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```
clear
clc
resloution = 0.001;
```

Inputs

%All in standard SI units unless noted

```
OperatingVoltage = 40           %V
TorqueConstant = 8.474E-3       %N.m/A
VoltageConstant = 1125          %RPM/V
ArmatureResistance = 0.072      %Ohms
reqOutputSpeed = 575            %RPM
reqOutputTorque = 55             %N.m
Efficiency = 90/100
Duty = 2000                      %Hours
```

%Gear Specs

```
PressureAngle = 20*pi/180       %Rad
FaceWidth = [1.5, 1.25, 1, 0.75, 0.5, 0.25, 0.188, 0.125] %inch
DiametralPitch = [8, 10, 12, 16, 20, 24, 32, 48] %teeth/inch
DiametralPitchSelected = 20;
k = 1                            %Teeth Depth (1 = full)
```

%Number of teeth variables:

```
P1 = 16
N1 = 70
P2 = 16
N2 = 70
```

OperatingVoltage =

40

TorqueConstant =

0.0085

VoltageConstant =

1125

ArmatureResistance =

0.0720

reqOutputSpeed =

575

reqOutputTorque =

55

Efficiency =

0.9000

Duty =

2000

PressureAngle =

0.3491

FaceWidth =

Columns 1 through 7

1.5000	1.2500	1.0000	0.7500	0.5000	0.2500	0.1880
--------	--------	--------	--------	--------	--------	--------

Column 8

0.1250

DiametralPitch =

8	10	12	16	20	24	32	48
---	----	----	----	----	----	----	----

k =

1

P1 =

16

N1 =

70

P2 =

16

N2 =

70

Motor Torque-Speed

%point 1:

NoLoadSpeed = VoltageConstant*OperatingVoltage;

NoLoadTorque = 0;

```

%point 2:
StallTorque = TorqueConstant*OperatingVoltage/ArmatureResistance;
StallSpeed = 0;

%determine Torque-speed graph
points = [StallSpeed, StallTorque;
          NoLoadSpeed, NoLoadTorque];
polynomialDegree = length(points)-1;

%fit a polynomial to the two data points
MotorLine = polyfit(points(:,1),points(:,2),polynomialDegree);

%generate figure
f1 = figure('Renderer', 'painters', 'Position', [10 10 1200 300])
subplot(1,3,1)
plot([StallSpeed:NoLoadSpeed], polyval(MotorLine,[StallSpeed:NoLoadSpeed]))
hold on
title("Motor Torque-Speed")
xlabel("Motor Speed (rpm)")
ylabel("Motor Torque (N.m)")
hold off

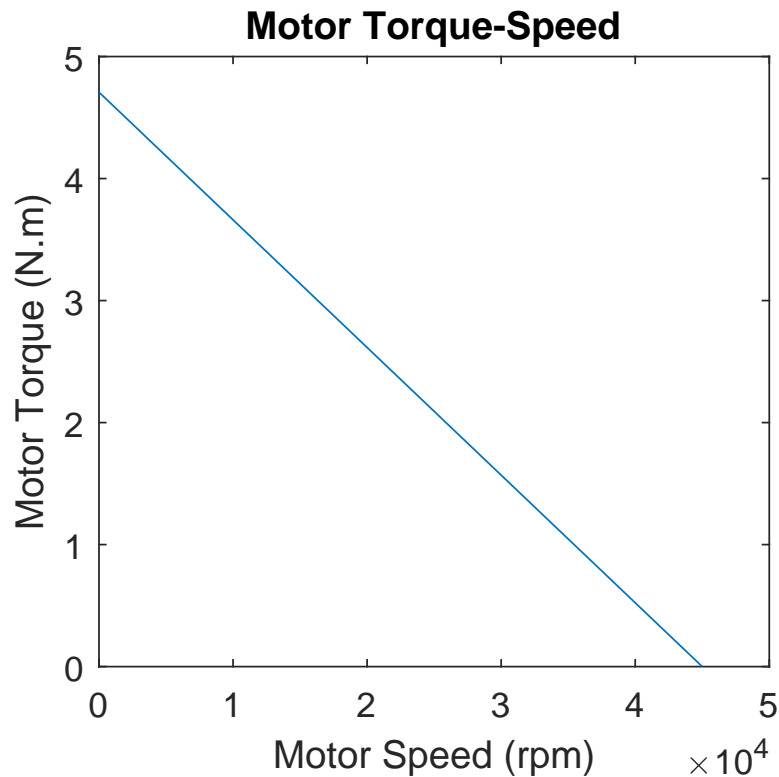
f1 =

Figure (1) with properties:

    Number: 1
      Name: ''
    Color: [0.9400 0.9400 0.9400]
  Position: [10 10 1200 300]
     Units: 'pixels'

Use GET to show all properties

```



Gear Ratio Target (minimization)

```
%setup equation to determine optimal ratio
syms r
eqn = MotorLine(1)*reqOutputSpeed*r^2 + MotorLine(2)*r == reqOutputTorque;
soln = double(solve(eqn,r))

%solve equation for tearget ratio
ratioTarget = soln(1);

%determine line for the target ratio
TargetLine = [MotorLine(1)*ratioTarget^2, MotorLine(2)*ratioTarget];

%generate figure
subplot(1,3,2)
plot([StallSpeed/ratioTarget: NoLoadSpeed/ratioTarget], polyval(TargetLine,[StallSpeed/ratioTarget: NoLoadSpeed/ratioTarget]), 'b')
hold on
plot(reqOutputSpeed, reqOutputTorque, '*')
title(["Drill Torque-Speed (Target), m = ", ratioTarget])
xlabel("Drill Speed (rpm)")
```

```

ylabel("Drill Torque (N.m)")
legend("Drill Output", "Given Requirement")
hold off

```

```

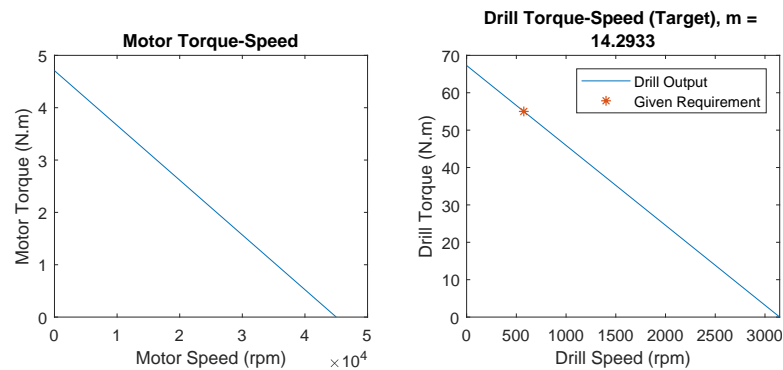
soln =

```

```

14.2933
63.9676

```



Gears

```

m1actual = N1/P1;
m2actual = N2/P2;
ratio = m1actual*m2actual;

```

```

%limits:

```

```

%pinions

```

```

P1l = ceil((2*k*(m1actual+(m1actual^2+(1-2*m1actual)*(sin(PressureAngle))^2)^0.5))/((1+2*m1actual)));
P2l = ceil((2*k*(m2actual+(m2actual^2+(1-2*m2actual)*(sin(PressureAngle))^2)^0.5))/((1+2*m2actual)));

```

