Did the apple fall far from the tree?
Uncertainty and learning about ability with family-informed priors

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Abstract

This paper examines the effect of uncertainty and learning about ability on inter-generational correlations in education and labor market outcomes, when children and their parents utilize family signals while forming initial beliefs about ability. I examine this question by using a cohort study and rich administrative data to estimate a dynamic discrete choice structural model of education and occupational decisions, incorporating multidimensional skills and ability endowments, as well as uncertainty and learning about ability, starting from a family-driven prior.

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1 Introduction

There is increasing recognition of the role that uncertainty and learning about ability plays in the choices and economic fortunes of individuals. Since Johnson (1978) and Jovanovic (1979), this line of inquiry has had a long history in the study of employment and the labor market. More recent work has emphasized that learning about ability is an important determinant of choices and outcomes in education as well. In analyses of both education and employment, structural analyses in particular have played a central part in estimating the importance of ability uncertainty and learning for economic inequality.

Since Becker and Tomes (1986), a nearly distinct literature has examined how innate ability affects education and labor market outcomes with a primary emphasis on the role of the family. A key insight from this intergenerational literature has been that families play a crucial role in the extent of human capital investments for new generations, both in terms of the resources families provide (that tend to be endogenous to ability), but also because of the genetic factors that bestow a correlation in ability from one generation to the next.

This analysis seeks to bridge these two literatures by considering how uncertainty and learning about ability affects human capital decisions in education and the labor market when families provide useful initial signals of ability (in addition to their potential role in financially supporting human capital investments). This research question may yield important insights into both literatures, as it suggests that while family priors may be helpful in forming priors about childrens' ability when faced with uncertainty, it may impede intergenerational mobility by causing families to underestimate the extent to which their children's ability differs from their own.

To examine this question, I integrate family-informed prior beliefs into a model of post-secondary educational and occupational choice similar to Arcidiacono et. al (2016), estimated using rich administrative data from Sweden. Using this model, I will estimate counterfactual differences in college attainment and occupational sorting between the family-informed ability beliefs setting and a setting in which individuals have no ability uncertainty.

2 Related Literature

> Human capital is by far the most important form of capital in modern
> economies. The economic successes of individuals, and also of whole
> economies, depends on how extensively and effectively people invest in
> themselves.

<div align="right">

Gary Becker

The Age of Human Capital

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Nobel Laureate Gary Becker championed the perspective that human capital investments stand at the heart of economics' central aims and interests — a primary basis for understanding not only poverty and inequality, but also economic efficiency and growth. As a result, Becker was quite naturally preoccupied with the question of why human capital investments differ so dramatically.

The critical role of parents in both the choice and financing of educational decisions of their children led Becker to consider intergenerational models of human capital investment and earnings mobility. "Human Capital and the Rise and Fall of Families", Becker's 1986 joint work with Nigel Tomes, not only formed the core of his contribution to the study of intergenerational dynamics of human capital accumulation and earnings mobility, but also stands as the basic framework from which nearly all intergenerational modeling derives.

The key insight from that framework is that parents transmit their fortunes in two key ways: first, through the biological inheritance of traits related to achievement (i.e. inherited ability), and second, by supplying the environments and investments that support children's achievement. The second channel, canonically stylized as financial investments, notably is affected by the first in that higher ability parents will tend to make more money and thus have greater financial resources with which to invest. The result of Becker-Tomes is that families make conditionally optimal human capital decisions, but because of the role of endogenous finances, higher and lower SES children of equal potential receive different

human capital investments – and so, different labor market outcomes.

Following Becker and Tomes (1986), a large amount of research has sought to test the model and quantify its implications. A central aim within this literature has been to understand broadly how much of intergenerational persistence is due to biological factors ("nature") and how much of it arises due to environmental factors ("nurture"). Empirical studies from both economics and behavioral genetics, traditionally using variation from twins and adoptees, have yielded a standard view that genetic factors generally account for between 40-60% of variation in cognitive ability, with broadly similar findings about educational attainment and earnings in most contexts (e.g. Björklund, Lindahl, & Plug 2006, Grönqvist, Öckert, and Vlachos 2017, Polderman et al 2015, Sacerdote 2011, Smith-Woolley et al 2018).

Recent advances have offered salient critiques to this literature (for instance, highlighting epigenetics and the lack of clear distinction between genetics and environment), as well as offering new ways of exploring genetic predictors of success (most notably, genome-wide association studies and polygenic score analysis). Yet the principal conclusions about the importance of genetic explanators have so far remained essentially intact (Harden 2020).[1]

In addition to exploration of the nature versus nurture decomposition implied by Becker and Tomes, a great deal of analysis has also pursued refinements to the model. Some of the most important extensions to emerge from this literature have focused on the multistage nature of human capital investment decisions, often emphasizing the importance of early human capital investments as emphasized most prominently by James Heckman (e.g. Caucutt and Lochner 2020, Cunha and Heckman 2007, Lee and Seshadri 2017, Restuccia and Urruttia 2004).[2]

The intergenerational modeling literature has so far not reckoned with the question of learning about ability, which has been a prominent (and growing) emphasis especially within the study of life cycle labor market inequality. This question has historically been most

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[1] In addition, popular perception about the heritability of traits and outcomes appears similar to the scientific consensus. (Willoughby et. al 2019).

[2] Other notable contributions have also considered multiple type of ability, as well as considering borrowing constraints and endogenous preferences.

associated with the study of labor search and matching, following the seminal theory contributions of Johnson (1978) and Jovanovic (1979, 1984).

Dynamic discrete choice structural methods have been the primary method of quantifying the importance of learning about ability on the labor market, including the work of James (2012), Miller (1984), Nagypál (2007), Pastorino (2015, 2019), and Sullivan (2010). A notable example of reduced form analysis of employee learning on the labor market is Fredriksson, Hensvik, & Skans (2018).

Recent evidence also suggest that learning about ability is also an important part of educational decisions. Learning about ability has recently been implicated in schooling effort (Azmat & Iriberri 2010, Bandiera, Larcinese, & Rasul 2015, Goulas & Megalokonomou 2015), course selection (Gonzalez 2017) and educational track choice (Bobba & Frisancho 2014), college application strategies (Bond et al 2018), college major choice (Arcidiacono 2004, Li 2018, Rury 2020), and college dropout decisions (Stinebrickner & Stinebrickner 2012, Arcidiacono et al 2016).

Estimated impacts about learning about ability are quite large, both in education and employment. Stinebrickner & Stinebrickner for example find that as much as 40% of the college dropout decisions in their sample frame were due to learning about ability, while the value of learning about multidimension ability across employment has been estimated at about one year (James 2012) and 32 months (Gorry, Gorry, & Trachter 2019).

To the extent that uncertainty and learning about multidimensional ability is sufficiently important, this poses a couple of significant concerns for traditional model of human capital investment in intergenerational settings. First, it may imply that the use of educational attainment to summarize the human capital investment behaviors of individuals mischaracterizes important parts of this dynamic. Whether or not one goes to college may not be an indicator of whether the returns to college are sufficiently high relative to employment, but rather an indicator of whether the perceived returns to college are greater than the returns to acquiring more information about occupational fits. Broadly, individuals maximize their

human capital by making the decisions that they believe to be optimal both in the acquisition of new skills (via learning in school or on the job), but also in terms of learning about what skills they already possess.

