$$F(-1,j)=0, \quad F(n,-1)=0, \quad F(0,1)=0, \quad F(0,2)=0, \quad F(1,2)=0,$$

$$F(n,j)= 3F(n-1,j-1)+ 2F(n-2,j-2)$$

$$F(0,0)=1, \quad F(1,0)=1, \quad F(1,1)=1,$$
$$F(2,0)=1, \quad F(2,1)=1$$

$$A(x,y)=\sum_{n,j\geq 0} F(n,j)\, x^n y^j$$

$$\sum_{n,j\geq 2} F(n,j)\, x^n y^j = \sum_{n,j\geq 2}\left(3F(n-1,j-1)\, x^n y^j\right) + \sum_{n,j\geq 2}\left(2F(n-2,j-2)x^n y^j\right)$$

$$* \qquad\qquad ** \qquad\qquad\qquad ***$$

$$* \quad A(x,y)-5$$

$$m=n-1 \qquad i=j-1$$
$$\ell=n-2 \qquad h=j-2$$

$$** \quad 3\sum_{m,i\geq 1} F(m,i)x^{m+1} y^{i+1} = 3xy\left(A(x,y)-2\right)$$

$$*** \quad 2\sum_{\ell,h\geq 0} F(\ell,h)x^{\ell+2} y^{h+2} = 2x^2 y^2\, A(x,y)$$

$$A-5= 3xy\left(A-2\right)+2x^2 y^2 A$$

$$A-3xyA-2x^2 y^2 A = 5-6xy$$

$$\boxed{A(x,y)=(5-6xy)\,\frac{1}{1-(3xy+2x^2 y^2)}}$$

$$A(x,y) = (5 - 6xy) \frac{1}{1 - xy(3 + 2xy)}$$

$$A(x,y) = (5 - 6xy) \sum_{m=0}^{\infty} x^m y^m (3 + 2xy)^m$$

$$A(x,y) = (5 - 6xy) \sum_{m=0}^{\infty} x^m y^m \sum_{k=0}^{m} \binom{m}{k} \cdot 3^{m-k} \cdot 2^k \cdot x^k \cdot y^k$$

$$A(x,y) = (5 - 6xy) \sum_{m=0}^{\infty} \sum_{k=0}^{m} \binom{m}{k} 3^{m-k} \cdot 2^k \cdot x^{m+k} y^{m+k}$$

$$A(x,y) = \sum_{m=0}^{\infty} \sum_{k=0}^{M} \binom{M}{k} 3^{m-k} \cdot 2^k \left(5 x^{m+k} y^{m+k} - 6 x^{m+k+1} y^{m+k+1}\right)$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$\frac{1}{1-r} = \sum_{k=0}^{\infty} r^k$$

$$n = j$$

$$n = m + k \qquad n = m+k+1$$
$$m = n - k \qquad m = n-k-1$$
$$k = n - m \qquad k = n-m-1$$