth is warming given that he is a conservative Republican?
  - Does it appear that whether or not a respondent believes the earth is warming is independent of their party and ideology? Explain your reasoning.
  - What is the probability that a randomly chosen respondent is a moderate/liberal Republican given that he does not believe that the earth is warming?
6. After an introductory statistics course, 80% of students can successfully construct box plots. Of those who can construct box plots, 86% passed, while only 65% of those students who could not construct box plots passed.
- Construct a tree diagram of this scenario.
  - Calculate the probability that a student is able to construct a box plot if it is known that he passed.
7. Lupus is a medical phenomenon where antibodies that are supposed to attack foreign cells to prevent infections instead see plasma proteins as foreign bodies, leading to a high risk of blood clotting. It is believed that 2% of the population suffer from this disease. The test is 98% accurate if a person actually has the disease. The test is 74% accurate if a person does not have the disease. There is a line from the Fox television show *House* that is often used after a patient tests positive for lupus: “It’s never lupus.” Do you think there is truth to this statement? Use appropriate probabilities to support your answer.
8. At a university, 13% of students smoke.
- Calculate the expected number of smokers in a random sample of 100 students from this university.
  - The university gym opens at 9 am on Saturday mornings. One Saturday morning at 8:55 am there are 27 students outside the gym waiting for it to open. Should you use the same approach from part (a) to calculate the expected number of smokers among these 27 students?
9. Consider the following card game with a well-shuffled deck of cards. If you draw a red card, you win nothing. If you get a spade, you win \$5. For any club, you win \$10 plus an extra \$20 for the ace of clubs.

- (a) Create a probability model for the amount you win at this game. Also, find the expected winnings for a single game and the standard deviation of the winnings.
- (b) What is the maximum amount you would be willing to pay to play this game? Explain your reasoning.
10. An airline charges the following baggage fees: \$25 for the first bag and \$35 for the second. Suppose 54% of passengers have no checked luggage, 34% have one piece of checked luggage and 12% have two pieces. We suppose a negligible portion of people check more than two bags.
- (a) Build a probability model, compute the average revenue per passenger, and compute the corresponding standard deviation.
- (b) About how much revenue should the airline expect for a flight of 120 passengers? With what standard deviation? Note any assumptions you make and if you think they are justified.
11. The relative frequency table below displays the distribution of annual total personal income (in 2009 inflation-adjusted dollars) for a representative sample of 96,420,486 Americans. These data come from the American Community Survey for 2005-2009. This sample is comprised of 59% males and 41% females.
- (a) Describe the distribution of total personal income.
- (b) What is the probability that a randomly chosen US resident makes less than \$50,000 per year?
- (c) What is the probability that a randomly chosen US resident makes less than \$50,000 per year and is female? Note any assumptions you make.
- (d) The same data source indicates that 71.8% of females make less than \$50,000 per year. Use this value to determine whether or not the assumption you made in part (c) is valid.

<i>Income</i>	<i>Total</i>
\$1 to \$9,999 or loss	2.2%
\$10,000 to \$14,999	4.7%
\$15,000 to \$24,999	15.8%
\$25,000 to \$34,999	18.3%
\$35,000 to \$49,999	21.2%
\$50,000 to \$64,999	13.9%
\$65,000 to \$74,999	5.8%
\$75,000 to \$99,999	8.4%
\$100,000 or more	9.7%

**Extra Challenge Problem:** A *chord* of a circle is a straight line segment whose endpoints both lie on the circle. For a fixed circle, what is the probability that the length of a randomly drawn chord will exceed that circle's radius?