Furthermore, the extent to which individuals are forced to make a long sequence of human capital decisions under uncertainty, and the critical dependence of contemporaneous choices on past decisions (characterized by greater ability uncertainty), may dramatically impede the type of efficient human capital investment typically assumed in models of intergenerational mobility.

Utilization of familial signals in the formation of initial beliefs about ability — although instrumental in improving beliefs over fully naive expectations, nevertheless is likely to exacerbate the extent to which ability uncertainty hampers intergenerational mobility. Although family-informed signals of ability may be unbiased in the limited information context in which true ability is unobserved, it is necessarily biased towards parental outcomes when considering the full-information context hitherto assumed in intergenerational models.

In contrast to the standard Becker-Tomes model, wherein investments are conditionally optimal responses to endowments of income and ability, uncertainty about ability and family-influenced priors suggest that households may mis-invest in a manner that reflects the bias of assumed ability persistence. Because of the dynamic structure of human capital investments and the higher relative return to earlier human capital investments (popularly known as the "Heckman Curve"), mis-investment that occurs during early life periods of substantial ability uncertainty can not easily be corrected in later periods.[3]

3 Data and Estimation Sample

Data used in estimation comes from primarily from Swedish administrative register sources collected by Statistics Sweden (*Statistiska centralbyrån*). Educational data for youth comes

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[3]Further, the self-productivity of human capital investments may imply that later life ability beliefs do not converge to potential at birth, but instead reflect the consequences of early childhood investments.

from the Educational Register. The Educational Register includes data on higher education enrollment, course completion, degree completion, and financial aid.

Data on income and occupation comes from a combination of register sources. The primary occupational and wage data source is the *Structural Wage Statistics*, an employment survey covering all public sector employees and a sample of firms in the private sector that accounts for about half of private sector employees each year. I further observe employment earnings by firm and workplace through the Register-based Labor Market Statistics (RAMS) data source. Finally, I observe a summary measure of total earnings from employment each year from the Longitudinal database on education, income and employment (LOUISE).

LOUISE is also the primary basis for data on the demographic characteristics of individuals. This includes an individual's age, sex, and country of birth. To facilitate identification of family linkages, I further draw on the Multigenerational Register, which identifies the parents of all individuals in the population since 1961.

Finally, to empirically validate the ability beliefs model and conduct follow-up analysis, I link the administrative data to a rich cohort study of youth: the Evaluation through Follow-Up (ETF) Survey, conducted by the Department of Education at the University of Gothenburg. The ETF Survey elicits (among other things) students' perception of ability, attitudes towards education, and future plans at multiple points during compulsory and upper secondary education. The population of the ETF Survey is a stratified random sample of students that corresponding to about 10,000 students, or about 10% of all Swedish students in the cohort.

In order to take advantage of the ETF Survey, I estimate the structural model for a sample of the Swedish population born in the same year as an ETF Cohort. Specifically, I estimate the model for male students born in 1992, coincident with the 1992 ETF birth cohort. I further limit the sample to individuals who have completed three-year upper secondary schooling by 2011. The final estimation sample corresponds to 17,234 individuals, 1,346 of which are observed in the ETF. Individuals are observed for the period 2011-2018,

corresponding to 137,523 person-year observations.

4   Model

Model Overview and Choice Structure

In the model, forward looking individuals make a series of decisions about education and employment to maximize the discounted present value of their lifetime utilities. Individuals start off having completed a three year upper secondary schooling degree, conferring eligibility to pursue higher education. They begin making choices about education and employment.

Educationally, individuals choose whether or not to pursue a STEM undergraduate degree, a non-STEM undergraduate degree, a four-year professional degree, or not pursue further education. If the students are already enrolled in education, they can choose whether to drop out or change programs. If a student has completed an undergraduate or professional degree, they can also choose whether or not to pursue graduate education (Masters/Doctoral degree). When choosing to participate in education, they can enroll either part-time or full-time.

In employment, individuals can choose whether or not to pursue a white collar or blue collar job. Designation of a job as either a white collar or blue collar occupation is based on the International Standard Classification of Occupations (ISCO) classification schema, ISCO-88. White collar jobs are those defined by ISCO-88 as having a skill level of at least 3 (out of 4), corresponding to skills "usually obtained as the result of study at a higher educational institution." The following Occupational Groups satisfy this skill level requirement: (1) Legislators, Senior Officials, and Managers, (2) Professionals, and (3) Technicians and Associate Professionals. Blue collar jobs are defined as all other jobs.

In addition to the choice of occupational sector, when pursuing employment an individual also chooses how much labor to supply, either working part-time or full-time. Finally, an

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individual also has the option to pursue neither education nor employment, a choice that I refer to as home production.

When making education and occupational decisions, the individual maximizes the expected discounted present value of their intended careers. Forming these expectations requires the agent to make assumptions about the way in which the economy functions: most notably, how educational achievement and earnings are "produced" as functions of one's investment decisions and other characteristics. I assume that the individual has complete information about the production functions of the economy, utilizing the same estimates that I produce econometrically. In the following sections, I specify the assumed production functions of education and employment outcomes in the economy.[4]

Production of educational achievement

I assume that achievement in a given period is a linear function of its determinants. In education, I treat credit completion rate G_{imt} as the relevant achievement outcome. Credit completion rate is defined as the number of credits the individual successfully completes (conditional on course registration pace) in higher education degree type m.[5]

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[4]Future work will extend the model considerably from the basic education and occupational choice structure outlined here, in two key ways: First, educational decisions will consider not only post-secondary schooling choices, but decisions from *mellanstadiet* (i.e. "middle school"), including time investment during compulsory schooling (captured by the ETF) and choice of upper secondary schooling track. Second, occupational choice will extend beyond the basic blue collar / white collar occupational choice outlined here, to instead include several occupations and multidimensional occupational ability. Similar to Sullivan (2010), future revisions will incorporate 6 occupational sectors, defined across 4 different ISCO skill levels. Occupational sector (1) is elementary labor, defined at ISCO Skill Level 1. At ISCO Skill Level 2, there are occupational sectors (2) Clerical, services, and sales workers, and (3) Craft, trade, and skilled agricultural workers. At ISCO Skill Level 3, there are occupational sectors (4) Non-STEM Technicians and Associate Professionals and (5) STEM Technicians and Associate Professionals. Finally, at ISCO Skill Level 5, there is occupational sector (6) Professional or managerial workers. Extending the model to this longer horizon and more multi-dimensional characterization of human capital offers great advantage in being able to capture the full life-cycle effect of ability uncertainty and highlights that this uncertainty entails an exploration-exploitation trade-off in human capital investment, which may heterogeneous effects across family background.

[5]In future revisions, I will have access to college grades, via the national study result system *Ladok*, managed by the consortium of the major higher education providers in Sweden. In the meantime, credit completion rate would appear to provide a good proxy for grade performance. In particular, in the Swedish setting an individual in principle has the ability to retake courses until they achieve their desired result. Consequently, pace of completion varies widely and is highly correlated with grade performance.

The assumed determinants of achievement are the child's background characteristics X_{it} (consisting of age and parental national origin), labor market participation (full or part-time), L_{it}, ability A_{im}, and idiosyncratic productivity shocks $\epsilon_{it} \sim N(0, \sigma_t^2)$.