```

%gear Limits

```

```

N1l = floor(((P1l^2)*(sin(PressureAngle)^2)-4*k^2)/(4*k-2*P1l*sin(PressureAngle)^2));
N2l = floor(((P1l^2)*(sin(PressureAngle)^2)-4*k^2)/(4*k-2*P2l*sin(PressureAngle)^2));

```

```

%generate ratio table

```

```

TRatio = table(P1, P1l, N1, N1l, m1actual, P2, P2l, N2, N2l, m2actual, ratio)

```

```

%old ratio determiner

```

```

%{

```

```

if(m1*P1l > N1l)

```

```

    N1 = N1l;    %Gear Ratio m is too large

```

```

    disp("Ratio m1 Too Large, increasing P1")

```

```

        P1l = P1l+1;
        N1 = floor(m1*P1l);
    else
        N1 = floor(m1*P1l);
    end

    if(m2*P2 > N2l);
        N2 = N2l;      %Gear Ratio m is too large
        disp("Ratio m2 Too Large, increasing P2")
        P2 = P2+1;
        N2 = floor(m2*P2);
    else
        N2 = floor(m2*P2);
    end

    %}

TRatio =

    117 table

         P1      P1l      N1      N1l      m1actual      P2      P2l      N2      N2l      m2actual      ratio
         --      ---      --      ---      -
16      16      70      101      4.375      16      16      70      101      4.375      19.141

```

Gear Ratio (Overall)

```

outputTorque = [0];
outputSpeed = [0];

%Adjust line with ratio
RealLine = [MotorLine(1)*ratio^2, MotorLine(2)*ratio];

for x = [StallSpeed:NoLoadSpeed]
    outputSpeed = [outputSpeed, x/ratio];
    outputTorque = [outputTorque, polyval(RealLine,x/ratio)];
end
outputSpeed = outputSpeed(2:end);
outputTorque = outputTorque(2:end);

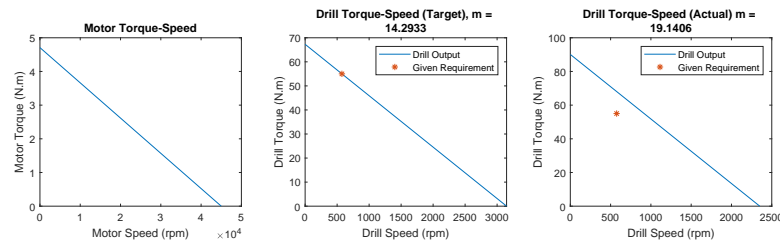
%generate figure
subplot(1,3,3)
plot(outputSpeed, outputTorque)

```

```

hold on
plot(reqOutputSpeed,reqOutputTorque,'*')
title(["Drill Torque-Speed (Actual) m = ", ratio])
xlabel("Drill Speed (rpm)")
ylabel("Drill Torque (N.m)")
legend("Drill Output", "Given Requirement")
hold off

```



Shaft Torque and Speed Estimation

```

%{
Motor - P1 : Shaft A
N1 - P2    : Shaft B
N2 - Chuck : Shaft C
}%

outputPower = reqOutputSpeed*reqOutputTorque*pi/30 ;
intermediatePower = outputPower/Efficiency^0.5;
inputPower = intermediatePower/Efficiency^0.5;
efficiency = outputPower/inputPower;

%Shaft C
SpeedC = reqOutputSpeed;
TorqueC = reqOutputTorque;
%Shaft B
SpeedB = SpeedC*m2actual;
TorqueB = intermediatePower/(SpeedB*pi/30);
%Shaft A
SpeedA = SpeedB*m1actual;
TorqueA = inputPower/(SpeedA*pi/30);

%Table for torque and speed estimations
TTorqueSpeedEstimation = table(TorqueA, SpeedA, TorqueB, SpeedB, TorqueC, SpeedC)

%table for power estimations
TPower = table(inputPower, intermediatePower, outputPower, efficiency)

```



```
TTorqueSpeedEstimation =
```

```
117table
```

TorqueA	SpeedA	TorqueB	SpeedB	TorqueC	SpeedC
-----	-----	-----	-----	-----	-----
3.1927	11006	13.251	2515.6	55	575

```
TPower =
```

```
117table
```

inputPower	intermediatePower	outputPower	efficiency
-----	-----	-----	-----
3679.7	3490.9	3311.8	0.9

Operating Gear Forces, Torques and Diameters

```
%initialize arrays
```

```
Force1t = [0];
```

```
Force1r = [0];
```

```
Force2t = [0];
```

```
Force2r = [0];
```

```
P1diameter = [0];
```

```
N1diameter = [0];
```

```
P2diameter = [0];
```

```
N2diameter = [0];
```

```
for x = DiametralPitch
```

```
    x = x/0.0254; %Teeth/inch -> teeth/meter
```

```
    P1dia = P1/x; %Teeth / (Teeth/meter) = meter
```

```
    N1dia = N1/x;
```

```
    P2dia = P2/x;
```

```
    N2dia = N2/x;
```

```
%record diameter
```

```
P1diameter = [P1diameter, P1dia];
```

```
N1diameter = [N1diameter, N1dia];
```

```
P2diameter = [P2diameter, P2dia];
```

```
N2diameter = [N2diameter, N2dia];
```

```

%Transmitted Load: Pinion 1 -> Gear 1 EQN 13-36
Wt1 = (60000*inputPower*10^-3)/(pi*(P1dia*1000)*SpeedA);
%Radial Load: Pinion 1 -> Gear 1
Wr1 = tan(PressureAngle)*Wt1;

%Transmitted Load: Pinion 2 -> Gear 2
Wt2 = (60000*outputPower*10^-3)/(pi*(N2dia*1000)*SpeedC);
%Radial Load: Pinion 2 -> Gear 2
Wr2 = tan(PressureAngle)*Wt2;

%Record Forces
Force1t = [Force1t, Wt1*1000]; %kN -> N
Force1r = [Force1r, Wr1*1000];
Force2t = [Force2t, Wt2*1000];
Force2r = [Force2r, Wr2*1000];
end

%Prepare arrays for table
Force1t = transpose(Force1t(2:end));
Force2t = transpose(Force2t(2:end));
Force1r = transpose(Force1r(2:end));
Force2r = transpose(Force2r(2:end));
P1diameter = transpose(P1diameter(2:end)).*100; %meters -> centimeters;
N1diameter = transpose(N1diameter(2:end)).*100; %meters -> centimeters;
P2diameter = transpose(P2diameter(2:end)).*100; %meters -> centimeters;
N2diameter = transpose(N2diameter(2:end)).*100; %meters -> centimeters;

