Single-variate Generating Functions

Let

$$F(n) = \sum_{i=1}^{k} C_i F(n-i) = C_1 F(n-1) + C_2 F(n-2) + \cdots + C_k F(n-k)$$

be a recursively defined function with base cases defined as

$$F(0) = F_0, F(1) = F_1, \ldots, F(k) = F_k.$$

Then the associated generating function $G_F(x)$ is given by

$$G_F(x) = \frac{\sum\limits_{i=0}^{k-1} \left(F_i - \sum\limits_{j=1}^{i} C_j F_{i-j} \right) x^i}{1 - \sum\limits_{i=1}^{k} C_i x^i}.$$

Multivariate Generating Functions

	1	x	x^2	x^3	x^4
1	$F(0,0)$	$F(1,0) - F(0,0)$	$F(2,0) - F(1,0)$	$F(3,0) - F(2,0)$	$F(4,0) - F(3,0)$
y	$F(0,1)$	$F(1,1) - F(0,1) - 2F(0,0)$	$F(2,1) - F(1,1) - 2F(1,0)$	$F(3,1) - F(2,1) - 2F(2,0)$	$F(4,1) - F(3,1) - 2F(3,0)$
y^2	$F(0,2)$	$F(1,2) - F(0,2) - 2F(0,1)$	$F(2,2) - F(1,2) - 2F(1,1) - F(0,0)$	$F(3,2) - F(2,2) - 2F(2,1) - F(1,0)$	$F(4,2) - F(3,2) - 2F(3,1) - F(2,0)$
y^3	$F(0,3)$	$F(1,3) - F(0,3) - 2F(0,2)$	$F(2,3) - F(1,3) - 2F(1,2) - F(0,1) - F(0,0)$	$F(3,3) - F(2,3) - 2F(2,2) - F(1,1) - F(1,0)$	$F(4,3) - F(3,3) - 2F(3,2) - F(2,1) - F(2,0)$
y^4	$F(0,4)$	$F(1,4) - F(0,4) - 2F(0,3)$	$F(2,4) - F(1,4) - 2F(1,3) - F(0,2) - F(0,1)$	$F(3,4) - F(2,4) - 2F(2,3) - F(1,2) - F(1,1)$	$F(4,4) - F(3,4) - 2F(3,3) - F(2,2) - F(2,1)$
y^5	$F(0,5)$	$F(1,5) - F(0,5) - 2F(0,4)$	$F(2,5) - F(1,5) - 2F(1,4) - F(0,3) - F(0,2)$	$F(3,5) - F(2,5) - 2F(2,4) - F(1,3) - F(1,2)$	$F(4,5) - F(3,5) - 2F(3,4) - F(2,3) - F(2,2)$