The production function of educational achievement can thus be expressed as follows:

$$G_{imt} = \gamma_1 X_{it} + \gamma_2 L_{it} + \gamma_3 A_{im} + \epsilon_{imt} \tag{1}$$

Production of income in employment

Employment is characterized by both occupation (blue-collar or white-collar) and employment intensity (full or part-time). The relevant outcome variable for employment in occupation l is log wages, w_{ilt}, which depends on the following features: Observable worker characteristics, X_{ilt}, occupation-specific ability, A_{il}, Sector-specific time dummies, δ_{lt}, and idiosyncratic productivity shocks, $e_{ilt} \sim N(0, \sigma_l^2)$.

Observable work characteristics characteristics are considered to include demographics (age and parental origin), educational characteristics (level of educational attainment and type of degree), and finally occupational characteristics (years of experience and full or part-time status). The assumed income production function is thus as follows:

$$w_{ilt} = \delta_{lt} + \gamma_1 X_{ilt} + \gamma_3 A_{il} + \epsilon_{ilt} \tag{2}$$

Ability Beliefs

In both education and work, a prominent component of the assumed production functions is ability. Yet, as I have motivated, ability is not a fully observable characteristic to either the individual or the econometrician. In the presence of ability uncertainty, I assume that individuals use their experiences and knowledge about the structure of the economy to form Bayesian beliefs about ability.

More specifically, a person's latent ability, A_i is a multi-dimensional vector consisting of

ability specific to each educational type and employment sector. Ability is assumed to be distributed multivariate normal, with unconstrained covariance matrix, Δ. Individuals are assumed to not know their own ability (...*how far away did the apple fall?*), but are assumed to understand the distribution of outcomes for people that are observational similar to themselves and use this to form rational expectations about their ability.[6] As they observe further signals of their ability, individuals update their beliefs about ability according to Bayes' Rule for the multivariate normal distribution. Notably, this design allows for correlated learning about ability across types.

To demonstrate how ability and beliefs about ability are incorporated into the model, I highlight here the example of someone in the 2nd year of a STEM undergraduate degree. Restating the assumed production function of academic achievement (and denoting STEM undergraduate education as CS):

$$G_{it}^{CS} = \gamma_1^{CS} X_{it} + \gamma_2^{CS} L_{it} + \gamma_3 A_i^{CS} + \epsilon_{it}^{CS}$$

$$A_i^{CS} = \frac{1}{\gamma_3} \left(G_{it}^{CS} - \gamma_1^{CS} X_{it} - \epsilon_{it}^{CS} \right)$$

From the student's performance in the first year of university, they are able to obtain an unbiased signal of their ability. In particular, note that there are in principle two unobserved components of the achievement process: ability and productivity shocks. But since the productivity shocks are random and mean zero:

$$\mathbb{E}\left[A_i^{CS}\right] = \mathbb{E}\left[\frac{G_{it}^{CS} - \gamma_1^{STEM} X_{it} - \gamma_2^{CS} L_{it} - \epsilon_{it}^{CS}}{\gamma_3}\right]$$

$$\mathbb{E}\left[A_i^{CS}\right] = \frac{G_{it}^{CS} - \gamma_1^{CS} X_{it} - \gamma_2^{CS} L_{it}}{\gamma_3} \equiv s_{itCS}$$

This is to say that if a person understands how grades are produced, and can observe all the components except for latent ability and some noise, then the person knows that on

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[6]Further analyses will consider minimal deviations from rationality

average, ability will equal the difference between the grades they obtained and what would be expected given their demographic characteristics. Grades therefore provide *signals* of ability, which an individual can use to update ability beliefs (via application of Bayes' Rule).

Although the previous example illustrates how individuals can update their beliefs about ability using their experiences, it yields no insight into how individuals form their beliefs before they've had the chance to learn from experience. Without knowing anything more than the production function of achievement and their observable characteristics that go into achievement, it is clear that their ex-ante expected ability would be zero (i.e. the average of ability in the overall population). And indeed, this is precisely how initial beliefs are specified in the canonical ability learning model of Arcidiacono et. al. (2016).

However, suppose that individuals know about some attributes they have, z_{it}, that are predictive of ability but don't otherwise have a direct affect on achievement. In this case, they can use that information about predictors of ability to form beliefs. In particular, note that for the "residual" determinants of grades, without conditioning on the information z_{it} then $\mathbb{E}\left[A_i^{STEM} + \epsilon_{it}^{STEM}\right] = 0$. However, with z_{it} in the information set, we instead have $\mathbb{E}\left[A_i^{STEM} + \epsilon_{it}^{STEM} \mid z_{it}\right] = \mathbb{E}\left[A_i^{STEM} \mid z_{it}\right]$. Put simply, if students know about personal attributes that are predictors of their ability, then they can form initial beliefs based on individuals with similar predictive attributes, rather than relying on "naive" initial beliefs that convey no precision.

In the model, I assume that households use the following parental signals of ability: each parents' educational level and subject (STEM or non-STEM), occupation (blue or white collar), and the logged average of the primary parental earners' disposable income over the five previous years.

Utility

Having demonstrated how individuals are assumed to produce educational achievement and incomes, I now turn to their objective function in making decisions.

Individuals are assumed to be forward looking and choose the sequence of education (j) and labor market (k) decisions (d_{it}) that maximizes the present value of expected lifetime utility:[7]

$$V_t \equiv \max \mathbb{E}\left[\sum_{t=1}^{T} \beta^{t-1} \sum_j \sum_k (u_{jk}(Z_{it})) + \epsilon_{ijkt})1\{d_{it} = (j,k)\}\right] \tag{3}$$

where $Z_{it} = (Z_{1it}, Z_{2it})$ denotes the variables that affect the utility of schooling and work respectively, β is the discount rate, and ϵ_{ijkt} is a choice-specific idiosyncratic shock, assumed distributed Type 1 Generalized Extreme Value (GEV).

Components of the utility term, u_{jk} include several features. During both schooling and work, utility includes controls for both demographics, ability measures, and controls for the previous choice (thereby incorporating switching costs). Unique aspects for educational choices include the expected ability in schooling option j. Unique aspects of the utility term for employment includes the expected log wages in occupational sector k. The home sector is set as the reference sector, hence utility of this option is normalized to 0.[8]

5 Estimation

To solve the dynamic programming problem of individuals (and their families), I use the conditional choice probability (CCP) estimation methods introduced by Hotz and Miller (1993), simplifying estimation by avoiding the need for full solutions methods.

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[7]For presentational purposes, I abstract here from the probabilities that an individual is able to choose a schooling or an employment option (i.e., there probability of admission into a degree program, as well as job offer arrival rates). Formally, however, I allow the choice set of an individual, i at time t to $D_{i,t}$ to depend on their accumulated skills until that time. Schooling choice probabilities are directly estimated using admissions data from the education register, while job offer arrival rates are treated as a latent value, which depends on the expected productivity of the individual in the occupation (discretized into 5 types based on the standard deviation of expected wages).

[8]In future revisions, utility weights will be allowed to vary by latent class of the individual, thereby incorporating permanent type-specific unobserved heterogeneity. Future revisions will also incorporate education-specific financial transfers, both student aid and expected parental transfers. Expected parental transfers are drawn from two large surveys, the Survey of Household Finances and the Level of Living Survey.