%Table with Forces and Diameters
TForceDiameter = table(transpose(DiametralPitch), Force1t, Force1r, P1diameter, N1diameter,

%{

Not confident that this is correct

%Wt, Tangential Forces
Force1t = [0];
Force2t = [0];
%Wr, Radial Forces
Force1r = [0];
Force2r = [0];
%Gear Diameters
P1diameter = [0];
N1diameter = [0];
P2diameter = [0];
N2diameter = [0];
%Shaft Torques

```

```

AT = [0];
BT = [0];
CT = [0];

for x = DiametralPitch
DiametralPitch1 = x/0.0254;
DiametralPitch2 = x/0.0254; %Teeth/inch -> teeth/meter

P1dia = P1/DiametralPitch1; %Teeth / (Teeth/meter) = meter
N1dia= N1/DiametralPitch1;
P2dia= P2/DiametralPitch2;
N2dia= N2/DiametralPitch2;

%Logging diameters for the table below
P1diameter = [P1diameter, P1dia];
N1diameter = [N1diameter, N1dia];
P2diameter = [P2diameter, P2dia];
N2diameter = [N2diameter, N2dia];

%Tangential Forces
Ctorque = CopTorque;
Wt2 = Ctorque/(cos(PressureAngle2)*N2dia/2);
Btorque = (Wt2*P2dia/2)/cos(PressureAngle2);
Wt1 = Btorque/(cos(PressureAngle1)*N1dia/2);
Atorque = (Wt1*P1dia/2)/cos(PressureAngle1);
Force2t = [Force2t, Wt2];
Force1t = [Force1t, Wt1];
%Radial Forces
Force1r = [Force1r, tan(PressureAngle1)*Wt1];
Force2r = [Force2r, tan(PressureAngle2)*Wt2];
%Shaft Torques
AT = [AT, Atorque];
BT = [BT, Btorque];
CT = [CT, Ctorque];

end

Force1t = Force1t(2:end);
Force2t = Force2t(2:end);
Force1r = Force1r(2:end);
Force2r = Force2r(2:end);

P1diameter = P1diameter(2:end);
N1diameter = N1diameter(2:end);
P2diameter = P2diameter(2:end);
N2diameter = N2diameter(2:end);

```

```

AT = AT(2:end);
BT = BT(2:end);
CT = CT(2:end);

DiametralPitch = transpose(DiametralPitch);
Force1t = transpose(Force1t);
Force2t = transpose(Force2t);
Force1r = transpose(Force1r);
Force2r = transpose(Force2r)
P1diameter = transpose(P1diameter).*100; %meters -> centimeters
N1diameter = transpose(N1diameter).*100;
P2diameter = transpose(P2diameter).*100;
N2diameter = transpose(N2diameter).*100;
AT = transpose(AT);
BT = transpose(BT);
CT = transpose(CT);

ForceTorqueTable = table(DiametralPitch, AT, Force1t, Force1r, BT, Force2t, Force2r, CT)

DiameterTable = table(DiametralPitch, P1diameter, N1diameter, P2diameter, N2diameter)

%}

```

TForceDiameter =

817table

Var1	Force1t	Force1r	P1diameter	N1diameter	Force2t	Force2r	P2diameter
----	-----	-----	-----	-----	-----	-----	-----
8	125.7	45.751	5.08	22.225	494.94	180.14	5.08
10	157.12	57.188	4.064	17.78	618.67	225.18	4.064
12	188.55	68.626	3.3867	14.817	742.41	270.21	3.3867
16	251.4	91.501	2.54	11.113	989.88	360.29	2.54
20	314.25	114.38	2.032	8.89	1237.3	450.36	2.032
24	377.1	137.25	1.6933	7.4083	1484.8	540.43	1.6933
32	502.79	183	1.27	5.5563	1979.8	720.57	1.27
48	754.19	274.5	0.84667	3.7042	2969.6	1080.9	0.84667

Stresses

(Calculated in USCS then converted to metric)

```

%Init Arrays
ContactPinion1 = [0];
BendingPinion1 = [0];
ContactGear1 = [0];
BendingGear1 = [0];
ContactPinion2 = [0];
BendingPinion2 = [0];
ContactGear2 = [0];
BendingGear2 = [0];

Pinion1 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0];
Gear1 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0];
Pinion2 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0];
Gear2 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0];

for x = [1:length(DiametralPitch)]

    p1 = stresses(DiametralPitch(x), P1, SpeedA, FaceWidth(x), Force1t(x), m1actual, Pressur
    g1 = stresses(DiametralPitch(x), N1, SpeedB, FaceWidth(x), Force1t(x), m1actual, Pressur
    p2 = stresses(DiametralPitch(x), P2, SpeedB, FaceWidth(x), Force2t(x), m2actual, Pressur
    g2 = stresses(DiametralPitch(x), N2, SpeedC, FaceWidth(x), Force2t(x), m2actual, Pressur

    Pinion1 = [Pinion1; p1];
    Gear1 = [Gear1; g1];
    Pinion2 = [Pinion2; p2];
    Gear2 = [Gear2; g2];

    if DiametralPitch(x) == DiametralPitchSelected
        PINION1 = p1;
        GEAR1 = g1;
        PINION2 = p2;
        GEAR2 = g2;
    end

end

Pinion1 = Pinion1(2:end,:);
Gear1 = Gear1(2:end,:);
Pinion2 = Pinion2(2:end,:);
Gear2 = Gear2(2:end,:);

%Appendix tables, DP varying
names = ["DP","ContactStress", "BendingStress", "ContactFOS", "BendingFOS", "Ko", "Kv", "Ks
Tpinion1 = array2table(Pinion1, 'VariableNames', names)
Tgear1 = array2table(Gear1, 'VariableNames', names)
Tpinion2 = array2table(Pinion2, 'VariableNames', names)

```

```
Tgear2 = array2table(Gear2, 'VariableNames', names)
```

```
%Prepare arrays for table
PINION1 = transpose(PINION1);
GEAR1 = transpose(GEAR1);
PINION2 = transpose(PINION2);
GEAR2 = transpose(GEAR2);
```

```
%Table for selected DP
Tselected = table(transpose(names), PINION1, GEAR1, PINION2, GEAR2)
```

```
Tpinion1 =
```

```
817 table
```

DP	ContactStress	BendingStress	ContactFOS	BendingFOS	Ko	Kv	
--	-----	-----	-----	-----	----	-----	---
8	198.31	8.3576	7.2734	52.846	1.25	1.4293	1
10	264.79	14.9	5.4473	29.642	1.25	1.3898	1
12	346.72	25.547	4.1601	17.288	1.25	1.36	1
16	516.52	56.698	2.7925	7.7897	1.25	1.317	1
20	764	124.05	1.8879	3.5605	1.25	1.287	0.
24	1241.9	327.75	1.1614	1.3475	1.25	1.2644	0.
32	1855.6	731.7	0.77733	0.60361	1.25	1.232	0.
48	3284.6	2292.7	0.43913	0.19263	1.25	1.1926	0.

