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COUNTING 3-UNIFORM HYPERPATH COLORINGS

Now we ask the question:

"How many ways are there to color the vertices of a size-N, 3-uniform hyperpath so that there are exactly J blue vertices, but no hyperedge contains all blue vertices?"

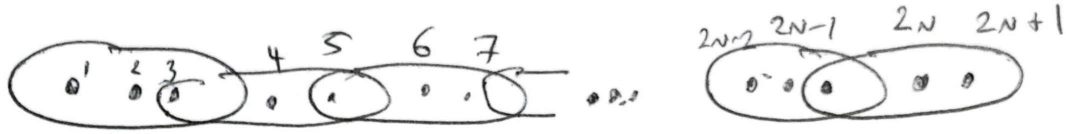
Let's denote this count with

$$\boxed{G(N,J).}$$

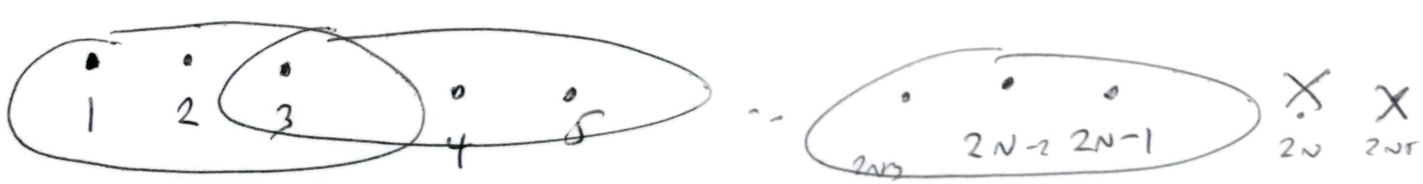
COUNTING 3-UNIFORM HYPERPATH COLORINGS

This time, we'll ignore the base/boundary conditions and jump straight to the recurrence:

~~Also~~ Begin with a size-N 3-uniform hyperpath.



This time, we "chop off" the last two vertices to look at a smaller case (because that's how recurrence works!)

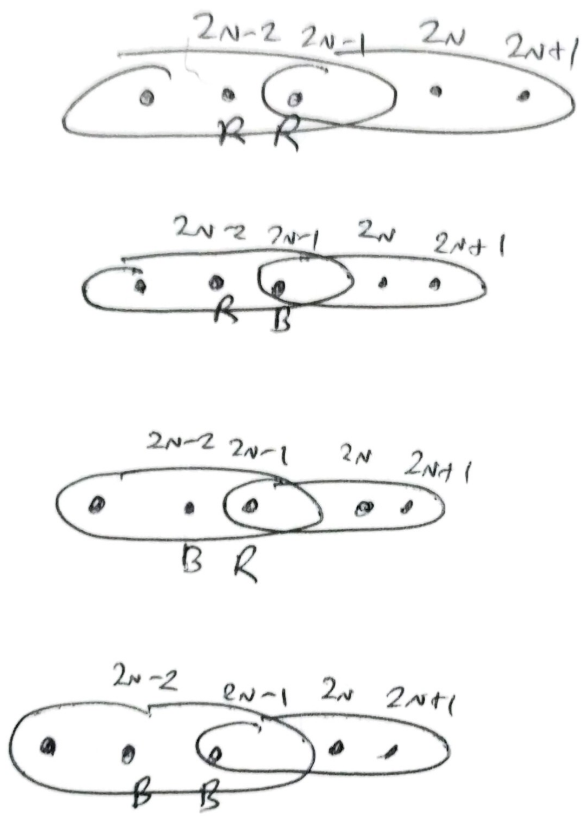


Again, when ~~coloring~~ considering the coloring of the vertices on the first (N-1) ~~edges~~ hyperedges, we have cases:

COUNTING 3-UNIFORM HYPERPATH COLORINGS

THE CASES ARE BASED ON TWO THINGS:

#1 THE COLORS IN THE NEXT-TO-LAST HYPEREDGE



#2 HOW MANY BLUE VERTICES ARE IN THE FIRST (N-1) HYPEREDGES?

(J, J-1, or J-2?)

AT FIRST GLANCE, THIS LOOKS LIKE 12 CASES, BUT WE CAN THROW OUT

$G_{RB}(N-1, J-2)$ and $G_{BB}(N-1, J-2)$:

both would create a blue hyperedge

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COUNTING 3-UNIFORM HYPERPATH COLORINGS

Letting G_{RR}, G_{RB}, etc. mean the obvious things, this gives:

eq. 1

$$\begin{aligned} G(N,J) = {} & G_{RR}(N-1,J) + 2\cdot G_{RR}(N-1,J-1) + G_{RR}(N-1,J-2) \\ & + G_{BR}(N-1,J) + 2\cdot G_{BR}(N-1,J-1) + G_{BR}(N-1,J-2) \\ & + G_{RB}(N-1,J) + 2\cdot G_{RB}(N-1,J-1) \\ & + G_{BB}(N-1,J) + 2\cdot G_{BB}(N-1,J-1) \end{aligned}$$

Since $G_{RR} + G_{BR} + G_{RB} + G_{BB} = G$, this becomes

eq. 2

$$\begin{aligned} G(N,J) = {} & G(N-1,J) + 2\cdot G(N-1,J-1) + G_{RR}(N-1,J-2) \\ & + G_{BR}(N-1,J-2). \end{aligned}$$

COUNTING 3-UNIFORM HYPERPATH COLORINGS

Now we need to look at G_{RR} and G_{BR} in greater depth.

- It isn't too hard to see that

$$G_{RR}(M,K) = G(M-1, K);$$

if you're going to end in RR, you introduce no new blue vertices, and you are safe on any K-BLUE coloring of the first $(M-1)$ hyperpaths (that avoids an all-blue hyperedge).

WRONG. IGNORE.

- ~~For $G_{BR}(M,K)$, we need to be a bit more careful. We can only safely add a BR onto RR, RB, or BR.~~

~~? ? B R~~

~~can't be BB~~

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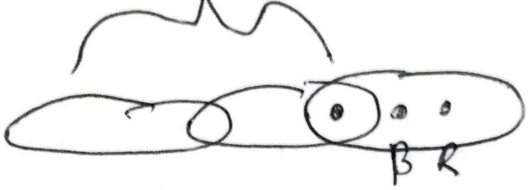
COUNTING 3-UNIFORM HYPERPATH COLORINGS

- similarly:

$$G_{BR}(M,K) = G(M-1,K-1).$$

anything legal works with as few B.



Plugging these back into ~~the~~ last eq (2) gives:

$$\boxed{G(N,J) = G(N-1,J) + 2\cdot G(N-1,J-1) + G(N-2,J-2) + G(N-2,J-3).}$$