Details of CCP Estimation

First, note the value function (eq 3) can be re-expressed as a Bellman equation:

$$V_t \equiv \max \mathbb{E} \left[\sum_{t=1}^{T} \beta^{t-1} \sum_j \sum_k (u_{jk}(Z_{it})) + \epsilon_{ijkt}) 1\{d_{it} = (j, k)\} \right] \quad (4)$$

$$V_t = \underbrace{u_{jk}(Z_{it}) + \epsilon_{ijkt}}_{\text{flow utility}} + \underbrace{\beta \mathbb{E} \left[V_{t+1}(Z_{i,t+1} \mid Z_{it}, d_{it} = (j, k)) \right]}_{\text{continuation value}} \quad (5)$$

Define the ex ante value function, $\bar{V}_t$ as the expected value of the value function at the beginning of time t, before ϵ_{ijkt} is revealed: $\bar{V}_t(Z_{i,t}) = \int V_t f(\epsilon) d\epsilon_t$.

The *conditional value function*, which is the expected discount present value of utility of a given choice at time, t, conditional on the history until t, can be expressed as:

$$v_{jkt}(Z_{it}) = u_{jk}(Z_{it}) + \beta \mathbb{E} \left[\bar{V}_{t+1} (Z_{i,t+1} \mid Z_{it}, d_{it} = (j, k)) \right] \quad (6)$$

The assumption of Type 1 GEV errors implies the choice structure follows the dynamic logit model. The ex ante value function, $\bar{V}_t$ is then:

$$\bar{V}_t(Z_{i,t}) = \ln \left(\sum_{d_{t+1} \in D} \exp(v_t(Z_t, d_{i,t+1})) \right) + \gamma \quad \text{(Euler's constant)} \quad (7)$$

For a given arbitrary reference choice, $d_{i,t}^*$, we can then multiply and divide by conditional value function of the choice inside the log:

$$\bar{V}_t(Z_{i,t}) = \ln \left(\exp(v_t(Z_t, d_{i,t+1}^*)) \frac{\sum_{d_{t+1} \in D} \exp(v_t(Z_t, d_{i,t+1}))}{\exp v_t(Z_{it}, d_{i,t+1}^*)} \right) + \gamma \quad \text{(Euler's constant)} \quad (8)$$

Notice again that by the logit structure of the choice that:

$$\frac{\sum_{d_{t+1} \in D} \exp(v_t(Z_t, d_{i,t+1})}{\exp v_t(Z_t, d_{i,t+1}^*)} = p(d_{i,t+1}^* \mid Z_{i,t})^{-1} \quad (9)$$

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Hence:

$$\bar{V}_t(Z_{i,t}) = \ln\left(\exp(v_t(Z_t, d^*_{i,t+1}))p(d^*_{it} \mid Z_{i,t})^{-1}\right) + \gamma = v_t(Z_t, d^*_{i,t+1}) - \ln p(d^*_{it}) + \gamma \qquad (10)$$

Using this result for the ex ante value function, the conditional value function becomes:

$$v_{jkt}(Z_{it}) = u_{jk}(Z_{it}) + \beta \int \left(v_t(Z_t, d^*_{i,t+1}) - \ln p(d^*_{i,t+1} \mid Z_{i,t})\right) dZ_{i,t+1} + \beta\gamma \qquad (11)$$

$$v_{jkt}(Z_{it}) = u_{jk}(Z_{it}) + \beta \int \left(v_t(Z_t, d^*_{i,t+1}) - \ln p(d^*_{i,t+1} \mid Z_{i,t+1})f(Z_{i,t+1} \mid Z_{i,t}, d_{it})\right) dZ_{i,t+1} + \beta\gamma$$
$$(12)$$

From this result, the state transitions and conditional choice probabilities can be directly estimated – the only component of the future value term that needs further simplification is the remaining conditional value term, $v_t(Z_t, d^*_{i,t+1})$.

The approach to handling this term is to work instead with *differences* in conditional value terms, by any given choice $d_{i,t} = (j, k)$ and home production, $d_{i,t} = (0, 0)$. The difference in values between these these two paths show finite dependence – that is, after a finite number of periods, the future values of both paths are the same. For example, consider two possible sequences starting from a period, t, in which an individual might choose to either work as in a blue collar occupation or stay home. Assuming the person chose to initially work in blue collar employment, the person might then choose home production. Meanwhile, for the sequence that began with home production, the person might then choose to work in blue collar employment in $t + 1$. Each sequence would result in the person having the same amount of experience as blue collar employment, but the future values would not yet line up do to switching costs from the current occupation in $t + 1$. Choosing the same occupation in $t + 2$, however, would result in the future value terms being equivalent between the two paths.

This finite dependence is used to achieve a cancellation of future value after two periods, hence by working with the difference in conditional value functions, $v_{jkt} - v_{00t}$, it is necessary

only to work with the conditional choice probabilities along the finite dependence path.

The conditional choice probability therefore results in a procedure where the differenced value functions can be estimated by a two-stage estimation process: first estimating the conditional choice probabilities, $p(d^*_{i,t+1} \mid Z_{i,t+1})$, then estimating flow utilities by re-expressing the conditional value functions as a function of flow utilities and the conditional choice probabilities along the finite dependence path.[9]

6   Results and counterfactual policy analysis

Descriptive evidence of parentally informed beliefs

Before turning to the estimates from the structural model, Table 1 documents preliminary descriptive evidence that parental education and labor market outcomes shape ability beliefs even after controlling for observed ability. Specifically, Table 1 estimates ability belief in Grade 9, a period prior to the human capital choice sequence estimated in the model.[10] Ability beliefs are elicited for ETF1992 cohort participants in grade 9 in multiple subjects, with math and language (Swedish) highlighted here.

The belief measure that is employed here is the sum of self-reported ability beliefs in a variety of subject-specific tasks for each of Swedish and mathematics. Ability beliefs for each sub-task is measured on a 1-5 Likert scale, with 5 being excellent and 1 and being very poor

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[9]The two-stage estimation process relies on separability between the choices and flow utility. Following Arcidiacono et al (2016), however, I will in future revisions account for permanent unobserved heterogeneity in preferences and ability incorporated by allowing for latent types which affect both utility parameters and education/wage production. With unobserved types, sequential estimation of production and utility parameters fails because the likelihood function is no longer additively separable between the two. To account for this, following Arcidiacono & Jones (2003) and Arcidiacono & Miller (2011), I will use an iterative application of the Expectation-Maximization algorithm to restore additive separability. In addition, I also adopt a latent variable approach to account for heterogeneous job offer arrival rates across occupations and skill types. Specifically, I allow the job offer probability for person i and occupation j, λ_{ij} to be a latent value which depends on the both observable determinants of worker productivity as well an unobserved ability.

[10]Ideally, the role of parental predictors would be estimated in the period immediately prior to the choice mode, i.e. in the final year of upper secondary schooling. This data is included in the ETF but is not yet available to the author (expected delivery from Statistics Sweden in May 2022). The ETF data will ultimately be used in a more formal way than the descriptives outlined here, however. Specifically, the structural model will be estimated for the ETF sample from Grade 6, allowing me to compare elicited ability beliefs to the rational expectation ability beliefs imposed by the model.