```
Tgear1 =
```

```
817 table
```

DP	ContactStress	BendingStress	ContactFOS	BendingFOS	Ko	Kv	
--	-----	-----	-----	-----	----	-----	---
8	92.82	5.2751	16.879	87.814	1.25	1.4293	1
10	123.82	9.3871	12.653	49.347	1.25	1.3898	1
12	162.26	16.121	9.6551	28.735	1.25	1.36	1
16	241.71	35.772	6.4815	12.949	1.25	1.317	1
20	358.75	78.799	4.367	5.8785	1.25	1.287	0.
24	587.26	211.16	2.6677	2.1937	1.25	1.2644	0.
32	877.42	471.37	1.7855	0.98272	1.25	1.232	0.
48	1553.2	1477.1	1.0087	0.31361	1.25	1.1926	0.

```
Tpinion2 =
```

817 table

DP	ContactStress	BendingStress	ContactFOS	BendingFOS	Ko	Kv	
--	-----	-----	-----	-----	----	-----	---
8	363.97	28.152	4.3044	16.454	1.25	1.2227	1
10	488.44	50.7	3.2075	9.1367	1.25	1.201	1
12	642.16	87.633	2.4397	5.286	1.25	1.1848	1
16	962.55	196.9	1.6276	2.3526	1.25	1.1616	1
20	1430.3	434.74	1.0954	1.0655	1.25	1.1455	0.
24	2333.3	1157	0.67144	0.40038	1.25	1.1335	0.
32	3505.1	2610.9	0.44696	0.17742	1.25	1.1165	0.
48	6248.1	8296.4	0.25074	0.055835	1.25	1.096	0.

Tgear2 =

817 table

DP	ContactStress	BendingStress	ContactFOS	BendingFOS	Ko	Kv	
--	-----	-----	-----	-----	----	-----	---
8	170.36	17.769	9.9888	27.342	1.25	1.2227	1
10	228.4	31.941	7.4502	15.21	1.25	1.201	1
12	300.52	55.297	5.6623	8.786	1.25	1.1848	1
16	450.44	124.23	3.7778	3.9109	1.25	1.1616	1
20	671.6	276.17	2.5337	1.7592	1.25	1.1455	0.
24	1103.4	745.38	1.5422	0.65181	1.25	1.1335	0.
32	1657.4	1682	1.0267	0.28885	1.25	1.1165	0.
48	2954.6	5344.8	0.57594	0.0909	1.25	1.096	0.

Tselected =

1517table

Var1	PINION1	GEAR1	PINION2	GEAR2
-----	-----	-----	-----	-----
"DP"	20	20	20	20
"ContactStress"	764	358.75	1430.3	671.6
"BendingStress"	124.05	78.799	434.74	276.17
"ContactFOS"	1.8879	4.367	1.0954	2.5337
"BendingFOS"	3.5605	5.8785	1.0655	1.7592
"Ko"	1.25	1.25	1.25	1.25

"Kv"	1.287	1.287	1.1455	1.1455
"Ks"	0.97972	0.97972	0.97972	0.97972
"Km"	1.0907	1.0521	1.0907	1.0521
"Kb"	1	1	1	1
"Kt"	1	1	1	1
"Kr"	1	1	1	1
"Cf"	1	1	1	1
"J"	0.27	0.41	0.27	0.41
"I"	0.1308	0.1308	0.1308	0.1308

Torque Calculation

```
Torque = zeros(3,1); %index 1 = Torque A, %index 2 = Torque B, %index 3 = Torque C,
Wt = zeros(4,1); %index 1 = P1, %index 2 = N1, %index 3 = P2, index 4 = N2
Wr = zeros(4,1);
```

```
Torque(3) = TorqueC;
Wt(4) = Torque(3)/(0.5*0.0254*N2/DiametralPitchSelected); %F = T/d, meters/inch* teeth / (t
Wr(4) = tan(PressureAngle)*Wt(4);
```

```
Torque(2) = TorqueB;
Wt(3) = Torque(2)/(0.5*0.0254*P2/DiametralPitchSelected); %F = T/d, meters/inch* teeth / (t
Wr(3) = tan(PressureAngle)*Wt(3);
Wt(2) = Torque(2)/(0.5*0.0254*N1/DiametralPitchSelected); %F = T/d, meters/inch* teeth / (t
Wr(2) = tan(PressureAngle)*Wt(2);
```

```
Torque(1) = TorqueA;
Wt(1) = Torque(1)/(0.5*0.0254*P1/DiametralPitchSelected); %F = T/d, meters/inch* teeth / (t
Wr(1) = tan(PressureAngle)*Wt(1);
```

```
TTorqueCalc = table(["Shaft A";"Shaft B";"Shaft C"], Torque)
TForceCalc = table(["Pinion 1";"Gear 1";"Pinion 2";"Gear 2"], Wt, Wr)
```

```
TTorqueCalc =
```

```
317table
```

Var1	Torque
-----	-----
"Shaft A"	3.1927
"Shaft B"	13.251
"Shaft C"	55


```
TForceCalc =
```

```
417table
```

Var1	Wt	Wr
-----	-----	-----
"Pinion 1"	314.25	114.38
"Gear 1"	298.12	108.51
"Pinion 2"	1304.3	474.72
"Gear 2"	1237.3	450.36

Shaft A BMD

