HW5 GU4710

Due Thursday October, 28

Question Borrower i defaults with probability 5%. If the borrower i does not default, the loan returns 1. Otherwise, it returns 0.5. To model the correlation of defaults across borrowers, suppose that each borrower i has a latent return X_i that is drawn as follows

$$X_i = \sqrt{\rho} S + \sqrt{1-\rho}\epsilon_i$$

where $S \sim N(0,1)$ is a normal variable that is common across all borrowers in the pool and $\epsilon_i \sim N(0,1)$ is a normal variable that is idiosyncratic to each borrower i. Borrower i default if $X_i \leq \Phi^{-1}(5\%)$ where Φ is the CDF of normal Gaussian variable (i.e. $\Phi^{-1}(5\%) \approx -1.6448$). $\rho \in [0,1]$ captures the correlation of defaults between different borrowers: if $\rho = 1$, all borrowers default at the same time.

A MBS is constructed from a pool of 100 loans, tranching the pool with attachement points displayed in the table below. The junior tranche is the first to absorb losses from the underlying collateral pool and does so until the portfolio loss exceeds 6% (i.e. the proportion of defaults exceeds 12%) at which point the junior tranche becomes worthless. The mezzanine tranche begins to absorb losses once the portfolio loss exceeds 6% and continues to do so until the portfolio loss reaches 12% (i.e. the proportion of defaults exceeds 24%). Finally, the senior tranche absorbs portfolio losses in excess of 12%.

Tranche	Attachment Point
Senior	12%-100%
Mezzanine	6%-12%
Junior	0%-6%

Question: Plot the expected payoff of the junior, mezzanine, senior tranche as a function of the correlation (To do so, for a sequence of correlation parameter ρ between 0 and 1, simulate 100 MBSs, and report the average payoff of each tranche across the simulations).