Table 1: Linear regression of Grade 9 ability beliefs
on national tests scores and parental predictors

	Swedish	Math
Swedish Standardize Test Result:		
... Part A: Fail	-0.276 (-4.202)	
... Pass A: Pass with Distinction	0.181 (4.876)	
... Part B: Fail	-0.323 (-2.456)	
... Part B: Pass with Distinction	0.345 (9.846)	
... Part C: Fail	-0.195 (-3.220)	
... Part C: Pass with Distinction	0.171 (5.076)	
Swedish Standardize Test Result:		
... Points with grade Pass		0.019 (6.823)
... Points with grade Pass with Distinction		0.049 (19.418)
Highest education, primary parental earner:		
... Less than 3 year Upp Sec	-0.089 (-2.105)	0.024 (0.618)
... Upper secondary: STEM	-0.075 (-1.417)	0.116 (2.296)
... Bachelors: Non-STEM	-0.052 (-0.934)	0.020 (0.393)
... Bachelors: STEM	-0.177 (-1.437)	0.118 (1.121)
... Professional: Non-STEM	0.007 (0.098)	-0.013 (-0.197)
... Professional: STEM	-0.154 (-2.095)	-0.062 (-0.925)
... Graduate: Non-STEM	-0.017 (-0.174)	-0.014 (-0.143)
... Graduate: STEM	-0.116 (-1.099)	-0.038 (-0.429)
Highest education, secondary parental earner:		
... Less than 3 year Upp Sec	-0.058 (-1.465)	-0.022 (-0.606)
... Upper secondary: STEM	-0.069 (-1.016)	0.034 (0.531)
... Bachelors: Non-STEM	0.053 (1.067)	0.013 (0.291)
... Bachelors: STEM	-0.140 (-0.966)	-0.065 (-0.440)
... Professional: Non-STEM	0.012 (0.194)	-0.100 (-1.626)
... Professional: STEM	-0.031 (-0.302)	0.289 (3.362)
... Graduate: Non-STEM	0.179 (1.497)	-0.032 (-0.263)
... Graduate: STEM	-0.057 (-0.463)	0.170 (1.339)
Log disposable income, primary parental earner	0.073 (2.155)	0.031 (0.971)
Female	0.008 (0.285)	-0.091 (-3.729)
Parent born in:		
... Europe (except former USSR)	-0.284 (-2.664)	-0.299 (-2.881)
... Former USSR (except Visegrád group[b]	0.279 (1.185)	0.528 (1.560)
... Latin America	-0.081 (-0.692)	-0.179 (-1.817)
... Africa	0.331 (2.516)	0.091 (0.743)
... Middle East	0.281 (2.584)	0.131 (1.205)
... Asia	-0.020 (-0.187)	-0.018 (-0.151)
Constant	-0.731 (-2.499)	-1.048 (-3.652)
Joint Significance: Parental Education	$p = 0.0927$	$p = 0.0276$
Joint Significance: Parental Education and Income	$p = 0.0101$	$p = 0.0249$
Observations	4363	4098

Robust t statistics in parentheses

[a] The educational reference category is a non-STEM 3 year upper secondary degree.

[n] Countries in the Visegrád group comprise the Czech Republic, Hungary, Poland and Slovakia, which are included in "Europe (except former USSR)".

[c] Parents born in North America (United States and Canada) and Oceania (primarily Australia and New Zealand) are also included within the Europe category. This is due to the small number of parents observed with these national origins and the high degree of cultural, social, and economic similarity between these countries and their European colonizers (popularly expressed in economics via the moniker "Neo-Europes," following Acemoglu, Johnson, & Robinson 2001).

ability.[11] Before conducting linear regression, composite ability beliefs are first normalized (results are similar for ordered logit on raw scores).

Linear regression is used to estimate ability beliefs as a function of performance on national standardized test results, gender, parental origin, and parental predictors of ability, which include education and the income of the primary parental earner. For performance in Swedish, the relevant standardized test results are the overall grade (Fail, Pass, Pass with Distinction) on each of three parts of the Swedish exam. In math, the exam is not broken into parts, but instead performance is measured by the number of questions scored at a level of Pass or Pass with Distinction on the test.

Parental education is estimated separately for each parent based on whether they are the primary or secondary earner (effectively allowing the signal value of education to depend on whether the parent is expected to be the primary source of income). The income measure is the log of disposable income (defined over the previous 5 years), for the primary earner. [12]

The results are consistent with a large effect of parental predictors of ability (education and income) affecting students' belief about ability. Parental predictors of ability are jointly significant across all specifications, although parental education alone is only marginally significant ($p = 0.0927$) for beliefs about Swedish ability. Additionally, ability beliefs are consistently higher for parental education that is matched to to the elicited subject, that is: parental STEM degrees for math ability beliefs, and parental non-STEM degrees for Swedish ability beliefs. This subject effect dominates the education level effect: although ability beliefs are increasing in educational level, a parental upper secondary degree that is matched to the ability belief subject is estimated to have a more impact on beliefs than a graduate or professional degree that is not matched to the subject. Overall, the estimated

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[11]Swedish ability sub-tasks include the ability to (1) read and understand a text, (2) read aloud for the whole class, (3) read the subtitles to a film, (4) write a story, (5) participate in a discussion, (6) give an oral presentation, and (7) be good at spelling. Ability sub-tasks are (1) mental arithmetic, (2) calculate sums and multiplication, (3) calculate percentages, (4) calculate area and circumference, (5) solve equations, (6) solve math problems, (7) explain math problems, and (8) work on large math assignments or projects.

[12]Model selection for both education and income was based on the penalized Bayesian information criterion. Alternatives included parental attainments measures by sex (i.e. father, mother), the maximum attainment between parents, and attainment for each parent sorted by income.

effects of parental education are quite large: a parent with two parents that both have graduate degrees that matches with the elicited subject are estimated to have ability beliefs that are 0.31 and 0.13 standard deviations higher than students whose parents have only upper secondary schooling that is not matched to the elicited subject.

Production parameters

Estimates of the structural model educational and earnings productions parameters are reported in Tables 2 and 3. Educational achievement (currently modelled as standardized score of the credit completion rate) is strongly increasing in parental predictors of ability in addition to GPA percentile in upper secondary education and having completed a university preparatory degree (rather than a vocational upper secondary degree). The effects of family predictors of ability are estimated via auxiliary regression as a basis for family-informed initial beliefs about ability, but crucially enter production only through prior ability beliefs. A one standard deviation increase in the family ability signal is estimated to increase educational performance by 0.06-0.10 standard deviations conditional on determinants of performance. Besides education and ability predictors, employment is also estimated to have a significant (albeit largely negative) effect on performance. Full-time unemployment reduces credit completion rates by 0.11 standard deviations in non-STEM degrees, while it is estimated to have a near zero effect on STEM bachelor and graduate degree completion rates, and positive effects on STEM professional degrees. STEM and Graduate degrees,

For earnings (Table 3), higher levels of education are associated with large increases in predicted log earnings. Similarly STEM degrees have large but heterogeneous (by degree level) effect on earnings. The main effect of a STEM degree is 13-14% increase in earnings. Family ability signals once predict large effects on achievement, with a 1 standard deviation increase in the ability signal estimates to increase earnings by 3.1-5.6%. Estimates are consistent with positive returns to experience, although results suggests that earnings are concave in occupational experience over the panel duration estimated.