```
%define Shaft A length and positions, units: meters
```

```
LengthA = 1.5*0.0254; %inch -> meter
```

```
BearingA2pos = 1.5*0.0254;
```

```
BearingA1pos = 0; %bearing supports shaft
```

```
Motor1pos = 0; %motor generates torque
```

```
Pinion1pos = 0.5*0.0254; %inch -> meter;
```

```
pinion = [Wr(1), Wt(1), Pinion1pos, P1, 1];
```

```
gear = [0, 0, 0, 0, 0];
```

```
moment = [Torque(1), Motor1pos];
```

```
bearing = [BearingA1pos, BearingA2pos];
```

```
[OUT] = BMDGenerator(pinion, gear, bearing)
```

```
FxyA = OUT(1,:);
```

```
FxyposA = OUT(2,:);
```

```
FxzA = OUT(3,:);
```

```
FxzposA = OUT(4,:);
```

```
%Shear Force and Bending Moment Diagrams
```

```
padding = 0.2
```

```
f4 = figure('Renderer', 'painters', 'Position', [10 10 900 600])
```

```
hold on
```

```
title("Shaft A")
```

```
hold off
```

```
subplot(3,2,1)
```

```
ShearXY = Shear(LengthA, FxyposA, FxyA);
```

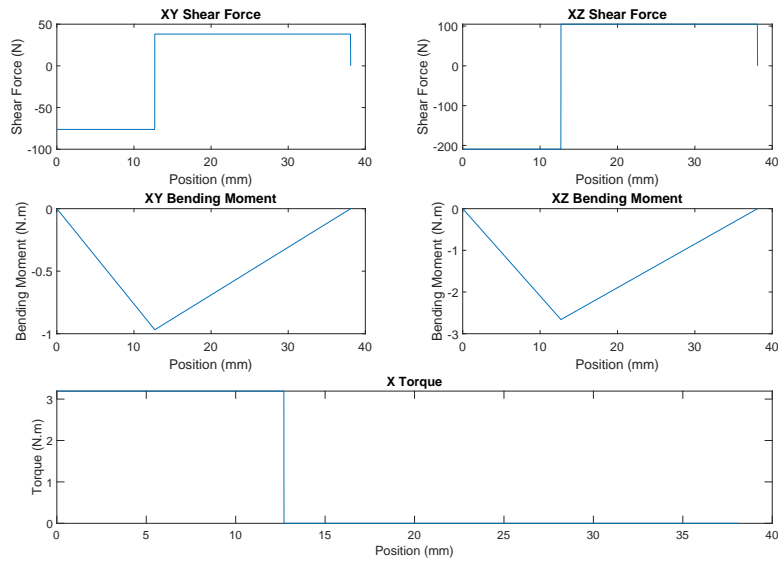
```
plot(ShearXY(:,1)*1000, ShearXY(:,2))
```

```
hold on
```

```

title("XY Shear Force")
xlabel("Position (mm)")
ylabel("Shear Force (N)")
hold off
subplot(3,2,2)
ShearXZ = Shear(LengthA, FxzposA, FxzA);
plot(ShearXZ(:,1)*1000, ShearXZ(:,2))
hold on
title("XZ Shear Force")
xlabel("Position (mm)")
ylabel("Shear Force (N)")
hold off
subplot(3,2,3)
BendingMomentXY = Moment(LengthA, FxyposA, FxyA);
plot(BendingMomentXY(:,1)*1000, BendingMomentXY(:,2))
hold on
title("XY Bending Moment")
xlabel("Position (mm)")
ylabel("Bending Moment (N.m)")
hold off
subplot(3,2,4)
BendingMomentXZ = Moment(LengthA, FxzposA, FxzA);
plot(BendingMomentXZ(:,1)*1000, BendingMomentXZ(:,2))
hold on
title("XZ Bending Moment")
xlabel("Position (mm)")
ylabel("Bending Moment (N.m)")
hold off
subplot(3,2,5:6)
TorqueX = TorquePlotter(LengthA, pinion, gear, moment);
plot(TorqueX(:,1)*1000, TorqueX(:,2))
hold on
title("X Torque")
xlabel("Position (mm)")
ylabel("Torque (N.m)")
hold off

```



Shaft B BMD

```
%define Shaft B length and positions, units: meters
LengthB = 1.5*0.0254; %inch -> meter
BearingB2pos = 1.5*0.0254;
BearingB1pos = 0; %bearing supports shaft
Gear1pos = 0.5*0.0254; %motor generates torque
Pinion2pos = 1*0.0254; %inch -> meter;

pinion = [Wr(3), Wt(3), Pinion2pos, P2, 1];
gear = [ Wr(2), Wt(2), Gear1pos, N1, 0];
moment = [0, 0];
bearing = [BearingB1pos, BearingB2pos];

[OUT] = BMDGenerator(pinion, gear, bearing)
FxyB = OUT(1,:);
FxyposB = OUT(2,:);
FxB = OUT(3,:);
FxBposB = OUT(4,:);

%Shear Force and Bending Moment Diagrams
padding = 0.2
f5 = figure('Renderer', 'painters', 'Position', [10 10 900 600])
hold on
title("Shaft B")
hold off
subplot(3,2,1)
```

```

ShearXY = Shear(LengthB, FxyposB, FxyB);
plot(ShearXY(:,1)*1000, ShearXY(:,2))
hold on
title("XY Shear Force")
xlabel("Position (mm)")
ylabel("Shear Force (N)")
hold off
subplot(3,2,2)
ShearXZ = Shear(LengthB, FxzposB, FxzB);
plot(ShearXZ(:,1)*1000, ShearXZ(:,2))
hold on
title("XZ Shear Force")
xlabel("Position (mm)")
ylabel("Shear Force (N)")
hold off
subplot(3,2,3)
BendingMomentXY = Moment(LengthB, FxyposB, FxyB);
plot(BendingMomentXY(:,1)*1000, BendingMomentXY(:,2))
hold on
title("XY Bending Moment")
xlabel("Position (mm)")
ylabel("Bending Moment (N.m)")
hold off
subplot(3,2,4)
BendingMomentXZ = Moment(LengthB, FxzposB, FxzB);
plot(BendingMomentXZ(:,1)*1000, BendingMomentXZ(:,2))
hold on
title("XZ Bending Moment")
xlabel("Position (mm)")
ylabel("Bending Moment (N.m)")
hold off
subplot(3,2,5:6)
TorqueX = TorquePlotter(LengthB, pinion, gear, moment);
plot(TorqueX(:,1)*1000, TorqueX(:,2))
hold on
title("X Torque")
xlabel("Position (mm)")
ylabel("Torque (N.m)")
hold off

SigmaFxy =

B1xy + B2xy + 2565049596265321/4398046511104 == 0

```

```

SigmaMxy =

(381*B2xy)/10000 + 60509782446090171/4503599627370496 == 0

SigmaFxz =

B1xz + B2xz + 7047415845520165/4398046511104 == 0

SigmaMxz =

(381*B2xz)/10000 + 41562315231686103/1125899906842624 == 0

eqns =

      B1xy + B2xy + 2565049596265321/4398046511104 == 0
(381*B2xy)/10000 + 60509782446090171/4503599627370496 == 0
      B1xz + B2xz + 7047415845520165/4398046511104 == 0
(381*B2xz)/10000 + 41562315231686103/1125899906842624 == 0

soln =

      struct with fields:

      B1xy: [117sym]
      B1xz: [117sym]
      B2xy: [117sym]
      B2xz: [117sym]

soln =

-230.5772
-633.5057
-352.6475
-968.8911

OUT =

1.0e+03 *

-0.2306    0.4747    0.1085   -0.3526

```

0	0.0000	0.0000	0.0000
-0.6335	1.3043	0.2981	-0.9689
0	0.0000	0.0000	0.0000

padding =

0.2000

f5 =

Figure (3) with properties:

Number: 3
 Name: ''
 Color: [0.9400 0.9400 0.9400]
 Position: [10 10 900 600]
 Units: 'pixels'

Use GET to show all properties

t =

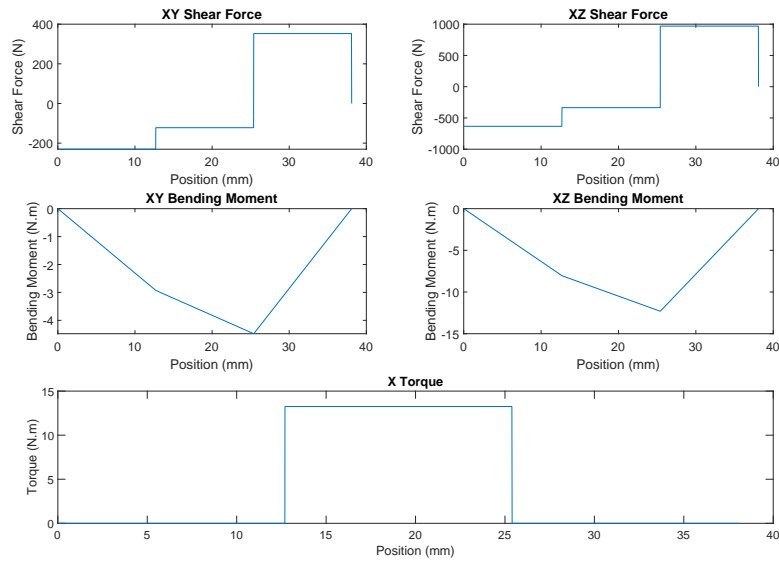
0

t =

13.2514

t =

-13.2514



Shaft C BMD

```
%define Shaft B length and positions, units: meters
LengthC = 1.5*0.0254; %inch -> meter
BearingC2pos = 1*0.0254;
BearingC1pos = 0; %bearing supports shaft
Gear2pos = 0.5*0.0254; %motor generates torque
Chuckpos = 1.5*0.0254; %inch -> meter;

pinion = [0,0,0,0,0];
gear = [Wr(4), Wt(4), Gear2pos, N2, 0];
moment = [-1*Torque(3), Chuckpos];
bearing = [BearingC1pos, BearingC2pos];

[OUT] = BMDGenerator(pinion, gear, bearing)
FxyC = OUT(1,:);
FxyposC = OUT(2,:);
FzxC = OUT(3,:);
FzposC = OUT(4,:);

%Shear Force and Bending Moment Diagrams
padding = 0.2
f6 = figure('Renderer', 'painters', 'Position', [10 10 900 600])
hold on
title("Shaft C")
hold off
subplot(3,2,1)
```

```

ShearXY = Shear(LengthC, FxyposC, FxyC);
plot(ShearXY(:,1)*1000, ShearXY(:,2))
hold on
title("XY Shear Force")
xlabel("Position (mm)")
ylabel("Shear Force (N)")
hold off
subplot(3,2,2)
ShearXZ = Shear(LengthC, FxzposC, FxzC);
plot(ShearXZ(:,1)*1000, ShearXZ(:,2))
hold on
title("XZ Shear Force")
xlabel("Position (mm)")
ylabel("Shear Force (N)")
hold off
subplot(3,2,3)
BendingMomentXY = Moment(LengthC, FxyposC, FxyC);
plot(BendingMomentXY(:,1)*1000, BendingMomentXY(:,2))
hold on
title("XY Bending Moment")
xlabel("Position (mm)")
ylabel("Bending Moment (N.m)")
hold off
subplot(3,2,4)
BendingMomentXZ = Moment(LengthC, FxzposC, FxzC);
plot(BendingMomentXZ(:,1)*1000, BendingMomentXZ(:,2))
hold on
title("XZ Bending Moment")
xlabel("Position (mm)")
ylabel("Bending Moment (N.m)")
hold off
subplot(3,2,5:6)
TorqueX = TorquePlotter(LengthC, pinion, gear, moment);
plot(TorqueX(:,1)*1000, TorqueX(:,2))
hold on
title("X Torque")
xlabel("Position (mm)")
ylabel("Torque (N.m)")
hold off

SigmaFxy =

B1xy + B2xy + 7922761848621797/17592186044416 == 0

```



```

SigmaMxy =

(127*B2xy)/5000 + 1609905207639949/281474976710656 == 0

SigmaFxz =

B1xz + B2xz + 5441902319701237/4398046511104 == 0

SigmaMxz =

(127*B2xz)/5000 + 110/7 == 0

eqns =

      B1xy + B2xy + 7922761848621797/17592186044416 == 0
(127*B2xy)/5000 + 1609905207639949/281474976710656 == 0
      B1xz + B2xz + 5441902319701237/4398046511104 == 0
                        (127*B2xz)/5000 + 110/7 == 0

soln =

      struct with fields:

          B1xy: [117sym]
          B1xz: [117sym]
          B2xy: [117sym]
          B2xz: [117sym]

soln =

-225.1784
-618.6727
-225.1784
-618.6727

OUT =

1.0e+03 *

-0.2252      0      0.4504      -0.2252

```

0	0	0.0000	0.0000
-0.6187	0	1.2373	-0.6187
0	0	0.0000	0.0000

padding =

0.2000

f6 =

Figure (4) with properties:

Number: 4
 Name: ''
 Color: [0.9400 0.9400 0.9400]
 Position: [10 10 900 600]
 Units: 'pixels'

Use GET to show all properties

t =

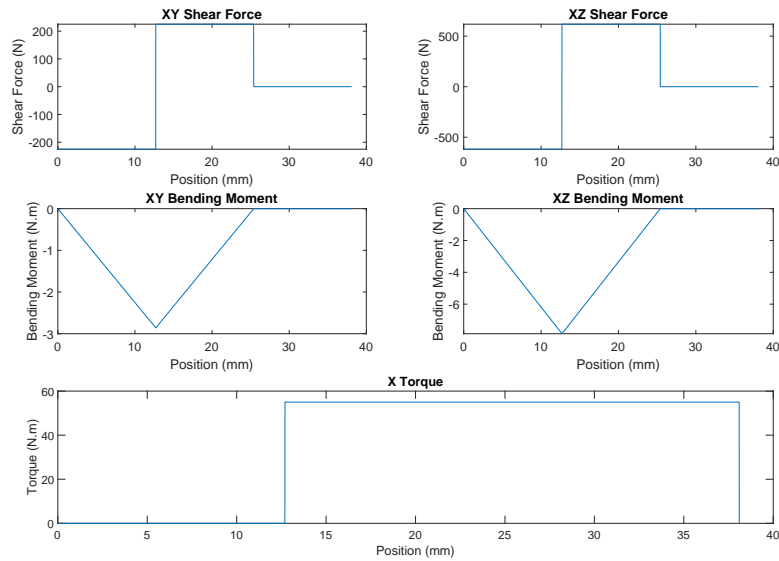
0

t =

55

t =

-55



BMD Tables

Endurance Limits

%Shaft Daimeters (meters) 1 = A, 2 = B...

Diameters = [5*10⁻³; 5*10⁻³; 5*10⁻³];

%Shaft Material Properties

%Ultimate Tensile Stress (MPa)

UltimateTensile = [800; 800; 800]; %assumed material from CES

%Endurance Limits (MPa)

EA = enduranceLimit(Diameters(1), UltimateTensile(1));

EB = enduranceLimit(Diameters(2), UltimateTensile(2));

EC = enduranceLimit(Diameters(3), UltimateTensile(3));

EnduranceLimit = [EA(1); EB(1); EC(1)];

MarinFactors = [EA; EB; EC];

TEnduranceLimit = table(Diameters, UltimateTensile, EnduranceLimit)

TMarinFactors = array2table(["A";"B";"C"],Diameters, UltimateTensile, MarinFactors], 'VariableNames',{'Diameter','UltimateTensile','MarinFactors'})

kb =

1.0461

kc =

1

kb =

1.0461

kc =

1

kb =

1.0461

kc =

1

TEnduranceLimit =

317table

Diameters	UltimateTensile	EnduranceLimit
-----	-----	-----
0.005	800	261.79
0.005	800	261.79
0.005	800	261.79

TMarinFactors =

317 table

Shaft	Diameters	UltimateTensile	EnduranceLimit	Ka	Kb
-----	-----	-----	-----	-----	-----
"A"	"0.005"	"800"	"261.7855"	"0.7671101"	"1.046115"
"B"	"0.005"	"800"	"261.7855"	"0.7671101"	"1.046115"
"C"	"0.005"	"800"	"261.7855"	"0.7671101"	"1.046115"

Test