Table 2: Educational Performance (standardized credit completion rate) production parameters:

	(1) Bachelors Non-STEM	(2) Bachelors STEM	(3) Prof. (4+ yr) Non-STEM	(4) Prof. (4+ yr) STEM	(5) Graduate
Family Ability Signal (Standardized)	0.059 (0.010)	0.075 (0.012)	0.095 (0.014)	0.055 (0.011)	0.068 (0.021)
Upper Secondary degree: Univ Prep	0.063 (0.065)	0.198 (0.075)	0.239 (0.113)	0.582 (0.154)	0.450 (0.303)
Upper Secondary GPA Percentile	0.559 (0.082)	1.047 (0.080)	0.846 (0.132)	1.685 (0.148)	1.070 (0.290)
Univ Prep degree × Upper Sec GPA Percentile	0.097 (0.091)	-0.187 (0.114)	-0.089 (0.149)	-0.434 (0.174)	-0.388 (0.341)
Part-time employment	-0.050 (0.024)	-0.036 (0.026)	-0.064 (0.030)	0.040 (0.032)	-0.016 (0.049)
Full-time employment	-0.113 (0.048)	0.000 (0.052)	-0.113 (0.055)	0.319 (0.059)	0.078 (0.074)
Full-time enrollment rate	-0.077 (0.023)	-0.065 (0.024)	-0.125 (0.028)	0.031 (0.025)	-0.009 (0.046)
Parent born in:					
... Europe (except former USSR)	0.317 (0.117)	-0.570 (0.186)	0.428 (0.129)	-0.108 (0.117)	0.240 (0.207)
... Former USSR (except Visegrád group)	0.231 (0.122)	-0.574 (0.194)	0.409 (0.151)	-0.138 (0.095)	0.181 (0.212)
... Latin America	-0.036 (0.083)	0.029 (0.115)	-0.034 (0.126)	-0.190 (0.099)	-0.282 (0.270)
... Africa	-0.116 (0.102)	-0.343 (0.118)	-0.127 (0.149)	-0.184 (0.155)	-0.017 (0.191)
... Middle East	-0.213 (0.047)	-0.080 (0.052)	0.013 (0.054)	-0.160 (0.050)	0.013 (0.079)
... Asia	-0.036 (0.065)	0.025 (0.069)	0.024 (0.086)	0.026 (0.066)	-0.035 (0.110)
Constant	-0.893 (0.072)	-1.174 (0.078)	-1.424 (0.128)	-1.630 (0.142)	-0.995 (0.526)
Observations	8526	6443	5943	6737	2152

Bootstrapped Standard errors in parentheses

Table 3: Log Earnings Production Parameters

	(1) Blue-Collar	(2) White-Collar
Family Ability Signal (Standardized)	0.056 (0.002)	0.031 (0.003)
Highest degree completed		
Highest Ed. Level: Univ Prep Upper Secondary	-0.056 (0.005)	-0.019 (0.010)
Highest Ed. Level: Bachelors	0.261 (0.022)	0.183 (0.012)
Highest Ed. Level: Professional degree (4+ yr)	0.343 (0.044)	0.483 (0.018)
Highest Ed. Subject: STEM	0.130 (0.004)	0.139 (0.010)
Univ Prep Upper Secondary × STEM	-0.134 (0.012)	-0.147 (0.016)
Bachelors × STEM	0.034 (0.036)	-0.065 (0.016)
Professional degree × STEM	-0.582 (0.070)	-0.103 (0.020)
Full-time employment	0.322 (0.003)	0.351 (0.008)
Age	0.040 (0.002)	0.084 (0.002)
Occupational experience	0.249 (0.006)	0.170 (0.006)
Occupational experience^2	-0.017 (0.001)	-0.015 (0.001)
Educational experience		
... Bachelors deg., 1 year	-0.290 (0.010)	
... Bachelors deg., 2 years	-0.417 (0.013)	
... Bachelors deg., 3 years	-0.337 (0.016)	
... Bachelors deg., 4+ years	-0.130 (0.012)	
... Graduate deg., 1 year		-0.382 (0.031)
... Graduate deg., 2 years		-0.290 (0.024)
... Graduate deg., 3 years		-0.095 (0.025)
... Graduate deg., 4+ years		0.015 (0.058)
... Professional deg., 1 year		-0.180 (0.018)
... Professional deg., 2 years		-0.341 (0.025)
... Professional deg., 3 years		-0.371 (0.019)
... Professional deg., 4+ years		-0.218 (0.013)
Constant	10.454 (0.031)	9.727 (0.041)
Observations	65029	24106

Bootstrapped Standard errors in parentheses

Utility parameters

Flow utility parameters for education are reported in Table 4. For both education and employment, the utility of home production (not working, not enrolled in school) is normalized to zero. Utility of education enrollment is increasing in ability belief priors in STEM degrees and in non-STEM bachelors, but does not appear to be affected by belief priors for non-STEM professional degrees or graduate degrees. Indicators for previous educational choices indicate that there are very large switching costs across all educational choices (that is, utility is strongly increasing in pursuing the same degree as in the previous period). Employment during education is also estimated to have consistently negative, significant effect across all degrees, although having previously worked part-time in a white collar occupation is estimated to increase the utility of higher education.

Table 5 reports flow utility estimates for occupational choices. Expected earnings (incorporating ability belief priors) is predicted to have a large positive effect on the utility of a given occupational choice, across occupations. Relative to a vocational upper secondary schooling, university preparatory upper secondary schooling is estimated to increase the utility of white collar work and blue collar part-time work, but has a near zero effect for full-time blue collar employment.

Having previously enrolled in school in the previous period has an estimated positive effect regardless of schooling option. Estimated effects for blue collar occupations and part-time white collar employment are highest for lower levels of schooling, with very large effects for previous vocational upper secondary enrollment in blue collar employment. For full-time white collar employment, however, utility estimates are highest for having been previously enrolled in a professional degree. Having completed a professional degree is similarly estimated to have a large positive effect on the utility of employment in white collar occupations, while having a bachelors degree has a positive effect (relative to upper secondary level education) for all occupations except for part-time white collar employment.

Table 4: Flow utility estimates: educational choices

	Bachelors Non-STEM	Bachelors STEM	Professional Non-STEM	Professional STEM	Graduate
Constant	−5.14 (0.13)***	−4.38 (0.13)***	−6.37 (0.17)***	−6.62 (0.19)***	−1.10 (0.26)***
Parent born in:					
... Europe, except former USSR	−0.23 (0.12)*	−0.31 (0.12)*	−0.37 (0.14)*	−0.33 (0.16)*	0.20 (0.24)
... Former USSR	−0.41 (0.23)	−0.29 (0.24)	−0.49 (0.31)	0.29 (0.26)	0.51 (0.41)
... Latin America	0.16 (0.15)	−0.29 (0.20)	−0.24 (0.20)	0.10 (0.23)	−0.62 (0.30)*
... Africa	0.03 (0.18)	0.33 (0.19)	−0.43 (0.24)	−0.14 (0.28)	0.21 (0.38)
... Middle East	−0.03 (0.12)	0.15 (0.12)	0.06 (0.14)	−0.10 (0.15)	0.25 (0.23)
... Asia	−0.04 (0.14)	0.11 (0.15)	−0.40 (0.19)*	0.19 (0.17)	0.14 (0.25)
Predicted completion rate	0.30 (0.04)***	0.29 (0.04)***	0.11 (0.06)	0.41 (0.05)***	0.10 (0.07)
High school GPA Percentile	1.84 (0.06)***	1.91 (0.07)***	3.38 (0.11)***	4.54 (0.13)***	
Previous Enrollment:					
... Upper Secondary Type: Univ Prep	1.22 (0.04)***	0.63 (0.04)***	1.81 (0.05)***	1.37 (0.06)***	
... Upper Secondary: Vocational	−0.62 (0.10)***	−0.39 (0.09)***	−1.18 (0.19)***	−0.00 (0.10)	
... Upper Secondary: Univ Prep	−0.47 (0.07)***	−0.46 (0.09)***	−0.71 (0.10)***	0.02 (0.07)	
... Bachelors: non-STEM	4.83 (0.04)***	0.03 (0.20)	0.84 (0.15)***	−1.12 (0.38)***	1.01 (0.13)***
... Bachelors: STEM	0.02 (0.17)	5.25 (0.05)***	−0.51 (0.32)	1.12 (0.15)***	0.78 (0.13)***
... Professional degree: non-STEM	0.50 (0.17)***	−0.74 (0.41)	5.91 (0.07)***	−0.52 (0.36)	0.17 (0.24)
... Professional degree: STEM	−1.17 (0.34)***	0.87 (0.17)***	0.50 (0.21)*	5.72 (0.07)***	2.38 (0.13)***
... Graduate degree					3.21 (0.12)***
Previous employment:					
... Blue collar PT	0.19 (0.05)***	−0.15 (0.06)*	−0.01 (0.07)	−0.07 (0.08)	−0.77 (0.12)***
... Blue collar FT	0.22 (0.08)*	−0.19 (0.10)	0.08 (0.12)	−0.67 (0.15)***	−1.65 (0.29)***
... White collar PT	0.52 (0.06)***	0.45 (0.07)***	0.49 (0.08)***	0.58 (0.09)***	−0.92 (0.10)***
... White collar FT	−0.58 (0.20)***	−1.23 (0.23)***	−0.74 (0.33)*	−1.60 (0.31)***	−2.36 (0.17)***
Work PT	−0.90 (0.04)***	−1.21 (0.05)***	−0.96 (0.06)***	−1.55 (0.06)***	−0.90 (0.10)***
Work FT	−2.30 (0.07)***	−2.46 (0.08)***	−2.57 (0.11)***	−2.48 (0.10)***	−2.38 (0.11)***

***$p < 0.001$; **$p < 0.01$; *$p < 0.05$.
Robust standard errors in paranthesis.

Table 5: Flow utility estimates: occupational choices

	Blue collar PT	Blue Collar FT	White collar PT	White collar FT
Constant	−53.10***	−61.08***	−53.86***	−68.53***
	(0.07)	(0.06)	(0.24)	(0.09)
Predicted earnings	4.54***	4.93***	4.34***	5.13***
	(0.01)	(0.00)	(0.02)	(0.01)
High school GPA Percentile	−0.57***	−0.81***	0.89***	0.58***
	(0.03)	(0.04)	(0.05)	(0.06)
Parent born in:				
... Former USSR	−0.48***	−0.59***	−0.33*	−0.15
	(0.11)	(0.16)	(0.14)	(0.18)
... Latin America	−0.02	−0.28*	−0.31*	−0.19
	(0.08)	(0.12)	(0.12)	(0.14)
... Africa	−0.16	−0.26*	−0.13	−0.27
	(0.09)	(0.13)	(0.12)	(0.17)
... Middle East	−0.46***	−0.45***	−0.18***	−0.30***
	(0.04)	(0.06)	(0.05)	(0.07)
... Asia	−0.21***	−0.18*	−0.48***	−0.19
	(0.06)	(0.09)	(0.08)	(0.11)
Upper Secondary Type: Univ Prep	0.34***	0.07*	0.53***	0.49***
	(0.02)	(0.03)	(0.03)	(0.04)
Previous Enrollment:				
... Upper Secondary: Vocational	1.85***	1.81***	1.37***	−0.14
	(0.03)	(0.05)	(0.05)	(0.17)
... Upper Secondary: Univ Prep	1.53***	1.63***	1.38***	−0.03
	(0.04)	(0.07)	(0.05)	(0.18)
... Bachelors: non-STEM	0.56***	0.29***	0.31***	0.73***
	(0.04)	(0.07)	(0.04)	(0.06)
... Bachelors: STEM	0.12*	0.07	0.11*	0.53***
	(0.05)	(0.09)	(0.05)	(0.06)
... Professional degree: non-STEM	−0.32***	−0.83***	0.75***	1.70***
	(0.05)	(0.11)	(0.06)	(0.07)
... Professional degree: STEM	−0.89***	−1.05***	0.33***	0.95***
	(0.05)	(0.12)	(0.05)	(0.07)
Previous employment:				
... Blue collar PT	2.17***	1.86***	−0.30***	0.37***
	(0.03)	(0.04)	(0.06)	(0.06)
... Blue collar FT	1.77***	3.54***	0.79***	1.82***
	(0.05)	(0.05)	(0.09)	(0.08)
... White collar PT	−0.62***	−0.37*	2.92***	2.11***
	(0.07)	(0.14)	(0.04)	(0.05)
... White collar FT	0.15	2.00***	3.15***	4.79***
	(0.17)	(0.15)	(0.10)	(0.10)
Graduated bachelors	−0.86***	−0.58***	0.08	1.25***
	(0.07)	(0.09)	(0.06)	(0.07)
Graduated professional degree	−0.05	0.18	−0.31***	1.23***
	(0.09)	(0.11)	(0.08)	(0.08)

***$p < 0.001$; **$p < 0.01$; *$p < 0.05$.
Robust standard errors in paranthesis.

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Finally, occupational flow utility estimates again indicate switching costs in deviating from the previous occupation, except for part-time white collar employment, wherein there is switching costs in moving outside of white-collar employment, but full-time white collar employment is estimated to be preferred over continued part-time employment.

Counterfactual Analysis

I am currently in the process of estimating counterfactuals from the structural model. By carrying out counterfactual analysis, it is possible to explore the ramification of uncertainty and beliefs about ability in a setting with family-informed priors.

Principally, I will consider a counterfactual that changes information in the model—supposing instead that households have no uncertainty about ability. Over the duration of the model, individuals resolve much of the uncertainty about ability, however ability beliefs may not converge to near certainty, particularly for paths that are explored. For example, if an individual never attends higher education, individuals can never learn from their own experience about their aptitude in these choices. Instead, without personal experience in higher education, an individual updates their beliefs about educational ability only through correlated learning from other pursuits. Thus, posterior beliefs of individuals in the model will vary in how precise they are, based on the extent of direct or correlated learning about ability that an individual has experienced.