```
%{
L = 61
Fpos = [0 14 34 49 61]
F = [82.24 -80 -110 142.76 -35]
figure()
subplot(2,1,1)
plot(Shear(L, Fpos, F))
subplot(2,1,2)
plot(Moment(L, Fpos, F))
%}

function [OUTPUT] = BMDGenerator(pinion, gear, bearing)

Pxy = pinion(1);
Pxz = pinion(2);
PPos = pinion(3);
P = pinion(4);

Gxy = gear(1);
Gxz = gear(2);
GPos = gear(3);
N = gear(4);

B1Pos = bearing(1);
B2Pos = bearing(2);

DiametralPitchSelected = 20;
%generate system of equations for the shaft
% Variable naming convention: B1xy
% B = Bearing, P = Pinion, G = Gear, M = motor or chuck
% 1 = first bearing from the origin, 2 = second from origin...
% xy, xz, yz = reference plane
% Prad = pinion radius, Grad = gear radius
syms B1xy B1xz B2xy B2xz

Prad = 0.5*0.0254*P/DiametralPitchSelected;
Grad = 0.5*0.0254*N/DiametralPitchSelected;

% xy equations
SigmaFxy = B1xy + B2xy + Pxy + Gxy == 0
SigmaMxy = B1xy*B1Pos + B2xy*B2Pos + Pxy*PPos + Gxy*GPos == 0
```

```

% xz equations
SigmaFxz = B1xz + B2xz + Pxz + Gxz == 0
SigmaMxz = B1xz*B1Pos + B2xz*B2Pos + Pxz*PPos + Gxz*GPos == 0

%Solve system of equations
eqns = [SigmaFxy; SigmaMxy; SigmaFxz; SigmaMxz]
soln = solve(eqns)
soln = structfun(@double,soln)

%define bearing values
B1xy = soln(1);
B1xz = soln(2);
B2xy = soln(3);
B2xz = soln(4);

Fxypos = [B1Pos, PPos, GPos, B2Pos];
Fxzpos = [B1Pos, PPos, GPos, B2Pos];
Fxy = [B1xy, Pxy, Gxy, B2xy];
Fxz = [B1xz, Pxz, Gxz, B2xz];

OUTPUT = [Fxy; Fxypos; Fxz; Fxzpos];
end

%Shear Force Diagram Plotter: Tlength = length of shaft, Fpos = array of force
%positions (3 decimal places or fewer), F = array for force vectors (Transverse Direction On
function V = Shear(Tlength, FPos, F)
    RES = 10^-6; %
    size = ceil(Tlength/RES + 1);
    V = zeros(size, 2); %array which shear is plotted
    k = 0;
    for i = 1:length(FPos)
        k = k+1;
        v = F(k);
        for pos = 0:RES:Tlength
            index = int64((pos/RES)+1);
            V(index, 1) = pos;
            if(pos >= FPos(i))
                V(index, 2) = V(index, 2) + v;
            end
        end
    end
end

end

%Bending Moment Plotter: Tlength = length of shaft, Fpos = array of force
%positions (3 decimal places or fewer), F = array for force vectors (Transverse Direction On

```

```

function M = Moment(Tlength, FPos, F)
    RES = 10^-6;
    size = ceil(Tlength/RES + 1);
    M = zeros(size, 2);
    k = 0;
    for i = 1:length(FPos)
        k = k+1;
        mx = F(k);
        for pos = 0:RES:Tlength
            index = int64((pos/RES)+1);
            M(index, 1) = pos;
            if(pos >= FPos(i))
                M(index, 2) = M(index, 2) + mx*(pos - FPos(i));
            end
        end
    end
end

%Bending Moment Plotter: Tlength = length of shaft, Fpos = array of force
%positions (3 decimal places or fewer), F = array for force vectors (Transverse Direction On
function T = TorquePlotter(Tlength, pinion, gear, moment)
    DiametralPitchSelected = 20;
    RES = 10^-6;
    size = ceil(Tlength/RES + 1);
    T = zeros(size, 2);

    Pxz = pinion(2);
    PPos = pinion(3);
    P = pinion(4);
    Prad = 0.5*0.0254*P/DiametralPitchSelected;
    Pdir = pinion(5);
    if Pdir == 1
        Pxz = Pxz*-1;
    end

    Gxz = gear(2);
    GPos = gear(3);
    N = gear(4);
    Grad = 0.5*0.0254*N/DiametralPitchSelected;
    Gdir = gear(5);
    if Gdir == 1
        Gxz = Gxz*-1;
    end

    Myz = moment(1);

```

```

Mpos = moment(2);

ActingTorques = [Myz, Mpos; Gxz*Grad, GPos; Pxz*Prad, PPos];
ActingTorques = sortrows(ActingTorques, 2); %sort by position

k = 0;
for i = 1:length(ActingTorques)
    k = k+1;
    t = ActingTorques(k,1)
    for pos = 0:RES:Tlength
        index = int64((pos/RES)+1);
        T(index, 1) = pos;
        if(pos >= ActingTorques(i,2))
            T(index, 2) = T(index, 2) + t;
        end
    end
end
end

end

%{
Pd = Diametral Pitch (teeth/inch)
N = Number of Teeth
n = Angular Velocity (rpm)
F = Face Width (inches)
WtM = Tangential load (Newtons)
m = gear Ratio
phi = pressure angle (Rad)
duty = number of revs
%}
function [OUTPUT] = enduranceLimit(shaftDiameter, Sut)
    %SI Units

    if Sut <= 1400 %Eq. 6-8
        Se_ = 0.5*Sut;
    else
        Se_ = 700;
    end