Nevertheless, posterior beliefs exhibit greater precision and less reliance on family ability signals than initial family-informed priors. Given this learning that takes place, it is instructive to compare educational choice that takes place in the benchmark equilibria against a counterfactual setting in which youth have no Bayesian uncertainty about ability, but instead take as given the posterior beliefs from the structural model. Having more precise ability beliefs from the start implies that individuals who initially under- or over-estimated their ability based on parental ability signals may choose a different human capital path in the counterfactual setting. Analysis will therefore compare educational and occupational

attainment between the benchmark uncertainty setting and the counterfactual setting with posterior ability beliefs and no uncertainty.

Next, I will then consider a further counterfactual setting in which the costs of human capital decisions are altered: namely, through the introduction of tuition fees to Swedish higher education, in line with average tuition fees observed in the United States. Tuition fees and similar educational policies which increase the costs of pursuing uncertain human capital paths may be expected to change the costs of learning about ability and place greater emphasis on prior beliefs, hence such a counterfactual setting may help illuminate important features of the interaction between ability uncertainty and the educational policy environment.[13]

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[13]In upcoming revisions of the model to include secondary education, further counterfactual analysis will consider changes to educational tracking, including the timing and stringency of tracking in conferring eligibility to university education. Much like educational policies that make learning about ability more costly, policies that shift educational decisions earlier or make them more difficult to reverse are similarly predicted to heighten the importance of family-driven initial priors about ability. Reforms made to the upper secondary tracking system in Sweden moreover allow for direct comparison of counterfactual predictions to observed educational sorting changes.

Appendix A. Descriptive Statistics

Table 6: Characteristics of Enrolled Students in Higher Education, by Type

	(1) Bachelors Non-STEM	(2) Bachelors STEM	(3) Prof. (4+ yr) Non-STEM	(4) Prof. (4+ yr) STEM	(5) Graduate
Credit Completion Rate	0.75	0.75	0.79	0.83	0.90
	(0.216)	(0.203)	(0.205)	(0.161)	(0.138)
Father's Birthplace: Sweden	0.79	0.79	0.81	0.83	0.80
	(0.407)	(0.409)	(0.390)	(0.377)	(0.402)
Mother's Birthplace: Sweden	0.81	0.79	0.82	0.82	0.80
	(0.396)	(0.409)	(0.383)	(0.383)	(0.402)
Vocational	0.33	0.53	0.19	0.30	0.27
... Upper Secondary (3 years)	(0.470)	(0.499)	(0.390)	(0.458)	(0.447)
Univ. Prep.	0.67	0.47	0.81	0.70	0.73
... Upper Secondary (3 years)	(0.470)	(0.499)	(0.390)	(0.458)	(0.447)
Upper Secondary GPA	0.58	0.60	0.70	0.78	0.76
... (Percentile)	(0.254)	(0.252)	(0.251)	(0.197)	(0.204)
Part-Time Employment	0.39	0.29	0.33	0.19	0.33
	(0.488)	(0.455)	(0.472)	(0.396)	(0.470)
Full-Time Employment	0.06	0.06	0.08	0.06	0.16
	(0.245)	(0.235)	(0.264)	(0.233)	(0.369)
Part-Time Studies	0.49	0.55	0.37	0.50	0.56
	(0.500)	(0.498)	(0.483)	(0.500)	(0.497)
Full-Time Studies	0.51	0.45	0.63	0.50	0.44
	(0.500)	(0.498)	(0.483)	(0.500)	(0.497)
Observations	8632	6483	5991	6766	2182

Table 7: Parental Characteristics of Enrolled Students: Father and Family Income

	(1) Bachelors Non-STEM	(2) Bachelors STEM	(3) Prof. (4+ yr) Non-STEM	(4) Prof. (4+ yr) STEM	(5) Graduate
Disposable Income per parent	12.61	12.56	12.68	12.71	12.71
	(0.384)	(0.445)	(0.429)	(0.446)	(0.451)
Father's Educ.: Primary or less	0.10	0.08	0.11	0.11	0.12
	(0.302)	(0.279)	(0.311)	(0.313)	(0.320)
Father's Educ.: Compulsory	0.01	0.02	0.01	0.01	0.01
	(0.113)	(0.152)	(0.113)	(0.099)	(0.093)
Father's Educ.: Upper sec., 2 years or less	0.07	0.06	0.05	0.03	0.05
	(0.263)	(0.237)	(0.219)	(0.178)	(0.212)
Father's Educ.: Upper sec., 3 years	0.25	0.28	0.20	0.17	0.18
	(0.435)	(0.449)	(0.403)	(0.378)	(0.384)
Father's Educ.: Vocation Higher ed	0.23	0.26	0.20	0.23	0.24
	(0.420)	(0.438)	(0.398)	(0.423)	(0.429)
Father's Educ.: Bachelors	0.09	0.08	0.09	0.07	0.07
	(0.284)	(0.275)	(0.280)	(0.253)	(0.263)
Father's Educ.: Professional	0.10	0.08	0.11	0.10	0.10
	(0.304)	(0.271)	(0.318)	(0.295)	(0.300)
Father's Educ.: Some higher ed	0.09	0.09	0.17	0.19	0.16
	(0.292)	(0.283)	(0.373)	(0.388)	(0.365)
Father's Education Subject: STEM'	0.40	0.52	0.40	0.56	0.50
	(0.490)	(0.499)	(0.491)	(0.496)	(0.500)
Father's Occupation: White Collar	0.65	0.62	0.72	0.76	0.72
	(0.477)	(0.486)	(0.451)	(0.429)	(0.448)
Observations	8632	6483	5991	6766	2182

Table 8: Parental Characteristics of Students: Mother

	(1) Bachelors Non-STEM	(2) Bachelors STEM	(3) Prof. (4+ yr) Non-STEM	(4) Prof. (4+ yr) STEM	(5) Graduate
Mother's Educ.: Primary or less	0.06	0.05	0.05	0.05	0.06
	(0.237)	(0.228)	(0.226)	(0.222)	(0.237)
Mother's Educ.: Compulsory	0.01	0.02	0.01	0.01	0.01
	(0.111)	(0.141)	(0.098)	(0.091)	(0.106)
Mother's Educ.: Upper sec., 2 years or less	0.04	0.04	0.02	0.02	0.02
	(0.195)	(0.191)	(0.135)	(0.133)	(0.139)
Mother's Educ.: Upper sec., 3 years	0.21	0.20	0.17	0.16	0.18
	(0.409)	(0.398)	(0.372)	(0.365)	(0.383)
Mother's Educ.: Vocation Higher ed	0.22	0.22	0.19	0.19	0.18
	(0.413)	(0.416)	(0.396)	(0.390)	(0.382)
Mother's Educ.: Bachelors	0.13	0.14	0.15	0.15	0.16
	(0.340)	(0.347)	(0.357)	(0.355)	(0.362)
Mother's Educ.: Professional	0.18	0.17	0.20	0.19	0.20
	(0.381)	(0.378)	(0.402)	(0.391)	(0.400)
Mother's Educ.: Some higher ed	0.10	0.09	0.14	0.15	0.13
	(0.298)	(0.289)	(0.345)	(0.362)	(0.336)
Mother's Education Subject: STEM'	0.06	0.07	0.07	0.12	0.10
	(0.234)	(0.263)	(0.255)	(0.326)	(0.296)
Mother's Occupation: White Collar	0.60	0.60	0.69	0.70	0.66
	(0.491)	(0.489)	(0.465)	(0.456)	(0.473)
Observations	8632	6483	5991	6766	2182