    %Ka - surface condition factor
    a = 4.51; %Machined/CD, T6-2
    b = -0.265; %T6-2
    ka = a*Sut^b; %Eq. 6-19

    %Kb - size modification factor
    d = shaftDiameter*10^3; %mm
    if d <= 51 %Eq. 6-20

```



```

        kb = (d/7.62)^-0.107
    else
        kb = 1.51*d^-0.157
    end

    %Kc - load modification factor
    kc = 1% Eq. 6-26, Bending

    %Kd - temperature modification factor
    Tf = 77; % room temp in degF, 25 degC
    kd = 0.975 + Tf*0.432*10^-3 - Tf^2*0.115*10^-5 + Tf^3*0.104*10^-8 - Tf^4*0.595*10^-12; %

    %Ke - reliability factor
    ke = 0.814; %T6-5, 99% reliability

    %Kf - misc effects modification factor
    kf = 1;

    Se = ka*kb*kc*kd*ke*kf*Se_;

    OUTPUT = [Se, ka, kb, kc, kd, ke, kf];
end

%{
Pd = Diametral Pitch (teeth/inch)
N = Number of Teeth
n = Angular Velocity (rpm)
F = Face Width (inches)
WtM = Tangential load (Newtons)
m = gear Ratio
phi = pressure angle (Rad)
duty = number of revs
%}
function [OUTPUT] = stresses(Pd, N, n, F, WtM, m, phi, duty)
    dp = N/Pd; %Diameter inches
    V = pi*dp*n/12; %Linear Velocity
    Wt = 0.224809*WtM ; % Newtons -> lbf

    %Ko - Overload Factor
    Ko = 1.25; %uniform - MODERATE SHOCK

    %Kv - Dynamic Factor
    Qv = 9; %Low end of precision quality gears, Catalog: pg. 48, AGMA class: 9
    B = 0.25*(12 - Qv)^(2/3);

```

```

A = 50 + 56*(1 - B);
Kv = ((A + V^0.5)/A)^B;

%Ks - Size Factor
x = 1.5708/Pd; %Tooth Width, equation from boston
Y = 2*x*Pd/3;
Ks = 1.192*((F*Y^0.5)/Pd)^0.0535;

%Km - Load Distribution Factor
Cmc = 0.8; %Crowned
Cpf = F/(10*dp) - 0.025;
Cpm = 1;
A = 0.0675; %PRECISION, enclosed, Table 14-9
B = 0.0167;
C = -0.926E-4;
Cma = A + B*F + C*F^2;
Ce = 1;
Km = 1 + Cmc*(Cpf*Cpm + Cma*Ce);

%Kb - Rim Thickness Factor
Kb = 1; %Assumption given by professor
%{
if Pd < 20 %Whole Depth Equation from Boston
    ht = 2.157/Pd;
else
    ht = 2.2/Pd + 0.002;
end

tr = %Cannot find equation or values specifying rim thickness

mb = tr/ht;
if mb < 1.2
    Kb = 1.6*log(2.242/mb);
else
    Kb = 1;
end
%}

%J - Bending Strength Geometry Factor Fig. 14-6, Very bad lookup table
%for selected gears
if N == 16
    J = 0.27;
else
    J = 0.41;
end

```

```

%Kt - Temperature Factor
Kt = 1; %Less than 250F

%Kr - Reliability Factor
Kr = 1; %0.99 - Eq. 14-38

%Cp - Elastic Coefficient
Cp = 2300; %Table 14-8 steel-steel

%I - Pitting Resistance Geometry Factor
mn = 1; %Load sharing ratio: 1 for spur gears
I = ((cos(phi)*sin(phi))/(2*mn))*(m/(m+1));

%Cf - surface condition factor
Cf = 1;

%Contact Stress
CS = (Cp*sqrt(Wt*Ko*Kv*Ks*(Km/(dp*F))*(Cf/I))); %Contact stress in psi

%Bending Stress
BS = (Wt*Ko*Kv*Ks*(Pd/F)*((Km*Kb)/J)); %Contact stress in psi

%Factor of Safety

%Table 14-6, Grade 3 steel, carburized and hardened
St = 75000;
Sc = 275000; %psi

if duty > 10^-4 %Fig 14-15
    Zn = 2.466*duty^-0.056;
else
    Zn = 1.5;
end

Ch = 1;

if duty > 10^-6 %Fig 14-14
    Yn = 1.6831*duty^-0.0323;
else
    Yn = 6.1514*duty^-0.1192;
end

%Contact FOS
CFOS = ((Sc*Zn*Ch)/(Kt*Kr))/CS;

%Bending FOS

```

```

BFOS = ((St*Yn)/(Kt*Kr))/BS;

CS = CS*0.00689476; %psi -> MPa
BS = BS*0.00689476;

OUTPUT = [Pd, CS, BS, CFOS, BFOS, Ko, Kv, Ks, Km, Kb, Kt, Kr, Cf, J, I];
end

SigmaFxy =

B1xy + B2xy + 8048519973203097/70368744177664 == 0

SigmaMxy =

(381*B2xy)/10000 + 6541837034219477/4503599627370496 == 0

SigmaFxz =

B1xz + B2xz + 2764140860800629/8796093022208 == 0

SigmaMxz =

(381*B2xz)/10000 + 1760/441 == 0

eqns =

    B1xy + B2xy + 8048519973203097/70368744177664 == 0
(381*B2xy)/10000 + 6541837034219477/4503599627370496 == 0
    B1xz + B2xz + 2764140860800629/8796093022208 == 0
    (381*B2xz)/10000 + 1760/441 == 0

soln =

struct with fields:

    B1xy: [117sym]
    B1xz: [117sym]
    B2xy: [117sym]
    B2xz: [117sym]

```

```
soln =
```

```
-76.2509  
-209.4976  
-38.1254  
-104.7488
```

```
OUT =
```

```
-76.2509  114.3763      0  -38.1254  
          0    0.0127      0    0.0381  
-209.4976  314.2464      0 -104.7488  
          0    0.0127      0    0.0381
```

```
padding =
```

```
0.2000
```

```
f4 =
```

```
Figure (2) with properties:
```

```
Number: 2  
Name: ''  
Color: [0.9400 0.9400 0.9400]  
Position: [10 10 900 600]  
Units: 'pixels'
```

```
Use GET to show all properties
```

```
t =
```

```
3.1927
```

```
t =
```

```
0
```

```
t =
```

-3.1927